Poster 9154

Modulo Event-Driven Sampling

System Identification and Hardware Experiments

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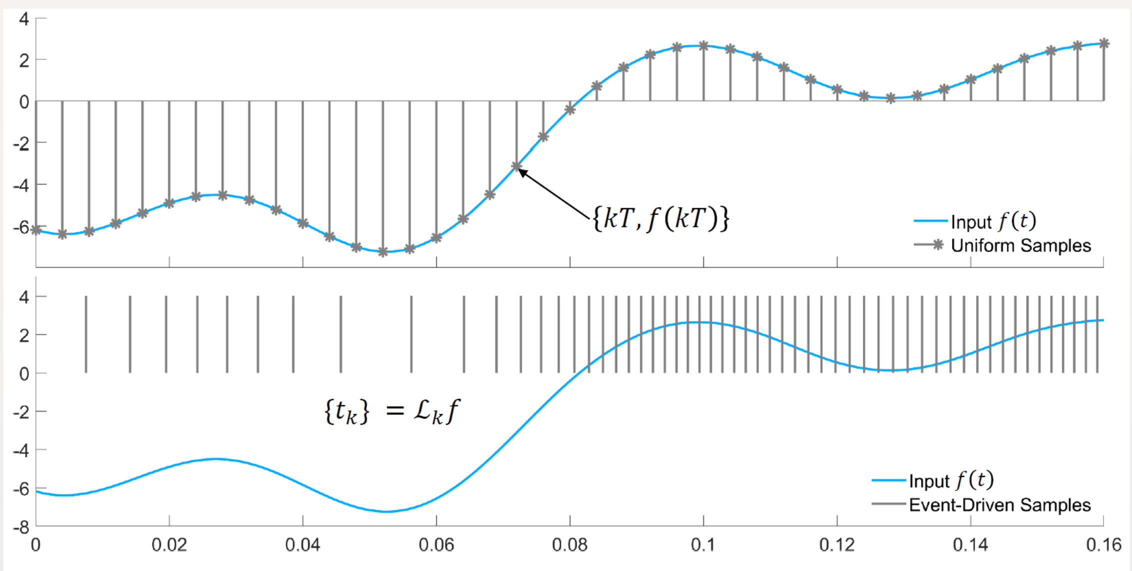
Summary

- **Event-driven sampling (EDS)** sensors are low-power and low-latency alternatives to traditional uniform sampling ADCs; they are characterized by a **restricted input dynamic range**, similarly to their traditional ADC counterparts;
- The Unlimited Sensing Framework (USF) introduced a **modulo nonlinearity** before the ADC that *folds* the input back into the desired dynamic range; at ICASSP 2021, we addressed input reconstruction for a generalized modulo called **modulo-hysteresis** in series with an integrate-and-fire EDS model.
- Here we consider a modulo event-driven sampling (MEDS) model consisting of a modulo-hysteresis in series with an **asynchronous sigma-delta modulator (ASDM)**.
- We go beyond the conventional ideal model assumption and propose the problem of **system identification**, where the input and output are known, and the unknowns are the model parameters; this is dual to **input recovery** and gives rise to a completely new mathematical problem.
- We provide theoretical guarantees for identification, and validate the result with hardware and synthetic data.

Event-Driven Sampling

- Uniform samples ⇒ two coordinates; Event-driven samples ⇒ one coordinate.



