Nearest Subspace Search in The Signed Cumulative Distribution Transform Space For 1D Signal Classification

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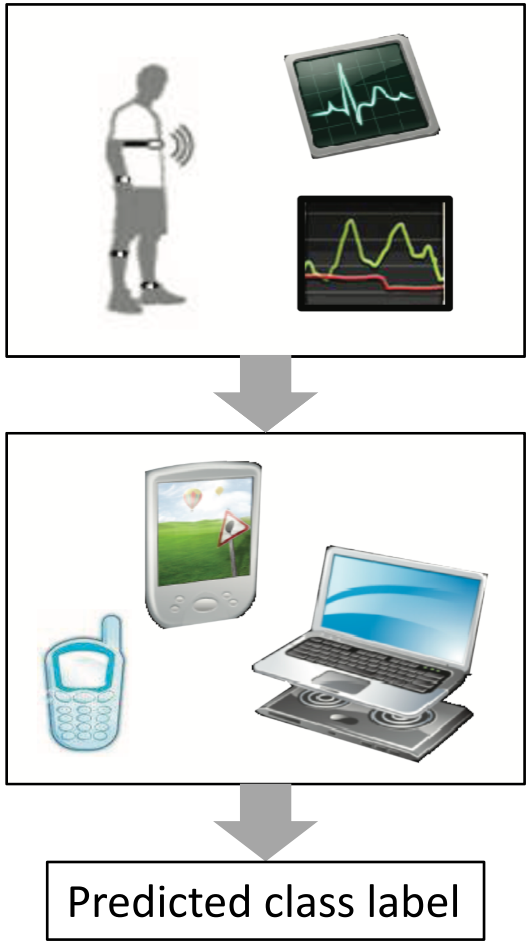
Signal Classification

- Automatic prediction of class label of an unknown signal
- Uses information extracted from signal values (data from sensors)
- Applications:
  - human activity recognition
  - physiological signal classification (e.g., ECG, EEG)
  - machine health monitoring systems
  - etc.

Source: Lara et. al. 2013



Signal Classification

Existing methods:

- Feature-based: train regression-based models with extracted numerical features [Bagnall et. al. 2017]
- End-to-end learning-based: convolutional neural networks (CNN) based classification methods [Fawaz et. al. 2019]
- Transport transform-based: CDT [Park et. al. 2018]/SCDT [Aldroubi et. al. 2022] in combination with linear classifiers

Proposed method: SCDT-NS

- A new classification technique for 1D signals that follow a specific generative model
- Uses signed cumulative distribution transform (SCDT) in combination with nearest subspace (NS) search algorithm
- Contributions:
  - Highly accurate
  - Data efficient
  - Robust to out-of-distribution samples

Problem Statement

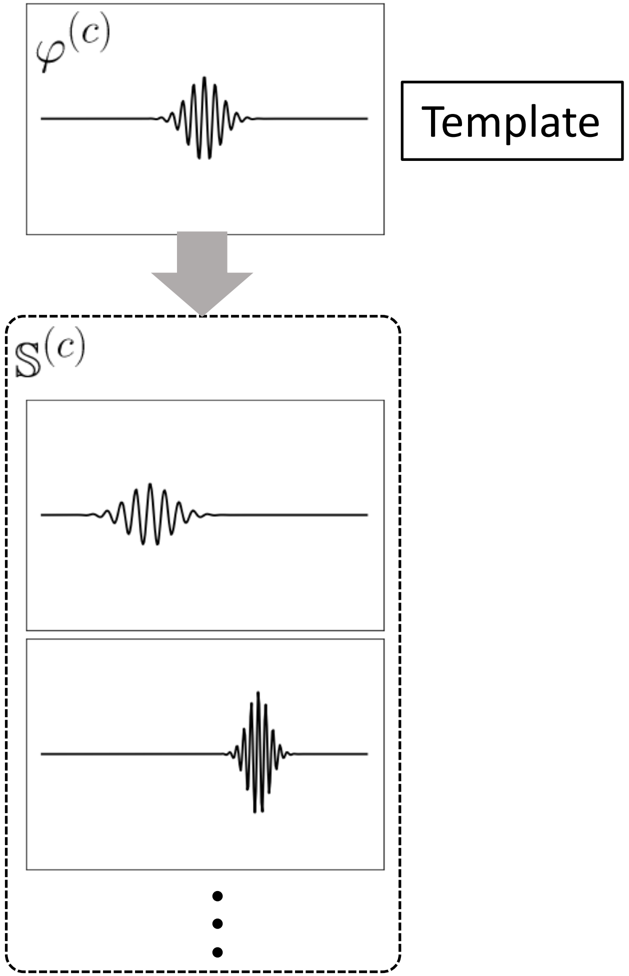
Generative model:

