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Model-Based Reconstruction for Collimated Beam Ultrasound Systems

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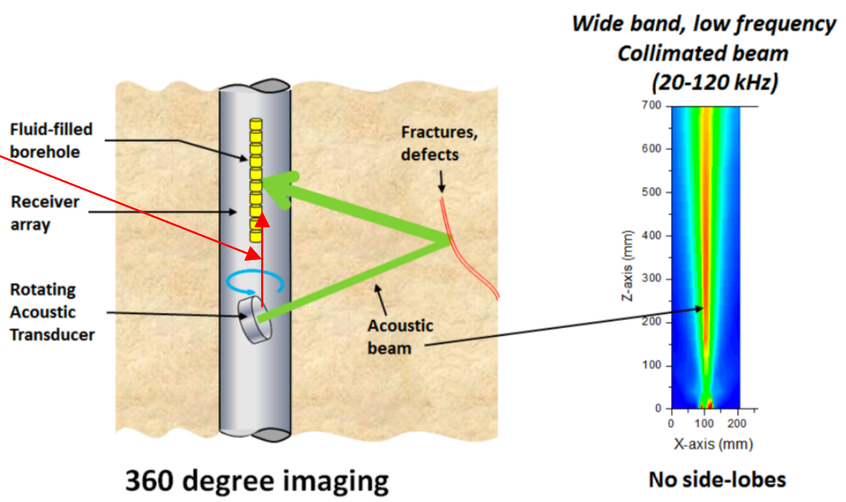
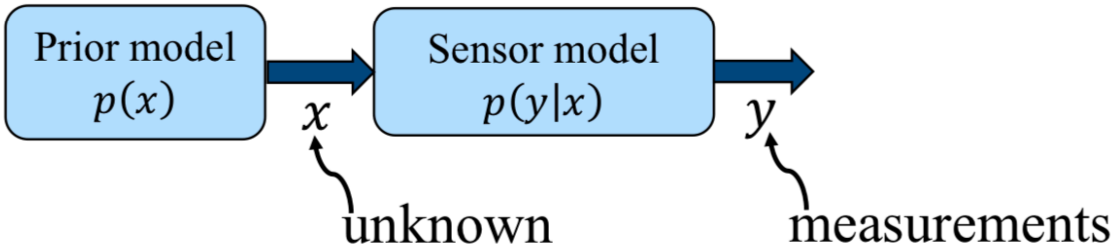
Challenges in NDT?

- Some of the major challenges:
  - Detecting flaws in multilayered objects that can be accessed from only one side.
  - Non-linear effects such reverberations.
  - Direct arrival signals.



Collimated-beam systems use carefully crafted acoustic beams with side lobes suppressed and transducer diffraction minimized to provide deep penetration and high spatial resolution

MAP or Regularized Inversion



- Forward model: $f(x) = -\log p(y|x)$
- Prior model: $h(x) = -\log p(x)$

- MAP or regularized inversion:

$$\hat{x} \leftarrow \arg\min_{x}\{f(x) + h(x)\}$$

System Model

- Assuming a linear system, we seek to reconstruct an image x using a mathematical model of the form

$$y = Ax + Dg + w$$

  - y is the observed data,
  - A is the system matrix,
  - D a matrix whose columns form a basis for the possible direct arrival signals,
  - g is a scaling coefficient vector for D,
  - w is a Gaussian random vector with distribution $N(0, \sigma^2 I)$.

Transfer Functions

- For the homogeneous medium shown in Fig. 1, the transfer function from point r_i to r_j is

$$G(v,f) = \tau \exp\left\{-(\alpha c|f| + j2\pi f)\left(\frac{\|v-r_i\|+\|r_j-v\|}{c}\right)\right\},$$

where

  - τ is the transmittance coefficient of the front surface of the medium,
  - α is the attenuation coefficient in s/m, and
  - c is the sound speed in m/s in the medium.

