22-27 MAY 2022
ICASSP 2022
SINGAPORE

Identification of Overlapping Echoes of Unknown Shape from Time-Encoding Machine Samples

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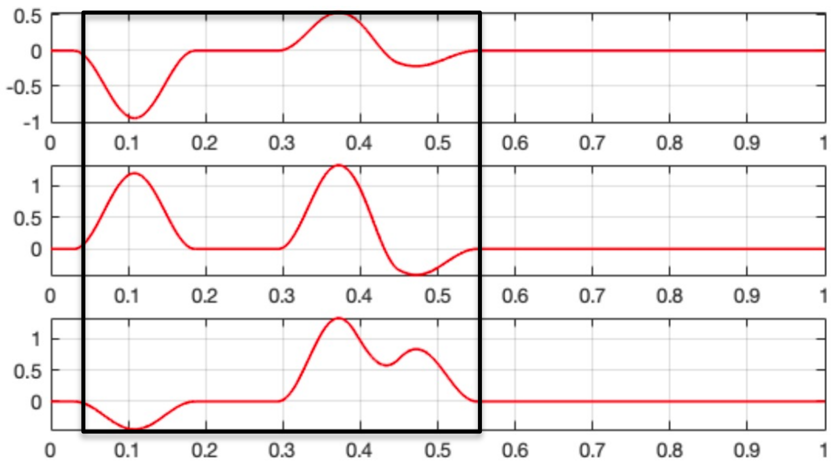
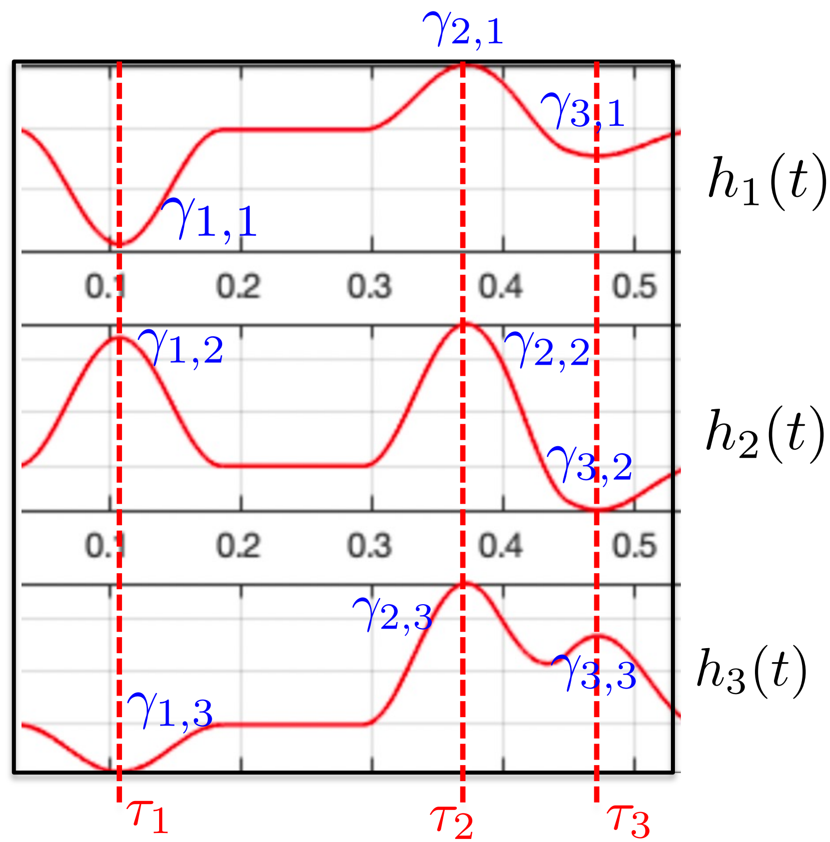
This work was supported in part by NSF CAREER Award CCF 19-43201

Problem statement

Goal: Recover the parameters of overlapping echoes (delays and pulse shape) from ultra-low power multi-channel measurements



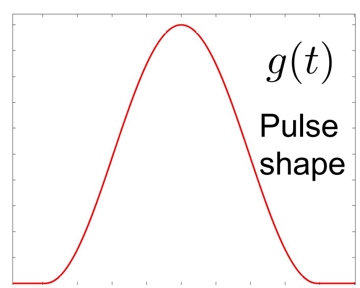
Overlapping pulse streams with a shared delay pattern and varying amplitudes