Given, a set of increasing 1D spatial deformations of a specific kind denoted as $\mathcal{G} \subset \mathcal{T}$.

Generative model for class-*c* is then defined to be the set:

$$\mathbb{S}^{(c)} = \{s_j^{(c)} | s_j^{(c)} = g_j' \varphi^{(c)} \circ g_j, g_j \in \mathcal{G}\}$$
$$\text{where} \quad g_j' > 0$$

Eq. (1)



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Eq. (1)

Classification problem:

Let $\mathcal{G} \subset \mathcal{T}$ be the set of spatial deformations and $\mathbb{S}^{(c)}$ be defined according to the generative model. Given training samples$\{s_1^{(c)}, s_2^{(c)}, ...\}$ for class-c $(c = 0, 1, 2, ..., etc.)$, determine the class label of an unknown signal s

Proposed Approach

- Assumptions:
  - Data were generated according to the defined generative model (compositions of a template signal)
  - The compositions form a convex group
  - Data space for a particular class does not overlap with data spaces corresponding to other classes
- **SCDT-NS**: Under these assumptions, we form a linear subspace for each class in the SCDT domain and employ a nearest subspace (NS) search algorithm

Signed Cumulative Distribution Transform (SCDT)

- Cumulative Distribution Transform (CDT) for a positive PDF $s(t)$ with respect to uniform reference:

$$s^*(y) = S^{-1}(y), \quad \text{where } S(t) = \int_{-\infty}^{t} s(u)du \qquad \text{Eq. (2)}$$

- SCDT of a non-negative signal with arbitrary mass:

$$\widehat{s}(y) = \begin{cases} \left(s^*(y), \|s\|_{L_1}\right), & \text{if } s \neq 0 \\ (0,0), & \text{if } s = 0, \end{cases} \qquad \text{Eq. (3)}$$

- SCDT of a signed signal is defined as:

$$\widehat{s}(y) = \left(\widehat{s}^+(y), \widehat{s}^-(y)\right) \qquad \text{Eq. (4)}$$

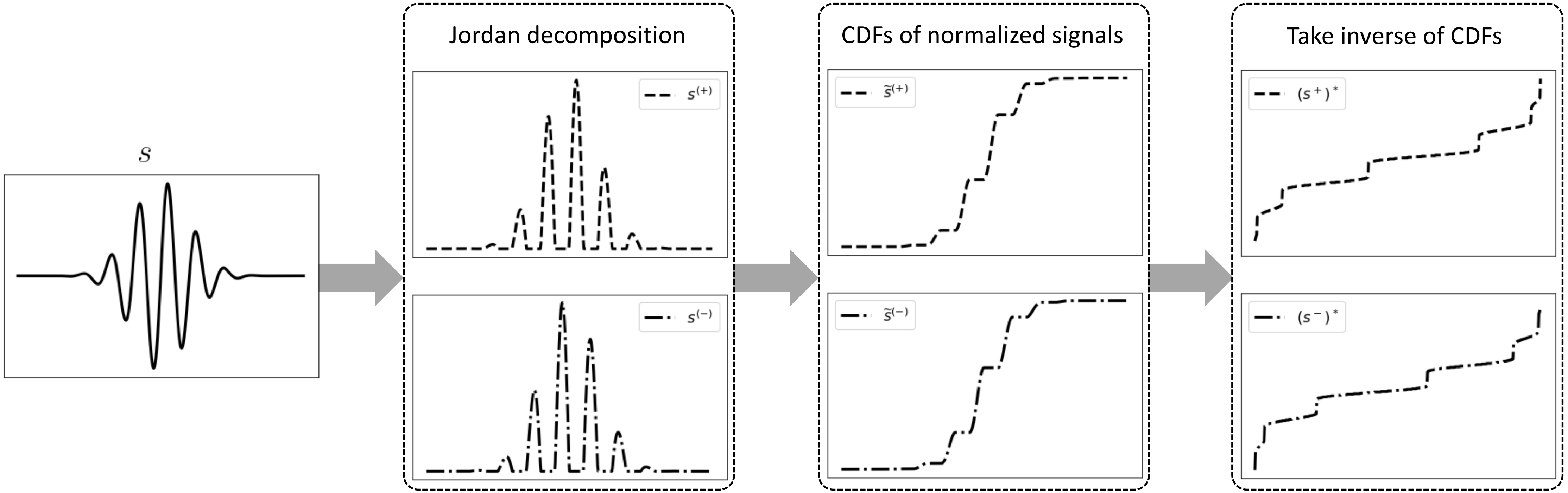
- SCDT ignoring the total mass terms:

$$\widehat{s}^\dagger(y) = \left((s^+(y))^*, (s^-(y))^*\right) \qquad \text{Eq. (5)}$$

Signed Cumulative Distribution Transform (SCDT)

- SCDT ignoring the total mass terms:

$$\hat{s}^{\dagger}(y) = \left((s^{+}(y))^{*}, (s^{-}(y))^{*}\right)$$



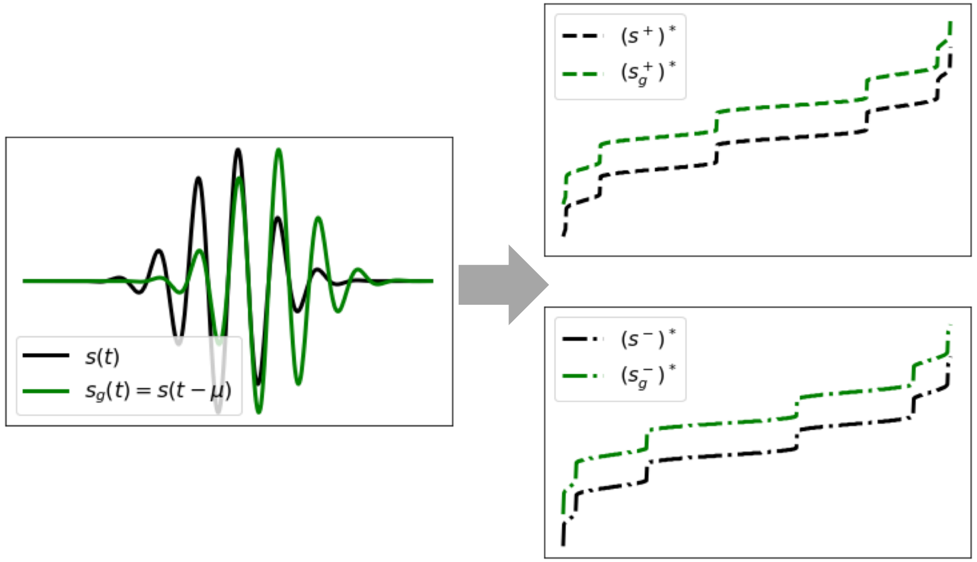
Signed Cumulative Distribution Transform (SCDT)

- SCDT ignoring the total mass terms:

$$\widehat{s}^{\dagger}(y) = \left((s^{+}(y))^{*}, (s^{-}(y))^{*}\right)$$

- Composition property: SCDT of $s_g = g' s \circ g$ is given by,

$$\widehat{s}_g^{\dagger} = g^{-1} \circ \widehat{s}^{\dagger}$$



- Convexity property: Given a set of signals,

$$\mathbb{S} = \{s_j | s_j = g_j' \varphi \circ g_j, g_j \in \mathcal{G}\}.$$

$\widehat{\mathbb{S}} = \{\widehat{s}_j | \widehat{s}_j = g_j^{-1} \circ \varphi, s_j \in \mathbb{S}\}$ is convex if and only if $\mathcal{G}^{-1} = \{g_j^{-1} : g_j \in \mathcal{G}\}$ is convex.