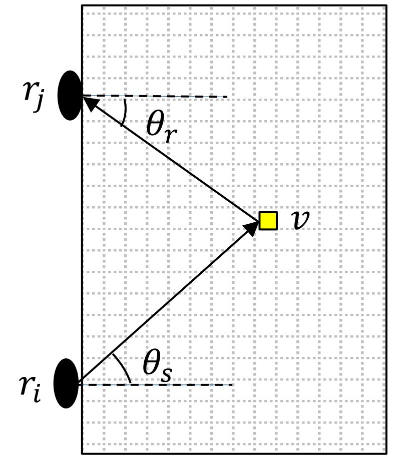


Fig.1 homogeneous medium

- So, the multi-layer media shown in Fig. 2, the transfer function from r_i to r_j

$$G(v,f) = \prod_{l=1}^{L} \tau_l e^{(\gamma_l(v)|f|+2j\pi f T_l(v))}$$

where

  - L is the total number of layers,
  - τ_l is the transmittance coefficient of the front surface of the l^{th} layer,
  - $\gamma_l(v) = c_l \alpha_l T_l(v)$,
  - c_l is the acoustic speed in m/s in the l^{th} layer,
  - α_l is the attenuation coefficient in s/m in the l^{th} layer,
  - and $T_l(v)$ is the travel time in seconds between the front and back interface of the l^{th} layer.

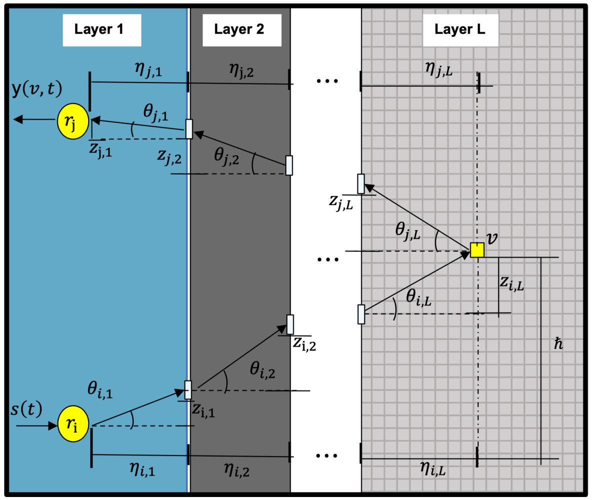
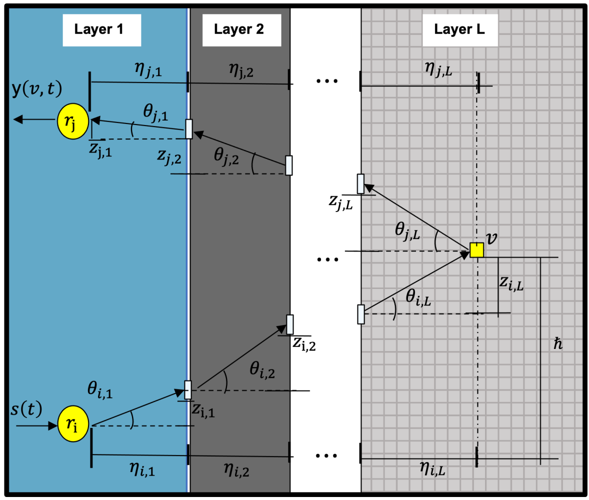


Fig.2 Multi-layer media

Time Delay Computation in Multi-layers



- Based on Snell's law, the time delay from r_i to v to r_j is given by

$$T(v) = \sum_{l=1}^{L} \frac{\sqrt{z_{i,l}^2 + \eta_{i,l}^2} + \sqrt{z_{j,l}^2 + \eta_{j,l}^2}}{c_l}$$

where $z_{i,l} = \eta_{i,l} \tan(\theta_{i,l})$ and $z_{j,l} = \eta_{j,l} \tan(\theta_{j,l})$, $l = 1, 2, \ldots, L$.