$$h_m(t) = \sum_{l=1}^{d} \gamma_{l,m} g(t - \tau_l), \quad m = 1, \ldots, M$$

Gain of l-th pulse at m-th sensor

Delays



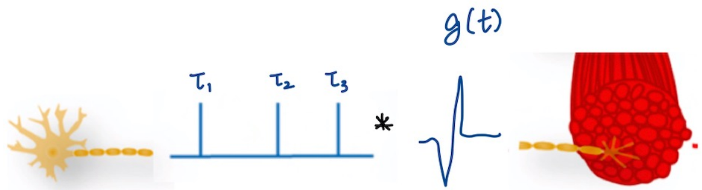
d = Total number of pulses

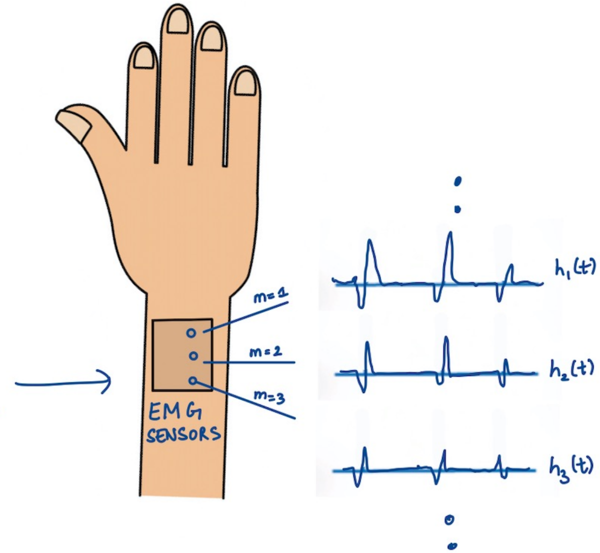
M = Total number of sensors/snapshots/channels

Electromyography (EMG) Application

$$h_m(t) = \sum_{l=1}^{d} \gamma_{l,m} g(t - \tau_l), \quad m \in [M],$$

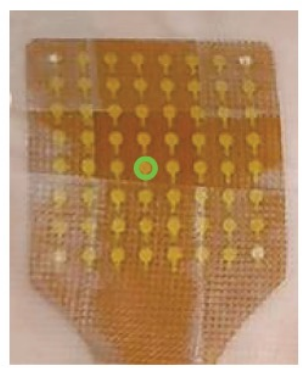
1. Decomposing the EMG signal. Recovering the locations of the impulses and shape of MUAP.





Signal with constant separation and attenuating amplitudes received at three different sensors (snapshots)

2. Limited power supply in wearable EMG devices → low power ADC



Example of wearable EMG skin tattoo [1]

[1] S. Chandra et al, IEEE. Trans. Biomed. Eng, 2020

Data Model

$$h_m(t) = \sum_{l=1}^{d} \gamma_{l,m} g(t-\tau_l) = g(t) * \sum_{l=1}^{d} \gamma_{l,m}\delta(t-\tau_l) \xrightarrow[\text{Transform}]{\text{Fourier}} H_m(\omega) = G(\omega)\sum_{l=1}^{d} e^{-j2\pi\tau_l}\gamma_{l,m}$$

Since $h_m(t) \in [0,T), \omega_0 = \frac{2\pi}{T}$

$$\underbrace{\begin{bmatrix} H_1(0\omega_0) & H_2(0\omega_0) & \cdots & H_M(0\omega_0) \\ H_1(1\omega_0) & H_2(1\omega_0) & \cdots & H_M(1\omega_0) \\ \cdots & \cdots & \cdots & \cdots \\ H_1((K-1)\omega_0) & H_2((K-1)\omega_0) & \cdots & H_M((K-1)\omega_0) \end{bmatrix}}_{\substack{\mathbf{H} \\ \text{Fourier measurements}}} =$$

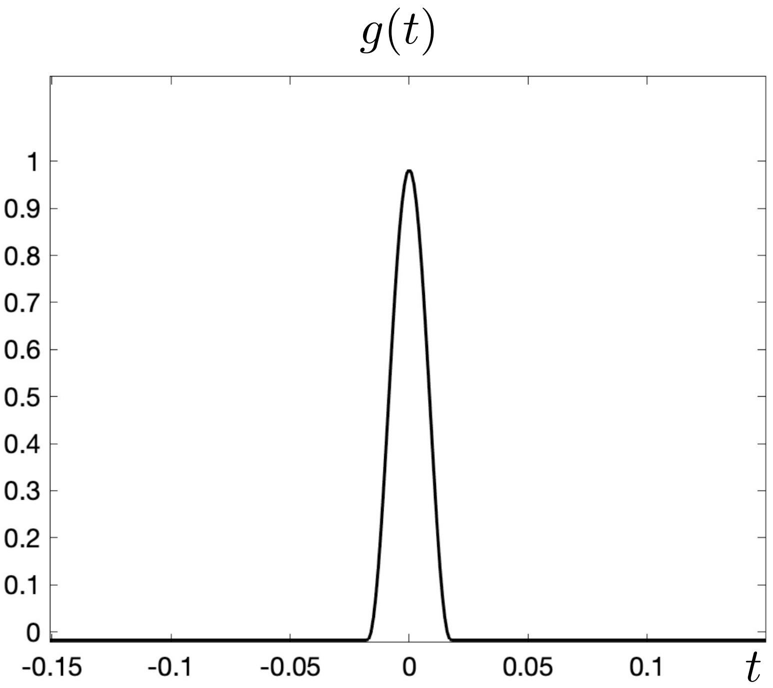
$$\underbrace{\begin{bmatrix} G(1\omega_0) & & & \\ & G(3\omega_0) & & \\ & & \ddots & \\ & & & G((K-1)\omega_0) \end{bmatrix}}_{\substack{\mathbf{G} \\ \textit{Pulse shape}}} \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j2\pi\tau_1} & e^{-j2\pi\tau_2} & \cdots & e^{-j2\pi\tau_d} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j2\pi(K-1)\tau_1} & e^{-j2\pi(K-1)\tau_2} & \cdots & e^{-j2\pi(K-1)\tau_d} \end{bmatrix}}_{\substack{\mathbf{V} \\ \textit{Vandermonde matrix}}} \underbrace{\begin{bmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,M} \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,M} \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{d,1} & \gamma_{d,2} & \cdots & \gamma_{d,M} \end{bmatrix}}_{\substack{\mathbf{\Gamma} \\ \textit{Amplitude matrix}}}$$