Signed Cumulative Distribution Transform (SCDT)

Generative model:

Generative model for class-*c* is defined to be the set:

$$\mathbb{S}^{(c)} = \{s_j^{(c)} | s_j^{(c)} = g_j' \varphi^{(c)} \circ g_j, g_j \in \mathcal{G}\}$$
$$\text{where} \quad g_j' > 0$$

SCDT:

$$\widehat{s}^{\dagger}(y) = \left((s^{+}(y))^{*}, (s^{-}(y))^{*}\right)$$

Generative model in SCDT domain:

$$\widehat{\mathbb{S}}^{(c)} = \{\widehat{s}_j^{(c)\dagger} | \widehat{s}_j^{(c)\dagger} = g_j^{-1} \circ \widehat{\varphi}^{(c)\dagger}, g_j \in \mathcal{G}\} \qquad \text{Eq. (6)}$$

- Forms a convex set, given $\mathcal{G}$ is a convex group

Proposed Solution

- Generative model in SCDT domain:

$$\widehat{\mathbb{S}}^{(c)} = \{\widehat{s}_j^{(c)\dagger} | \widehat{s}_j^{(c)\dagger} = g_j^{-1} \circ \widehat{\varphi}^{(c)\dagger}, g_j \in \mathcal{G}\}$$

- Define a subspace generated by the convex set $\widehat{\mathbb{S}}^{(c)}$

$$\widehat{\mathbb{V}}^{(c)} = \text{span}\left(\widehat{\mathbb{S}}^{(c)}\right) = \left\{\sum_{j\in\mathbf{J}} \alpha_j \widehat{s}_j^{(c)\dagger} | \alpha_j \in \mathbb{R}, \mathbf{J} \text{ is finite}\right\}$$

Eq. (7)

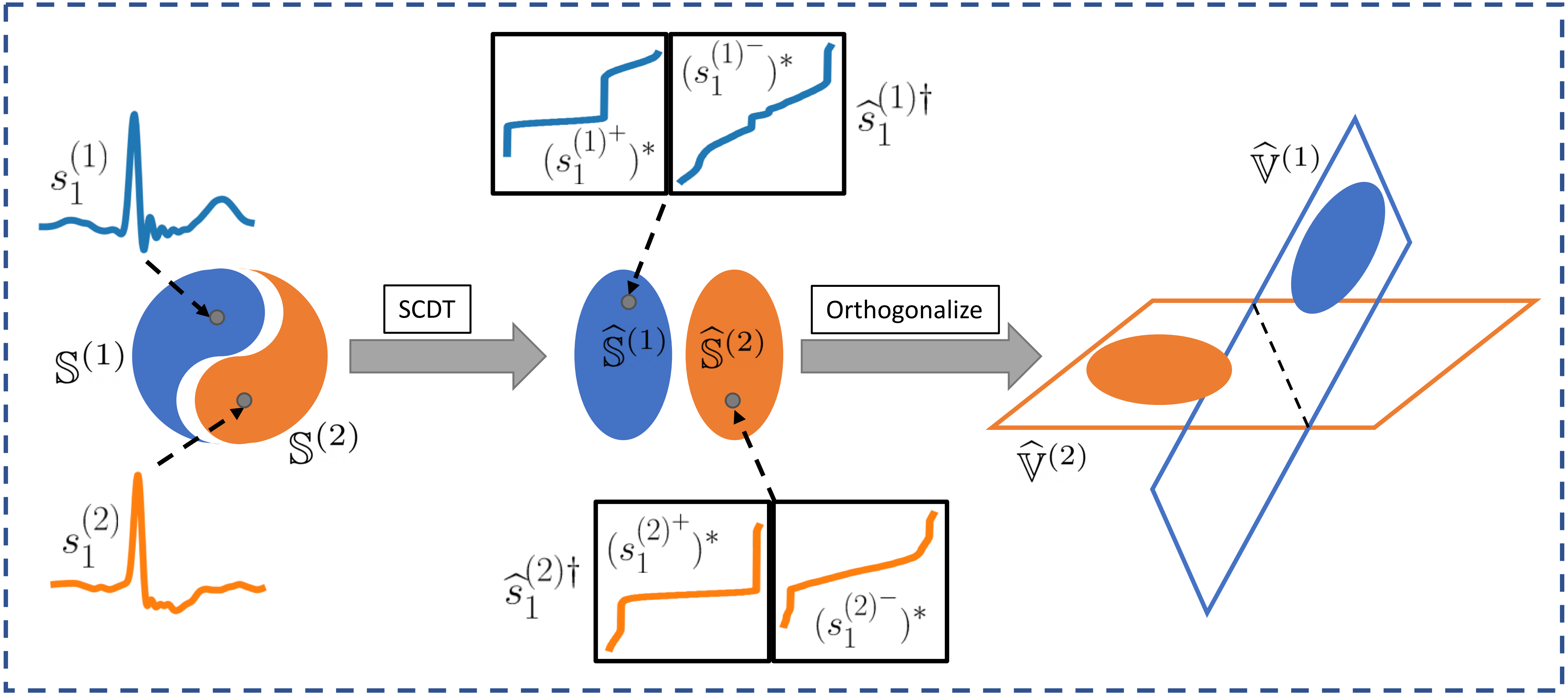
- Class of the test sample s can be predicted by solving