The Unlimited Sensing Framework (USF)

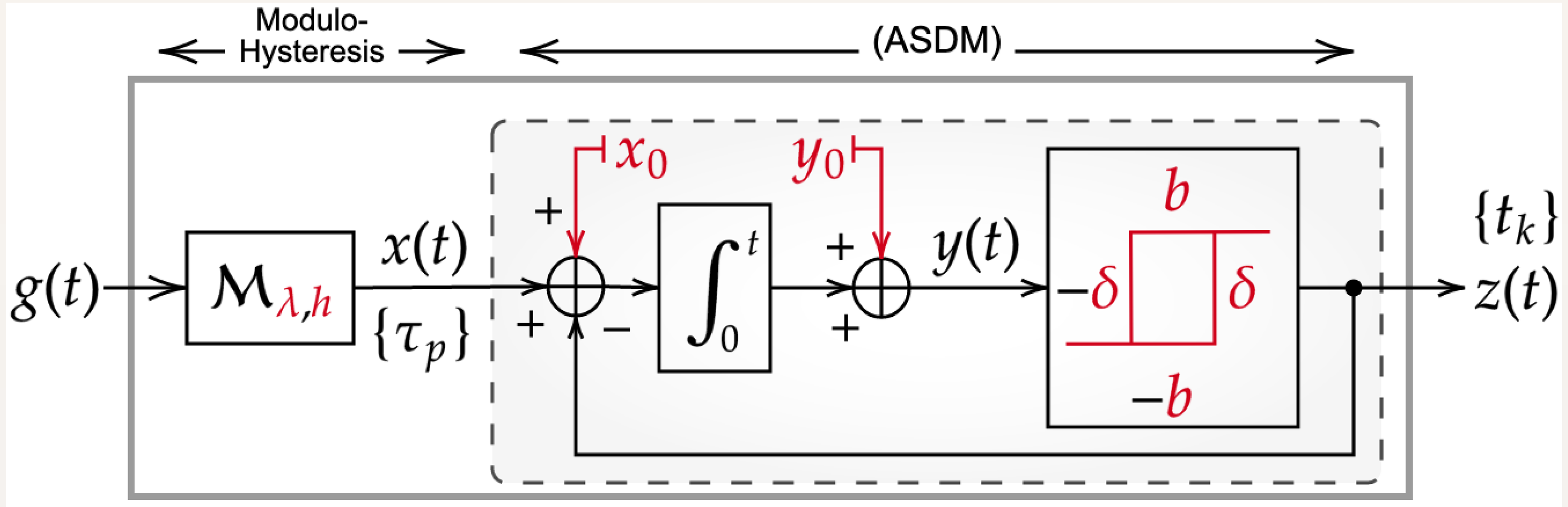
- A conventional ADC saturates for inputs outside its dynamic range ⇒ permanent information loss; modulo encoding addresses this [1]: $z(t) = \mathscr{M}_\lambda(g(t)), \quad k \in \mathbb{Z}$.

Theorem (Unlimited Sampling Theorem – [1]). *Let $g(t) \in \mathrm{PW}_\Omega$ and $y_k = \mathscr{M}_\lambda(g(t))|_{t=kT}, k \in \mathbb{Z}$ be the modulo samples of $g(t)$ with sampling period T. Then, a sufficient condition for recovery of $g(t)$ from the $\{y_k\}$ up to additive multiples of 2λ is*