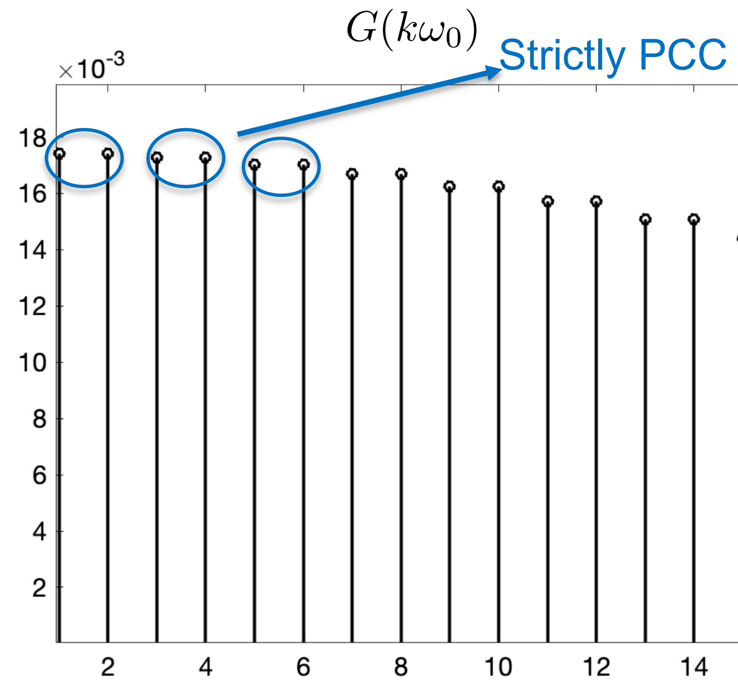
$$\mathbf{H} = \mathbf{G}\mathbf{V}\mathbf{\Gamma}$$

Special Assumption on Pulse Shape

PCC (Pairwise Constant Condition) : Pairs of consecutive Fourier samples are identical