$$\arg\min_c \; d^2(\widehat{s}^\dagger, \widehat{\mathbb{V}}^{(c)})$$
$$\Rightarrow \arg\min_c \; \|\widehat{s}^\dagger - A^{(c)}\widehat{s}^\dagger\|_{L_2}^2$$

$A^{(c)} = B^{(c)}{B^{(c)}}^T$ is an orthogonal projection matrix onto subspace $\widehat{\mathbb{V}}^{(c)}$ spanned by columns of $B^{(c)}$

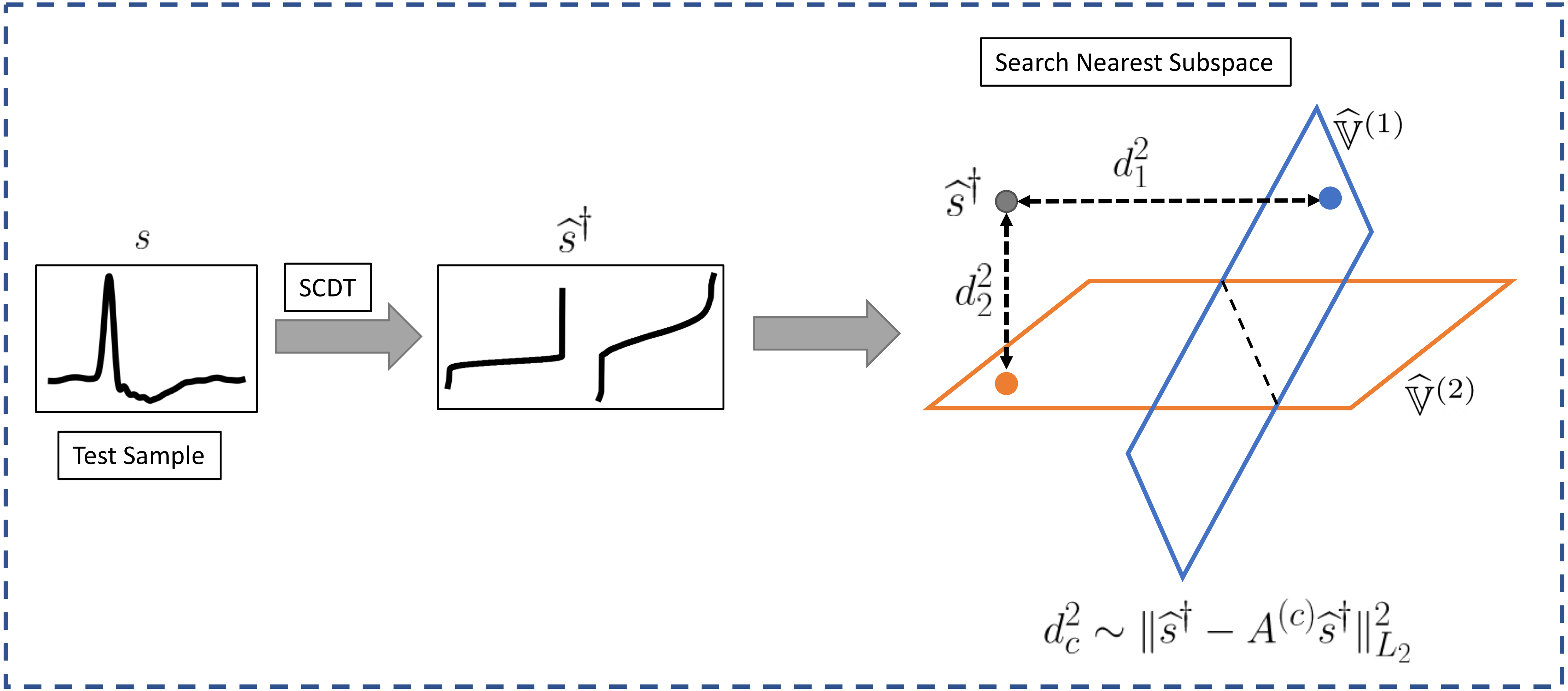
Algorithm

Training Phase:



Algorithm

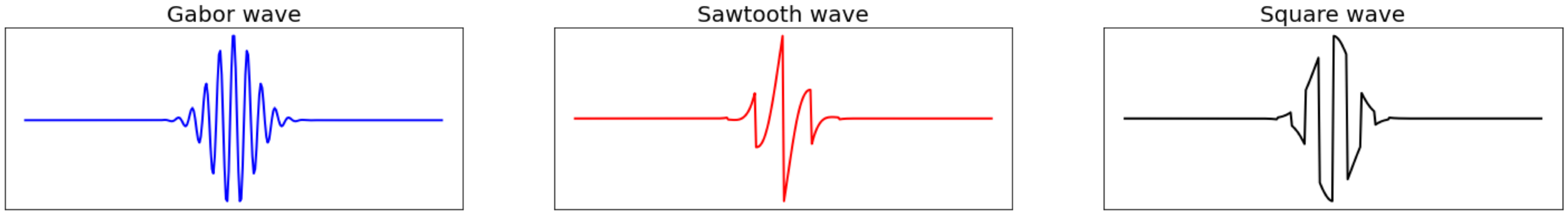
Testing Phase:



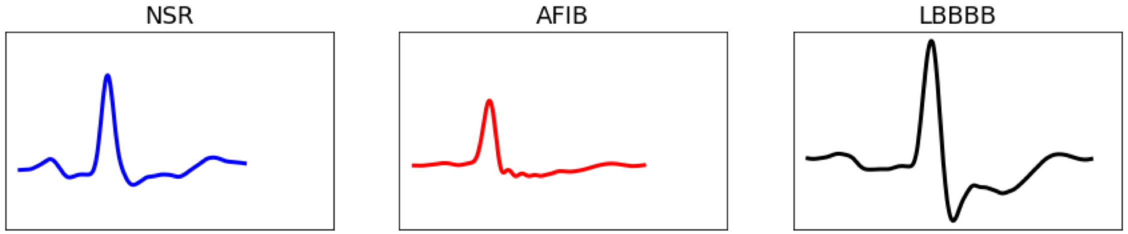
Experiments and Results

Experimental setup:

- Synthetic data:



- ECG data:

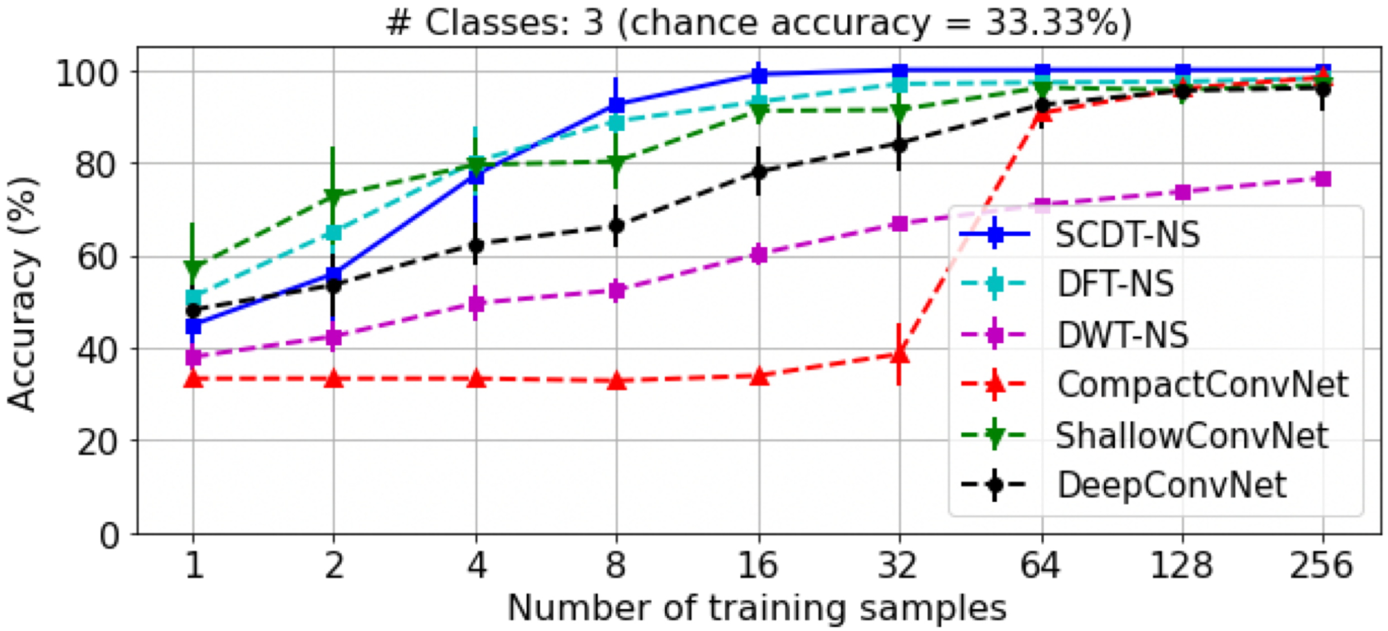


- Compared against:
  - CNNs: shallow [Schirrmeister et. al. 2017], compact [Lawhern et. al. 2018], deep [Schirrmeister et. al. 2017]
  - NS with other signal transform: DFT-NS, DWT-NS

Experiments and Results

Evaluation: Effective and Data Efficient