- The height of v as a function of $\theta_{i,l}$ is

$$\sum_{l=1}^{L} z_{i,l} = \eta_{i,1} \tan(\theta_{i,1}) + \cdots + \eta_{i,L} \tan(\theta_{i,L})$$

- From Snell's law, we know that

$$\theta_{i,k} = \sin^{-1}\left(\sin(\theta_{i,k-1}) \frac{c_k}{c_{k-1}}\right), \forall k \in \{2, 3, \ldots, L\}.$$

- The effective time delay is then computed using **Binary Search** by finding the angle of refraction and solving for the minimum distance.

Received signal & system matrix

- In frequency space, the received signal is proportional to

$$Y(v,f) = -x(v)S(f)\prod_{l=1}^{L} \tau_l e^{-(\gamma_l(v)|f|+2j\pi f T_l(v))}$$

where $x(v)$ in m^{-3} is the reflection coefficient for the voxel v and $S(f)$ the Fourier transform of the transmitted signal.

- Then the time-domain **received signal** for a reflection from location v is given by

$$y(v,t) = x(v)h(\gamma(v), t - T(v)),$$

where

$$h(\gamma(v),t) = \mathcal{F}^{-1}\{-S(f)e^{-\gamma(v)|f|}\}$$

and $\mathcal{F}^{-1}$ is the inverse Fourier transform.

- In order to reduce computation, we make the approximation that

$$\tilde{h}(\gamma,t) = h(\gamma,t)\, rect\left(\frac{t}{t_0} - \frac{1}{2}\right)$$

where t_0 is a constant based on the assumption that $h(\gamma,t)$ is equal to zero for $t > t_0$.

- The signal received at time t by transducer r_j in response to the transmission from r_i is computed by summing over all voxels v to obtain

$$\tilde{y}_{i,j}(t) = \sum_{v} \tilde{h}\big(\gamma(v), t - T(v)\big)x(v)$$

- This linear relationship between $x(v)$ and $y(t)$ determines a single row of the **system matrix** A in the time domain.

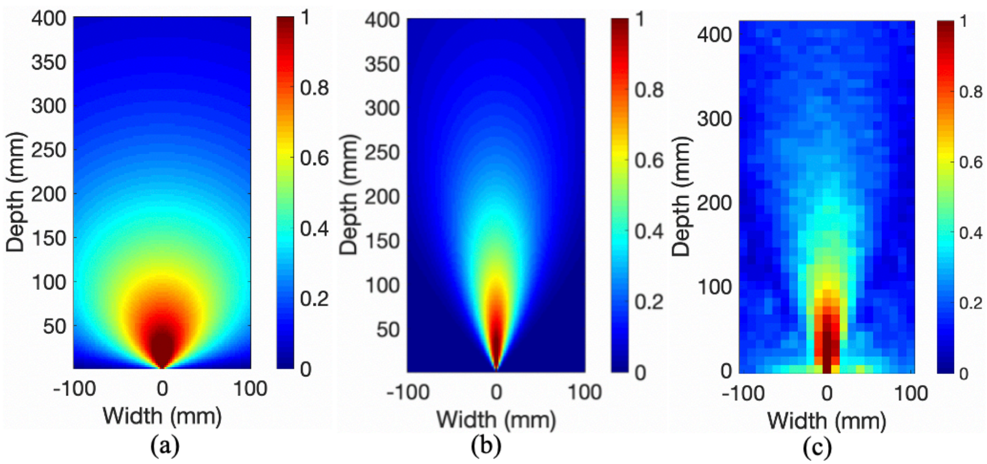
Collimated Beams

- Define a function $\phi_{s,r}(v)$ that has a value ranging from 0 to 1. Then, we modify $\tilde{y}_{i,j}(t)$ to

$$\tilde{y}_{i,j}(t) = \sum_{v} \tilde{h}\big(\gamma(v), t - T(v)\big)\phi(v)^{(\beta)}x(v)$$

- The function $\phi_{s,r}(v)$ depends on the incident and reflected angles and given by

$$\phi(v)^{(\beta)} = \cos^{\beta}\left(\sum_{p=1}^{L} \theta_{i,p}\right)\cos^{\beta}\left(\sum_{q=1}^{L} \theta_{j,q}\right)$$



(a) A simulated beam profile, $\phi(v)^{(\beta)}$, with (a) β = 1 and (b) β = 8.
(c) A real beam profile for a well-collimated source.