$$G((2k-1)\omega_0) = G(2k\omega_0), \quad k = 1, 2, \ldots, K$$

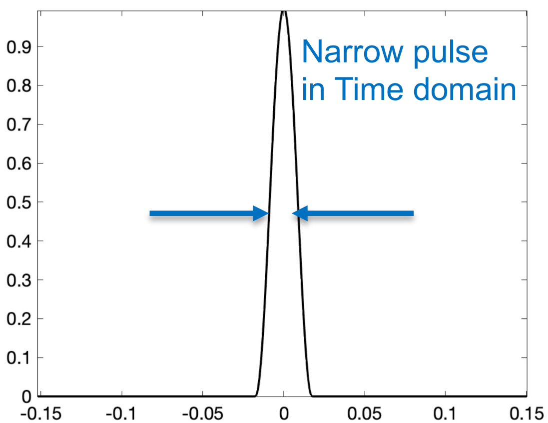


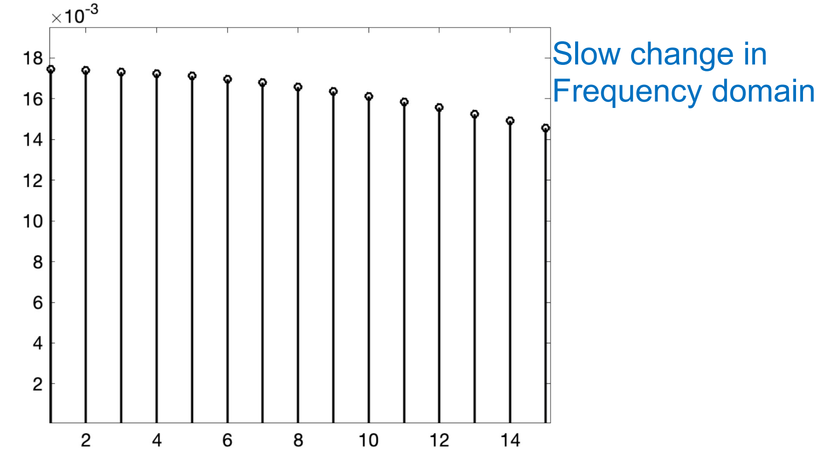


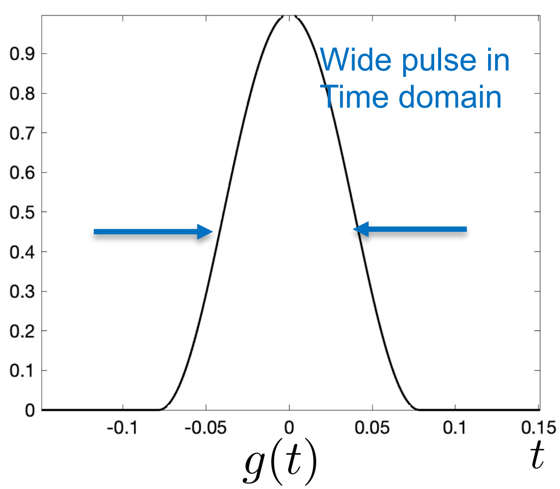
Approximate PCC of Narrow Pulse

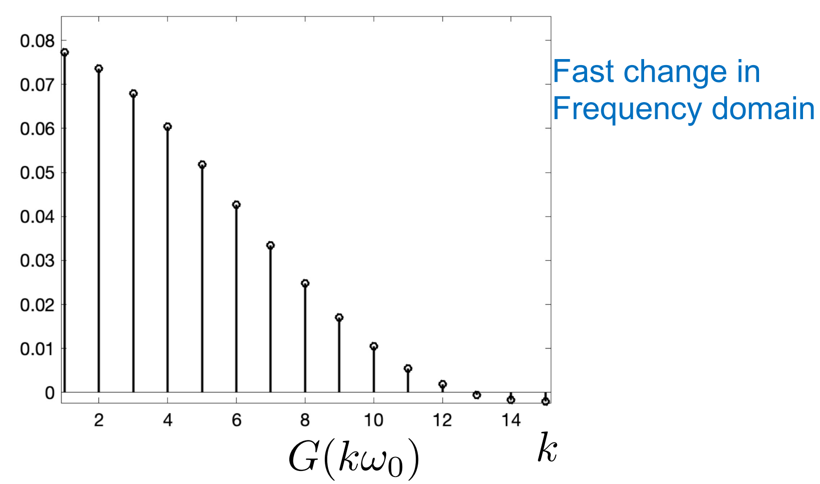
Bernstein's inequality $\frac{|\dot{G}(\omega)|}{\sup_\omega |G((\omega)|} \leq 2\pi R$ → PCC is approximately satisfied if g(t) is supported on narrow interval of $[\frac{-R}{2}, \frac{R}{2})$

Narrow pulse in time domain → Slow change in frequency domain









Dividing data into subarrays

$$\underbrace{\begin{bmatrix} H_1(0\omega_0) & H_2(0\omega_0) & \cdots & H_M(0\omega_0) \\ H_1(1\omega_0) & H_2(1\omega_0) & \cdots & H_M(1\omega_0) \\ \cdots & \cdots & \cdots & \cdots \\ H_1((K-2)\omega_0) & H_2((K-2)\omega_0) & \cdots & H_M((K-2)\omega_0) \\ H_1((K-1)\omega_0) & H_2((K-1)\omega_0) & \cdots & H_M((K-1)\omega_0) \end{bmatrix}}_{\mathbf{H}} = \underbrace{\begin{bmatrix} G(0\omega_0) & & & & \\ & G(1\omega_0) & & & \\ & & \ddots & & \\ & & & G((K-2)\omega_0) & \\ & & & & G((K-1)\omega_0) \end{bmatrix}}_{\mathbf{G}} \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j2\pi\tau_1} & e^{-j2\pi\tau_2} & \cdots & e^{-j2\pi\tau_d} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j2\pi(K-2)\tau_1} & e^{-j2\pi(K-2)\tau_2} & \cdots & e^{-j2\pi(K-2)\tau_d} \\ e^{-j2\pi(K-1)\tau_1} & e^{-j2\pi(K-1)\tau_2} & \cdots & e^{-j2\pi(K-1)\tau_d} \end{bmatrix}}_{\mathbf{V}} \mathbf{\Gamma}$$

Subarray X (Even rows)

$$\underbrace{\begin{bmatrix} H_1(0\omega_0) & H_2(0\omega_0) & \cdots & H_M(0\omega_0) \\ H_1(2\omega_0) & H_2(2\omega_0) & \cdots & H_M(2\omega_0) \\ \cdots & \cdots & \cdots & \cdots \\ H_1((K-2)\omega_0) & H_2((K-2)\omega_0) & \cdots & H_M((K-2)\omega_0) \end{bmatrix}}_{\mathbf{X}} = \underbrace{\begin{bmatrix} G(0\omega_0) & & & \\ & G(2\omega_0) & & \\ & & \ddots & \\ & & & G((K-2)\omega_0) \end{bmatrix}}_{\mathbf{G_X}} \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j2\pi 2\tau_1} & e^{-j2\pi 2\tau_2} & \cdots & e^{-j2\pi 2\tau_d} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j2\pi(K-2)\tau_1} & e^{-j2\pi(K-2)\tau_2} & \cdots & e^{-j2\pi(K-2)\tau_d} \end{bmatrix}}_{\mathbf{V_X}} \mathbf{\Gamma}$$