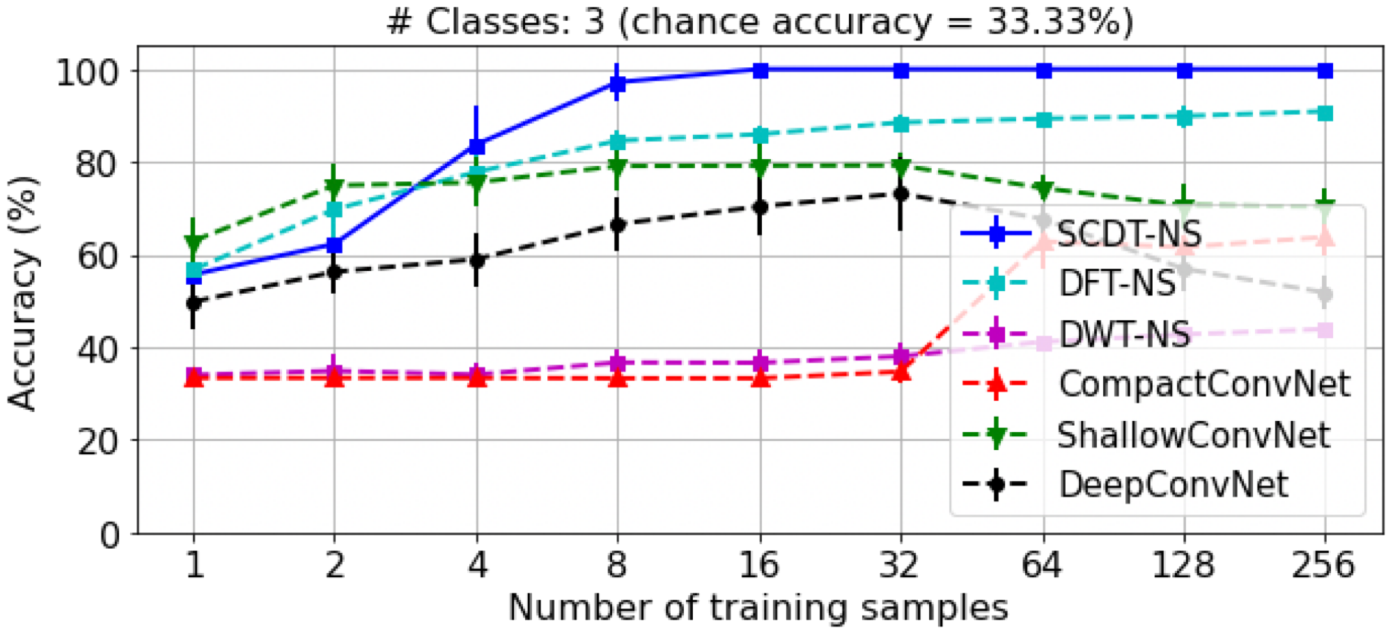
- Three classes: Gabor wave, apodized sawtooth wave, apodized square wave
- Synthetic dataset was generated by applying 4th degree polynomials on three prototype signals
- Polynomial coefficients were randomly chosen