Forward Model

- Finally, the discretized version of the forward model will be

$$-\log p(y|x, g) = \frac{1}{2\sigma^2} \|y - Ax - Dg\|^2 + \text{constant},$$

where

- $y \in \mathbb{R}^{MK\times 1}$ is the measurement,
- σ^2 is the variance of the measurement,
- $A \in \mathbb{R}^{MK\times N}$ is the system matrix,
- $x \in \mathbb{R}^{N\times 1}$ is the image,
- $D \in \mathbb{R}^{MK\times K}$ is the direct arrival signal matrix,
- $g \in \mathbb{R}^{K\times 1}$ is a vector that scales the columns of D independently,
- M is the number of measurement samples, and
- N is the number of pixels.

Prior Model

- We adopt the q-generalized Gaussian Markov Random Field (qGGMRF) for the prior model. With this design, the prior model is

$$p(x) = \frac{1}{z}\exp\left(-\sum_{\{s,r\}\in C} b_{s,r}\rho(x_s - x_r)\right)$$

where z is a normalizing constant, C is the set of pair-wise cliques, and

$$p(\Delta) = \frac{|\Delta|^p}{p\sigma_{g_{s,r}}^p}\left(\frac{\left|\frac{\Delta}{T\sigma_{g_{s,r}}}\right|^{q-p}}{1+\left|\frac{\Delta}{T\sigma_{g_{s,r}}}\right|^{q-p}}\right),$$

where $\sigma_{g_{s,r}} = \sigma_0\sqrt{m_s m_r}$ and $m_s = 1 + (m-1) * \left(\frac{\text{depth of pixel } s}{\text{maximum depth}}\right)^a$

Hence,

$$-\log p(x) = \sum_{\{s,r\}\in C} b_{s,r}\rho(x_s - x_r) + \text{constant}\ .$$

Optimization of MAP cost function

- After combining the forward and prior models, the MAP estimate is given by

$$(x, g)_{MAP} = \arg\min_{x\geq 0, g} \left\{ \frac{1}{2\sigma^2} \|y - Ax - Dg\|^2 + \sum_{\{s,r\}\in C} b_{s,r}\rho(x_s - x_r) \right\}$$

ICD Algorithm Using Majorization Technique

Initialize x, $e \leftarrow y - Ax$

For k iterations {

$g = (D^t D)^{-1} D^t e$

$e \leftarrow e - Dg$

For each pixel $s \in S$ {

$\tilde{b}_{s,r} \leftarrow \frac{b_{s,r}\rho'(x_s - x_r)}{2(x_s - x_r)}$

$\theta_1 \leftarrow -e^t \mathrm{A}_{*,s} + \sum_{r\in\partial s} \tilde{b}_{s,r}(x_s - x_r)$

$\theta_2 \leftarrow \mathrm{A}_{*,s}^{\mathrm{t}} \mathrm{A}_{*,s} + \sum_{r\in\partial s} \tilde{b}_{s,r}$

$\alpha^* \leftarrow \mathrm{clip}\left\{-\frac{\theta_1}{\theta_2}, [-x_s, \infty)\right\}$