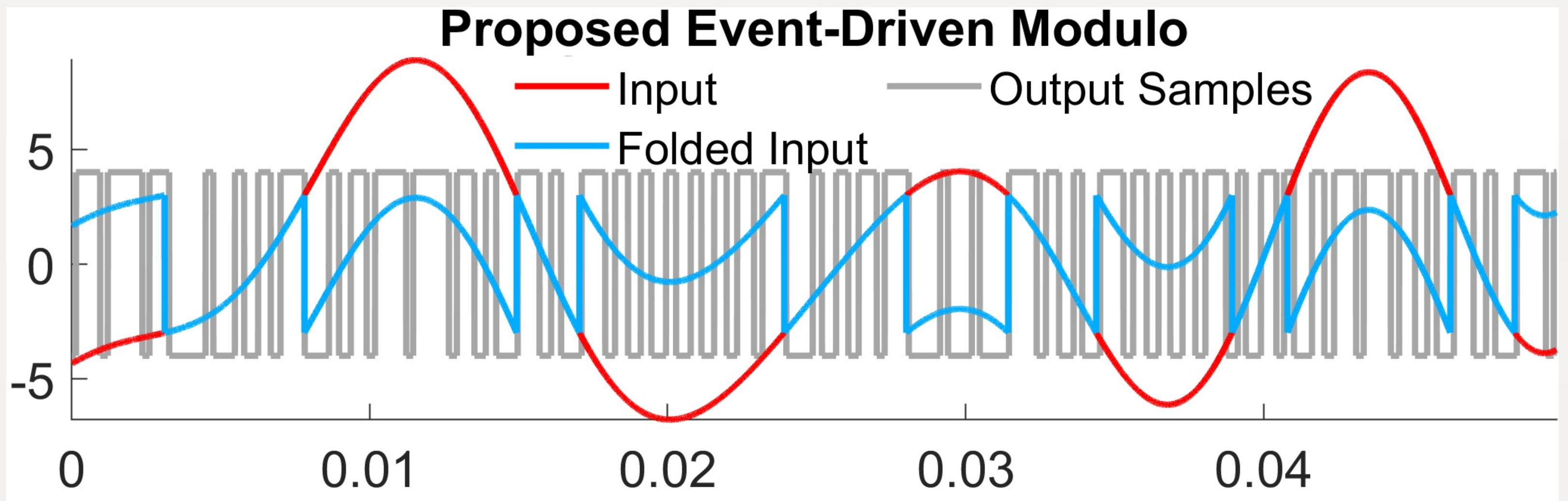
$$T < \frac{1}{2\Omega e}.$$

- If the sampling model is not ideal or point-wise (such as EDS sampling), then the USF approach is not applicable.

Modulo Event-Driven Sampling (MEDS)



Modulo-Hysteresis Event-Driven Sampler (MH-ASDM)



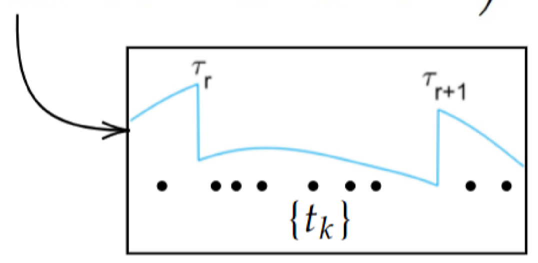
Two main *dual* inverse problems:

1. **Input reconstruction**: Compute $g(t)$ from $\{t_k\}$ using the MEDS parameters (**ICASSP 2021** – for an integrate-and-fire EDS)
2. **System identification**: Compute the MEDS parameters (in **red** in the diagram) using $g(t)$ and $\{t_k\}$ (**this presentation**)

The MEDS model equations are given below

$$y_0 + \int_0^{t_1} \left(\mathscr{M}_{\lambda,h} g(t) + x_0 + b\right) dt = \delta$$

$$\int_{t_k}^{t_{k+1}} \left(\mathscr{M}_{\lambda,h} g(t) + x_0 + (-1)^k b\right) dt = (-1)^k 2\delta, k \geq 1$$