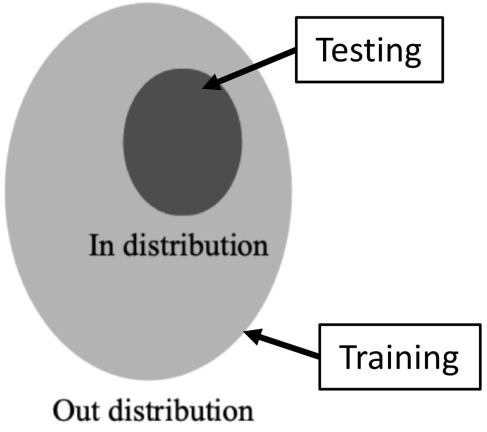


Experiments and Results

Evaluation: Robust to Out-of-distribution Samples

- Three classes: Gabor wave, apodized sawtooth wave, apodized square wave
- Synthetic dataset was generated by applying 4th degree polynomials on three prototype signals
- Polynomial coefficients were chosen in such a way that there exists a gap between training and testing distributions





Experiments and Results

Application:

- ECG data: collected from MIT-BIH arrhythmia database hosted at PhysioNet
- Three classes with three highest number of ECG fragments were used:
  - Normal sinus rhythm (NSR),
  - Atrial fibrillation (AFIB), and
  - Left bundle branch block beat (LBBBB)
- Data from same patients were not included in both training and test sets

	Accuracy (%)	**F1 score**
DeepConvNet	47.57	0.4065
ShallowConvNet	33.68	0.2618
CompactConvNet	29.59	0.2466
DFT-NS	37.93	0.3124
DWT-NS	35.00	0.2306
SCDT-NS	**61.50**	**0.5979**

Summary

- Introduced a new end-to-end 1D signal classification technique
- Generative model-based problem formulation
- Employs a nearest subspace search algorithm in SCDT space to produce a non-iterative solution to the classification problem
- Effective, data efficient, and robust to out-of-distribution samples
- **Future works** involve studying ways to learn general mathematical categories for the space of signal deformations $\mathcal{G}$

Acknowledgement:

- This work was supported in part by NIH grants GM130825, GM090033

Source code:

- Comes with PyTransKit package: https://github.com/rohdelab/PyTransKit
- Python code: https://github.com/rohdelab/PyTransKit/blob/master/pytranskit/classification/scdt_ns.py
- Tutorial: https://github.com/rohdelab/PyTransKit/blob/master/tutorials/11_tutorial_SCDT-NS_classifier.ipynb

- Lab website: http://imagedatascience.com/transport/

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Thank You