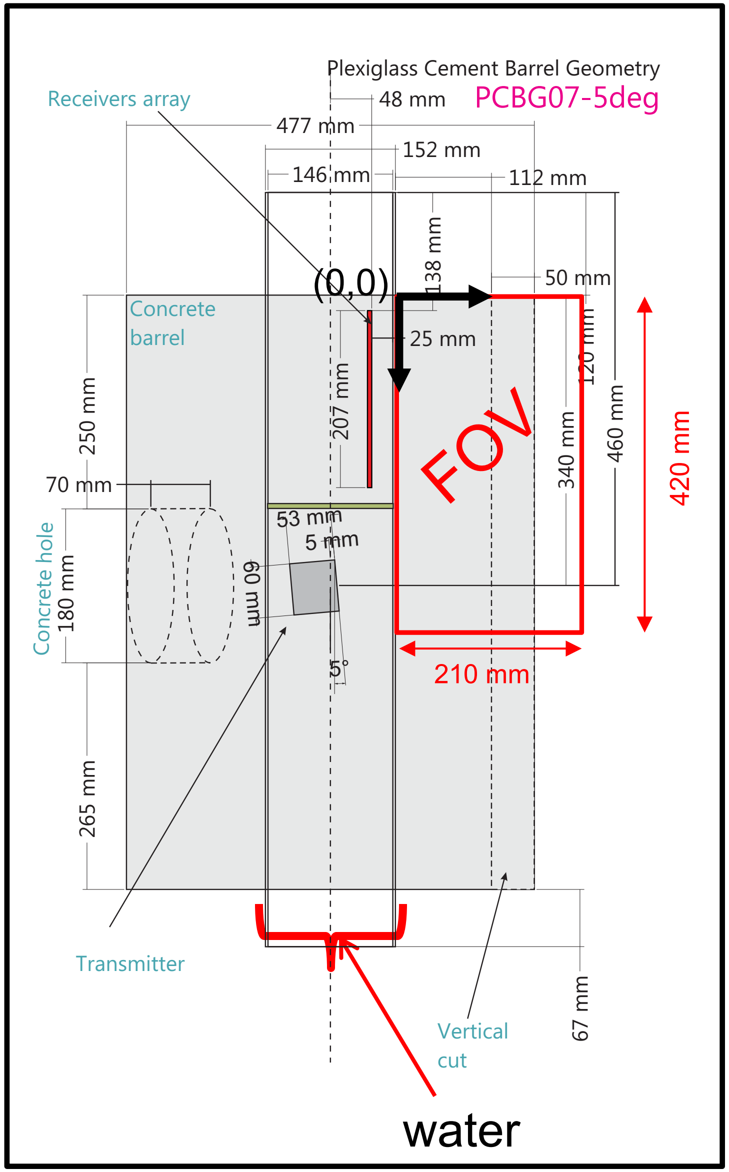
$x_s \leftarrow x_s + \alpha^*$

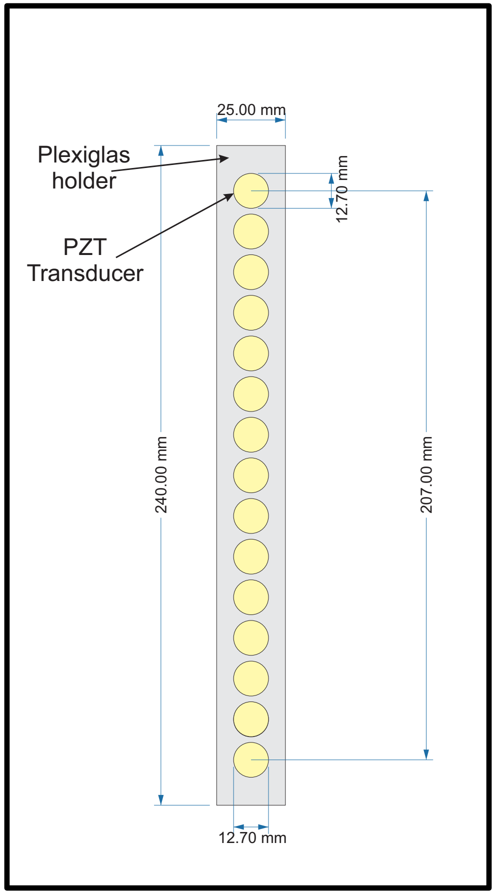
$e \leftarrow e - A_{*,s}\alpha^*$

}

}

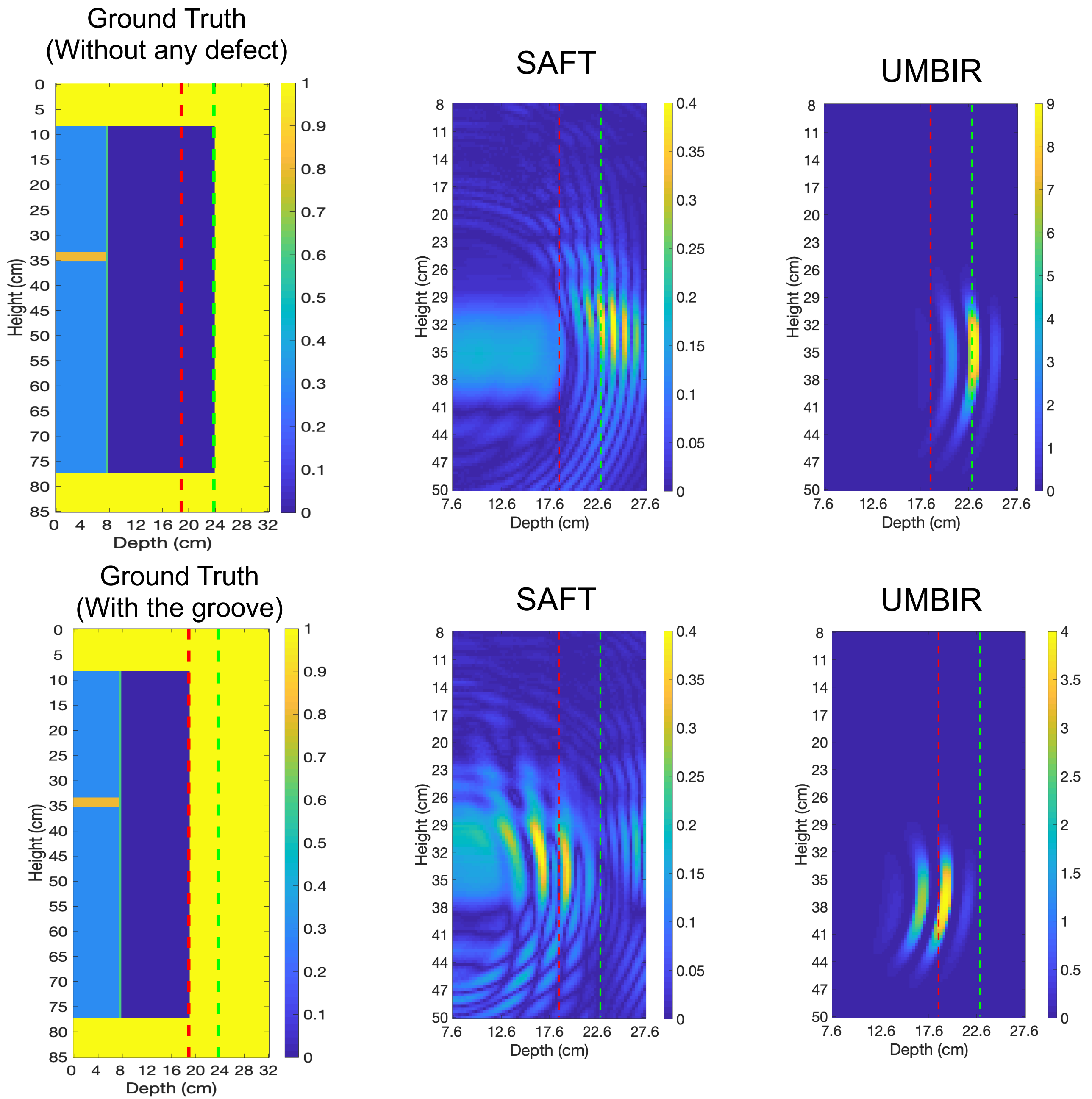
Experimental Results: System geometry





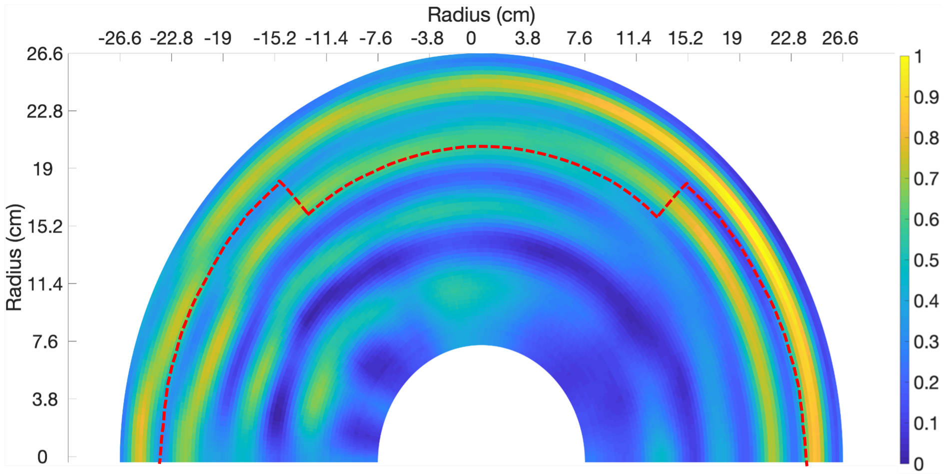
Experimental Results: Synthetic Data Results