Subarray Y (Odd rows)

$$\underbrace{\begin{bmatrix} H_1(1\omega_0) & H_2(1\omega_0) & \cdots & H_M(1\omega_0) \\ H_1(3\omega_0) & H_2(3\omega_0) & \cdots & H_M(3\omega_0) \\ \cdots & \cdots & \cdots & \cdots \\ H_1((K-1)\omega_0) & H_2((K-1)\omega_0) & \cdots & H_M((K-1)\omega_0) \end{bmatrix}}_{\mathbf{Y}} = \underbrace{\begin{bmatrix} G(1\omega_0) & & & \\ & G(3\omega_0) & & \\ & & \ddots & \\ & & & G((K-1)\omega_0) \end{bmatrix}}_{\mathbf{G_Y}} \underbrace{\begin{bmatrix} e^{-j2\pi 1\tau_1} & e^{-j2\pi 1\tau_2} & \cdots & e^{-j2\pi 1\tau_d} \\ e^{-j2\pi 3\tau_1} & e^{-j2\pi 3\tau_2} & \cdots & e^{-j2\pi 3\tau_d} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j2\pi(K-1)\tau_1} & e^{-j2\pi(K-1)\tau_2} & \cdots & e^{-j2\pi(K-1)\tau_d} \end{bmatrix}}_{\mathbf{V_Y}} \mathbf{\Gamma}$$

Consequence of PCC

$$\mathbf{X} = \mathbf{G_X}\mathbf{V_X}\mathbf{\Gamma}$$
$$\mathbf{Y} = \mathbf{G_Y}\mathbf{V_Y}\mathbf{\Gamma}$$

1.) Consequence of PCC condition: $\mathbf{G_X} = \mathbf{G_Y}$

$$\underbrace{\begin{bmatrix} G(0\omega_0) & & & \\ & G(2\omega_0) & & \\ & & \ddots & \\ & & & G((K-2)\omega_0) \end{bmatrix}}_{} = \underbrace{\begin{bmatrix} G(1\omega_0) & & & \\ & G(3\omega_0) & & \\ & & \ddots & \\ & & & G((K-1)\omega_0) \end{bmatrix}}_{}$$

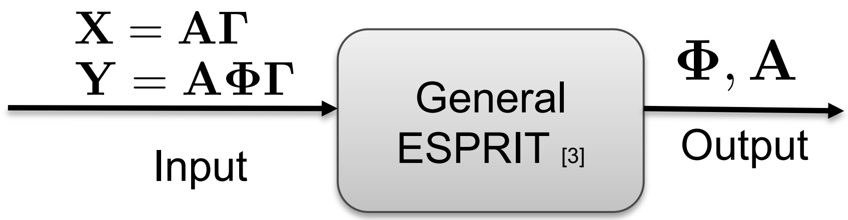
2.) Vandermonde structure: $\mathbf{V_Y} = \mathbf{V_X}\mathbf{\Phi}$

$$\underbrace{\begin{bmatrix} e^{-j2\pi 1\tau_1} & e^{-j2\pi 1\tau_2} & \cdots & e^{-j2\pi 1\tau_d} \\ e^{-j2\pi 3\tau_1} & e^{-j2\pi 3\tau_2} & \cdots & e^{-j2\pi 3\tau_d} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j2\pi(K-1)\tau_1} & e^{-j2\pi(K-1)\tau_2} & \cdots & e^{-j2\pi(K-1)\tau_d} \end{bmatrix}}_{} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j2\pi 2\tau_1} & e^{-j2\pi 2\tau_2} & \cdots & e^{-j2\pi 2\tau_d} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j2\pi(K-2)\tau_1} & e^{-j2\pi(K-2)\tau_2} & \cdots & e^{-j2\pi(K-2)\tau_d} \end{bmatrix}}_{} \underbrace{\begin{bmatrix} e^{-j2\pi\tau_1} & & & \\ & e^{-j2\pi\tau_2} & & \\ & & \ddots & \\ & & & e^{-j2\pi\tau_d} \end{bmatrix}}_{}$$

$$\mathbf{X} = \mathbf{G_X}\mathbf{V_X}\mathbf{\Gamma}$$
$$\mathbf{Y} = \mathbf{G_X}\mathbf{V_X}\mathbf{\Phi}\mathbf{\Gamma}$$

PCC provides rotation invariance of “steering matrices” in subarrays → **Enables reconstruction of $(\tau_l)_{l=1}^{d}$ from $\mathbf{\Phi}$ by ESPRIT** [2]

[2] Y. Bresler and A.H. Delaney, “Resolution of overlapping echoes of unknown shape,” in ICASSP, 1989



$$\mathbf{\Phi} = \begin{bmatrix} e^{-j2\pi\tau_1} & & & \\ & e^{-j2\pi\tau_2} & & \\ & & \ddots & \\ & & & e^{-j2\pi\tau_d} \end{bmatrix}$$