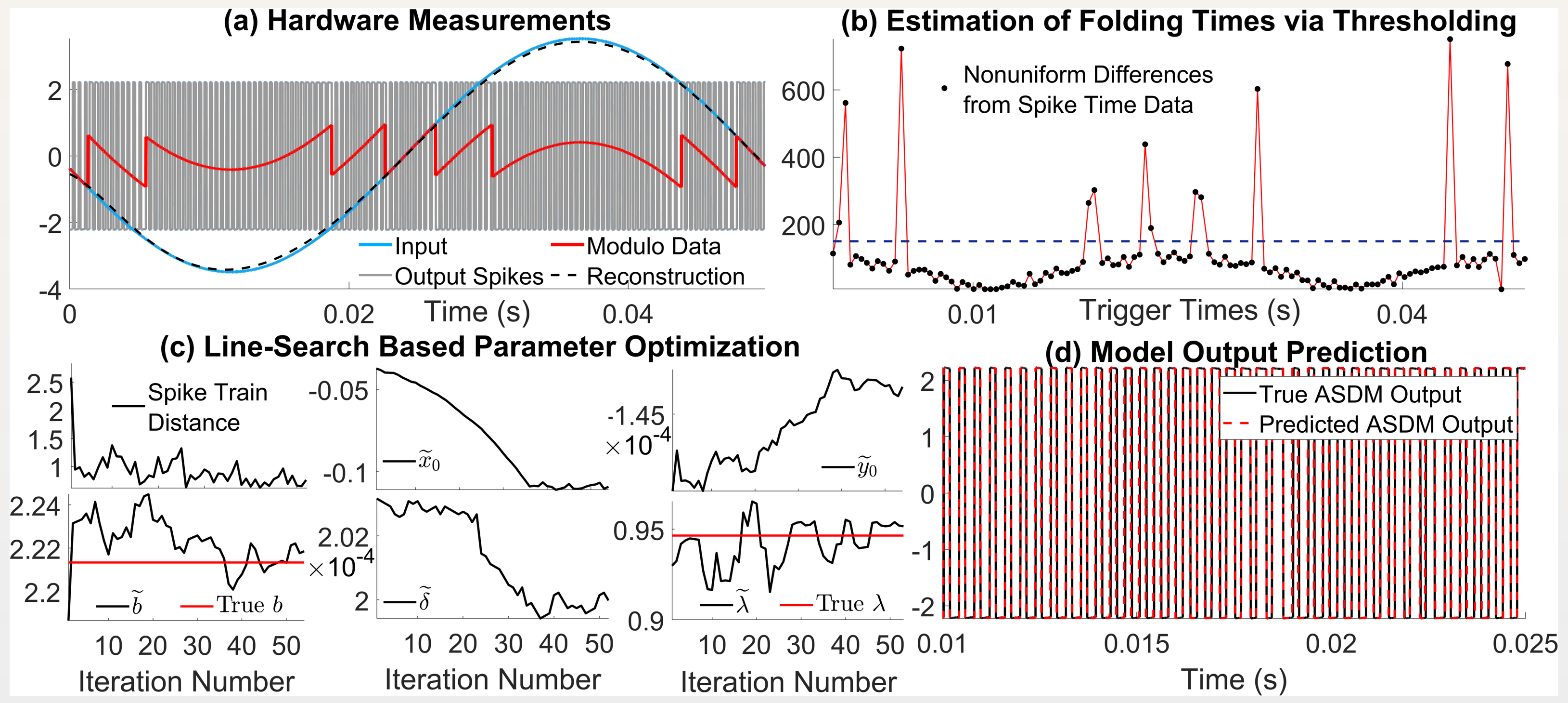
We perform identification using $\{t_k\}$ *in-between* consecutive folds τ_r, τ_{r+1}

- Generally, the system above is **linear** in x_0, y_0, **b**, **delta**, but **non-linear** in λ and **h**.
- However, in between two consecutive folding times the system is linear.

Theorem 1 (System Identification). *Let $\{t_k\}$ be the output of a MEDS model with parameters $\{\lambda, h, x_0, y_0, b, \delta\}$ in response to bandlimited input g. Then the identification problem can be solved uniquely if $x_0 < \lambda_h$, $g(0) \in [-\lambda + h, \lambda - h]$ and*