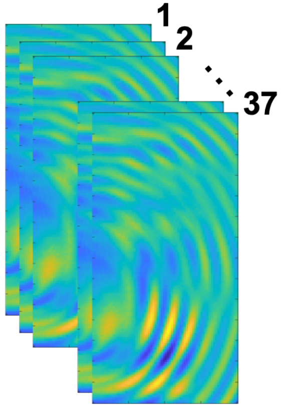
- Synthetic data was generated using the K-Wave simulator.
- The red and green dashed lines demonstrate the groove and backwall locations, respectively.



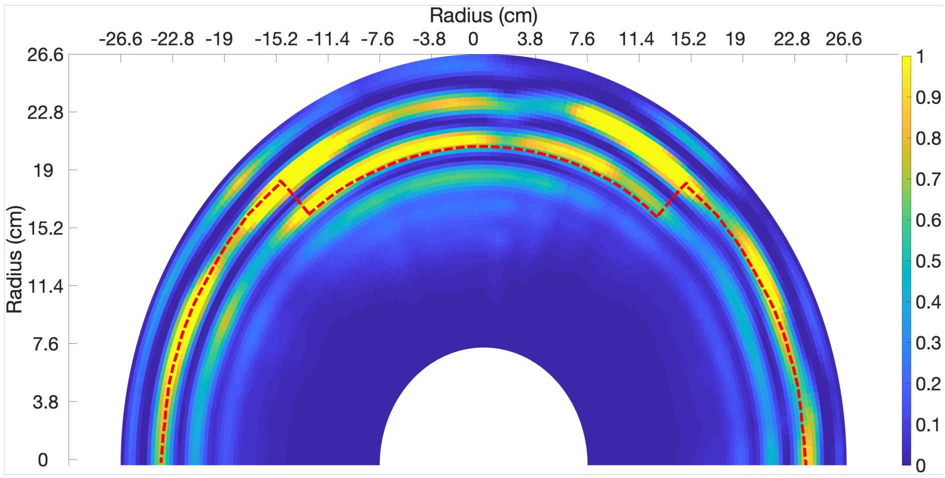
Experimental Results: Real Data Results

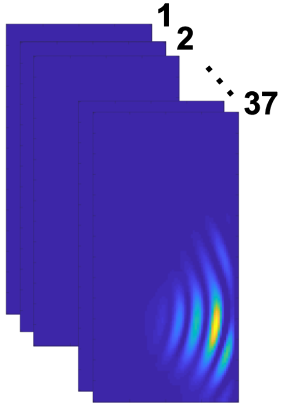
SAFT





UMBIR





Conclusion

- We proposed our multi-layer UMBIR algorithm designed for ultrasonic collimated beam systems.
- We showed the derivation of our modified forward model for multilayered structures and collimated ultrasonic-transducers.
- Our results demonstrated that our UMBIR shows clear improvements over SAFT and is effective for real data applications.