Our model:

1. Recover $\mathbf{V_X} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j2\pi 2\tau_1} & e^{-j2\pi 2\tau_2} & \cdots & e^{-j2\pi 2\tau_d} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j2\pi(K-2)\tau_1} & e^{-j2\pi(K-2)\tau_2} & \cdots & e^{-j2\pi(K-2)\tau_d} \end{bmatrix}}$ from $\mathbf{\Phi}$

2. Recover $\mathbf{G_X}$ from $\mathbf{A} = \mathbf{G_X}\mathbf{V_X}$

[3] R. Roy and T. Kailath, “Esprit-estimation of signal parameters via rotational invariance techniques,” *IEEE trans. Acoust. speech, and signal process.* 1989

Estimation from Minimal Measurements

Previous Approach [2]

Estimate $g(t)$ and $(\tau_l)_{l=1}^{d}$ from

$$h_m(t) = g(t) * \sum_{l=1}^{d} \gamma_{l,m}\delta(t-\tau_l), \quad m = 1, \ldots, M$$

from samples of Fourier transform $H_m(\omega)$ on fine and wide grid.
These Fourier samples are obtained from Nyquist-rate samples in the time domain.
This is power-hungry!

Our Modified goal

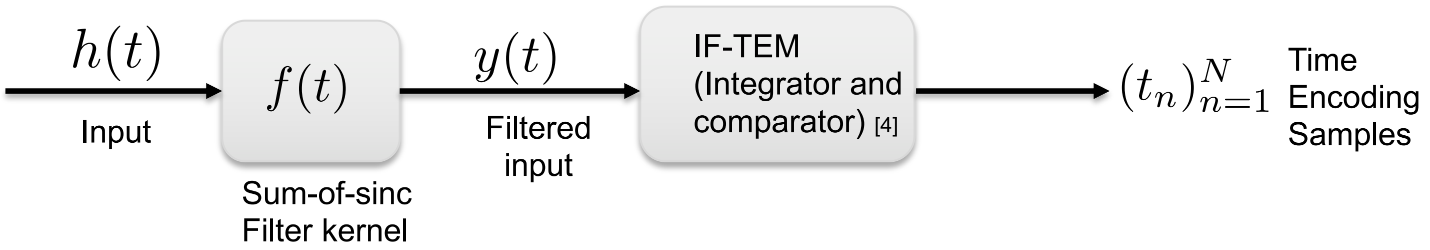
Estimate $(\tau_l)_{l=1}^{d}$ and $(G(k\omega_0))_{k=1}^{K}, (K > d)$ from fewer Fourier samples

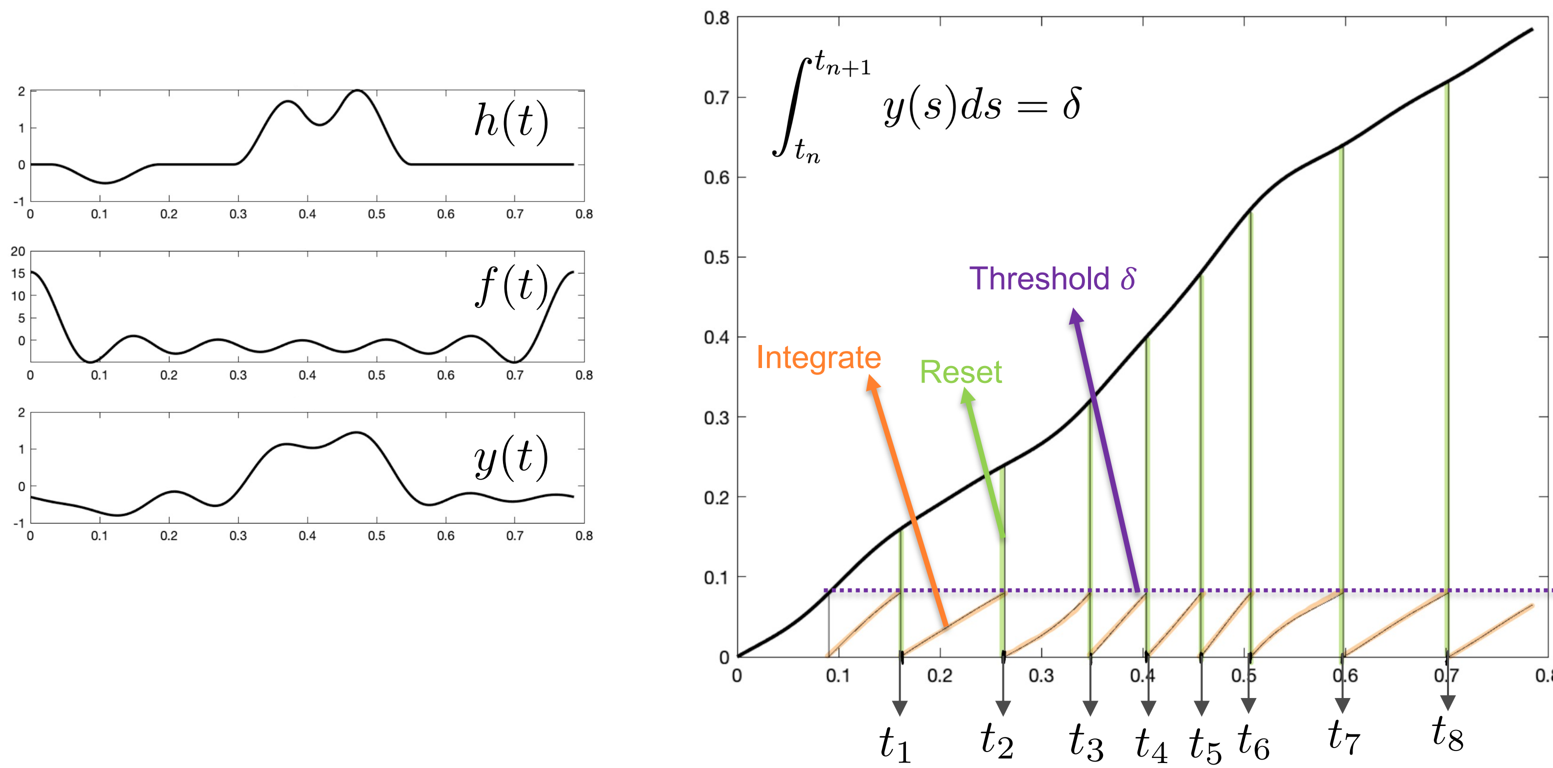
$$H_m(k\omega_0) \text{ for } k = 1, \cdots K \text{ and } m = 1, \cdots M$$