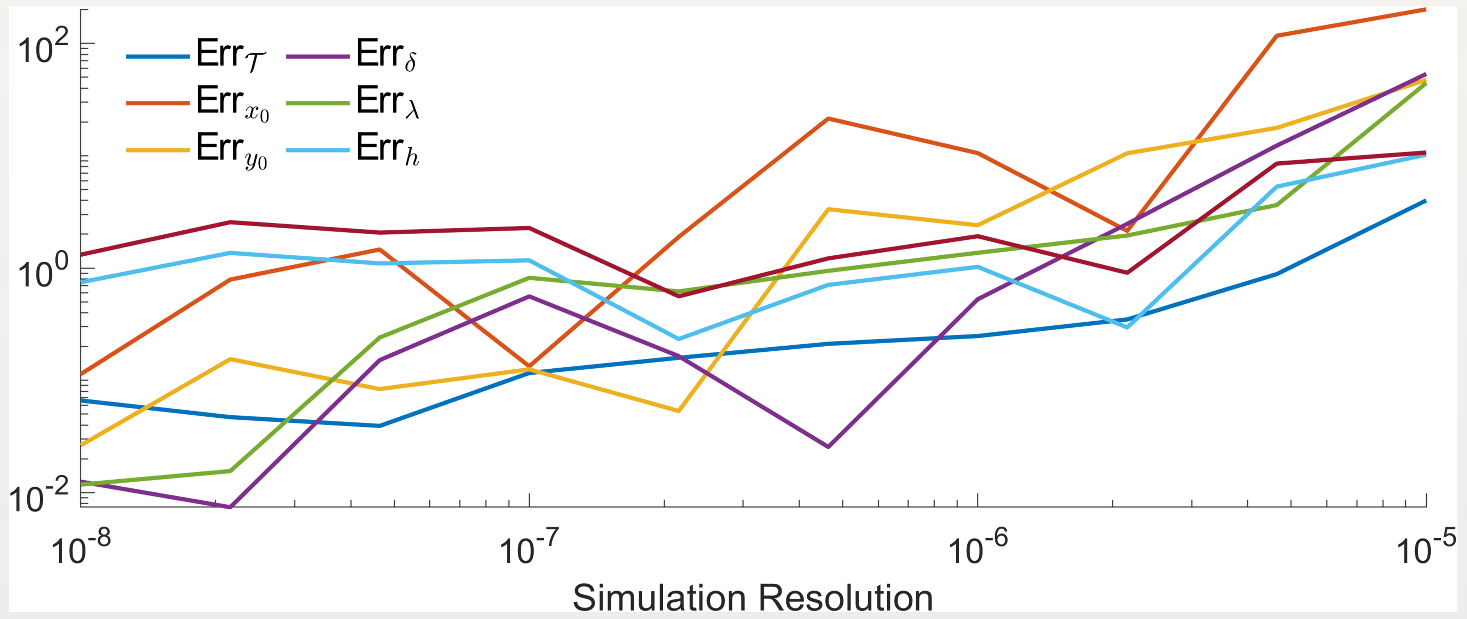
$$\frac{10\delta}{b - \lambda} \leq \frac{\min\{h, 2\lambda_h\}}{\Omega \|g\|_\infty}. \quad (1)$$

Hardware Experiment



Synthetic Data Experiment

- Due to non-idealities, the MEDS parameters don't always match perfectly the hardware components
- To evaluate the correctness of the identified parameters, we repeat the experiment with synthetic data generated with known parameters



References

[1] A. Bhandari, F. Krahmer and R. Raskar, "On Unlimited Sampling and Reconstruction," in *IEEE Trans. Signal Process.*, 2020.

[2] A. Lazar and L. Toth, "Perfect recovery and sensitivity analysis of time encoded bandlimited signals," *IEEE Trans. Circuits Syst. I*, 2004.

[3] D. Florescu, F. Krahmer and A. Bhandari, "Event-Driven Modulo Sampling," in *IEEE Intl. Conf. on Acoustics, Speech and Sig. Proc. (ICASSP)*, 2021.

[4] D. Florescu, F. Krahmer and A. Bhandari, "The Surprising Benefits of Hysteresis in Unlimited Sampling: Theory, Algorithms and Experiments," in *IEEE Trans. Signal Process.*, 2021.

[5] R. Alexandru and P. Dragotti, "Reconstructing classes of non-bandlimited signals from time encoded information," *IEEE Trans. Signal Process.*, 2019.