Can we obtain $(H_m(k\omega_0))_{k=1}^{K}$ from fewer observations?

[2] Y. Bresler and A.H. Delaney, "Resolution of overlapping echoes of unknown shape," in ICASSP, 1989

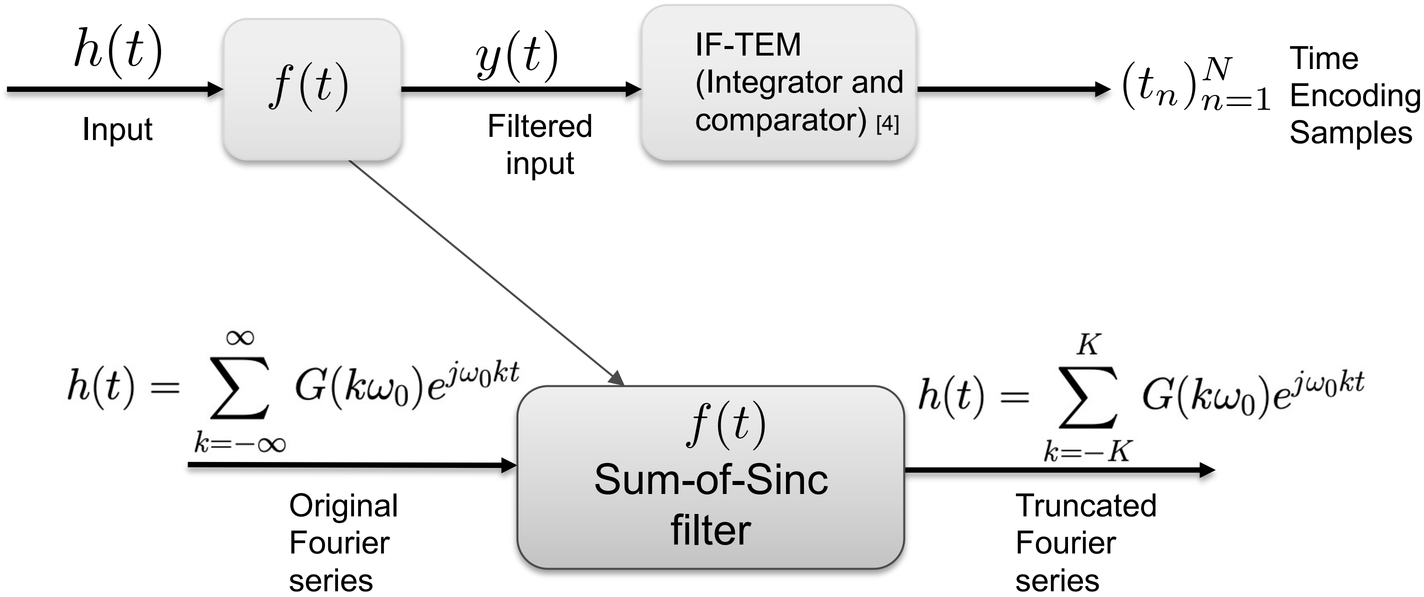
Acquisition of Fourier measurements from Integrate and Fire Time Encoding Machine (IF-TEM)





[4] A.A. Lazar, and L.T. Tóth. "Perfect recovery and sensitivity analysis of time encoded bandlimited signals." IEEE Trans. Circuits and Systems 2004

IF-TEM continued



Fourier measurements($\mathbf{h_m}$) are related to the IF-TEM samples ($\mathbf{t_n}$) via a linear equation with known matrix $\mathbf{B}$ [5]

$$\underbrace{\begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix}}_{\mathbf{t_n}} = \mathbf{B} \underbrace{\begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{K-1} \end{bmatrix}}_{\mathbf{h_m}}$$

[5] S. Rudresh et. al, "A Time-Based Sampling Framework for Finite-Rate-of-Innovation Signals," ICASSP 2020

Theorem

Suppose:

1. Fourier transform of shape $G(\omega)$ satisfies PCC condition.

2. Each channel has enough TEM samples $N_m > 2d + 1$ (where d is number of pulses)

3. Number of channels satisfies $M \geq d$,

Then:

the proposed algorithm can recover the,

- delays or pulse locations $(\tau_l)_{l=1}^{d}$ up to an inherent global shift within $[-R/2, R/2)$
- where R is the support of the pulse shape.
- Fourier coefficients of the pulse shape (up to some inherent global ambiguity).

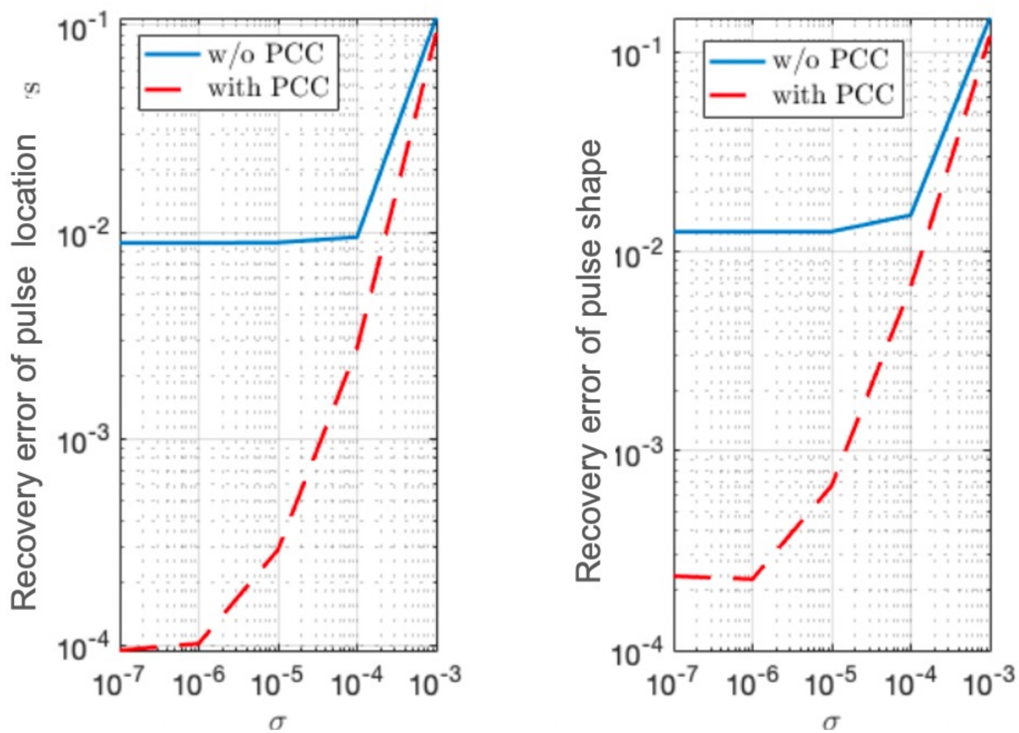
Experimental Setup and Results

- Pulse shape: $g(t) = rect(20t/\pi)cos^2(20t)$
- Error metrics:
  - Pulse locations: worst case error $||\hat{\tau}_l - \tau_l||_\infty$
  - Pulse shape: sine of angle between $\hat{g}$ and g
  - Median of $L = 50$ Monte Carlo reported.

$$\boldsymbol{\tau_l} \triangleq [\tau_1, \ldots, \tau_d]^T$$
$$\boldsymbol{g} \triangleq [G(0\omega_0), \ldots, G((K-1)\omega_0)]^T$$

Noisy Case

Noise model: jitter in TEM samples $\sim U[\frac{-\sigma}{2}, \frac{\sigma}{2}]$



Recovery Error with Noisy Measurements

Scenario:

d = 3 pulses

K = *4*d Fourier measurements.

M = *100* channels

Conclusions

Contributions

- Proposed algorithm can determine the parameters of overlapping echoes of unknown shape from a few TEM samples (under PCC or approximate PCC).
- Theoretical guarantee in noiseless case
- Empirical success with noise and PCC model error

Future Work

- Theoretical analysis with noise
- Extension to a mixture of multiple unknown pulse shapes
- Interdisciplinary collaboration to build physical system including sensors, circuits
- Validate this method on actual EMG and EEG data