Thank You!

References

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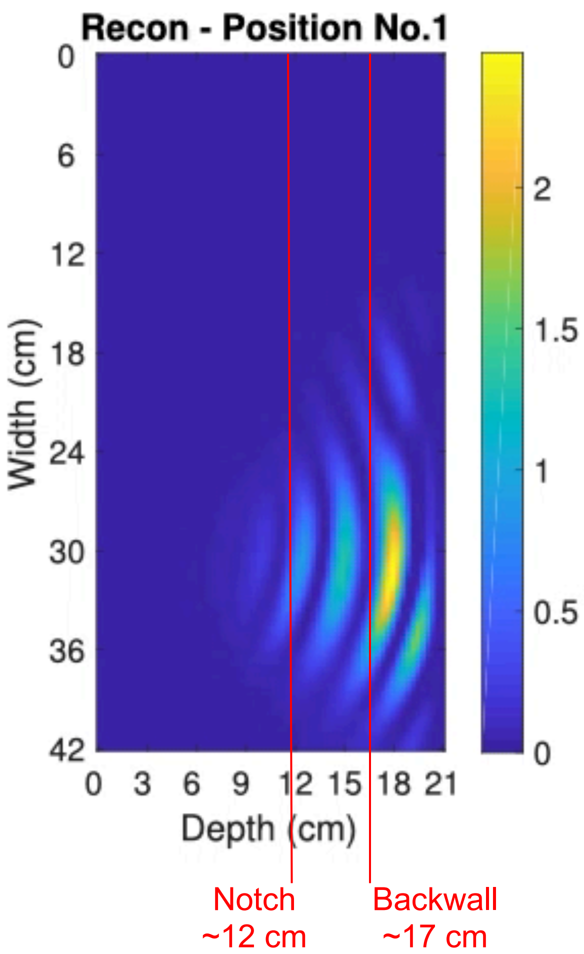
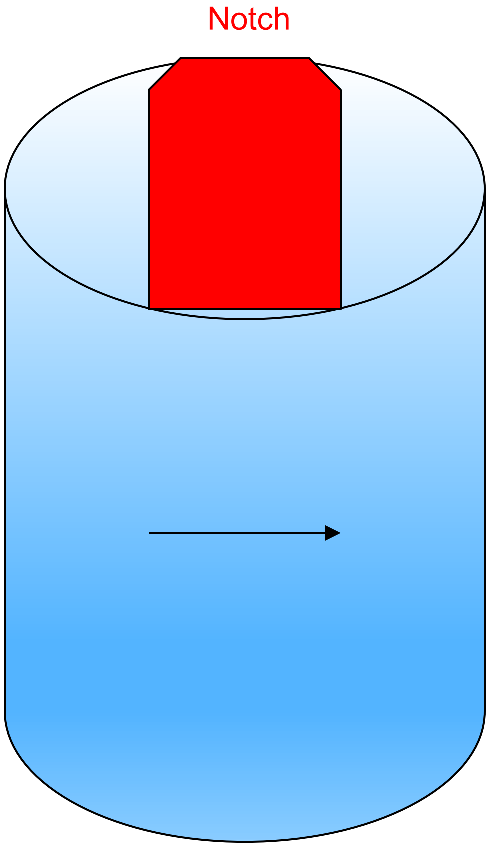
Reconstructions of all views





Recons of the 37 positions. The notch can be seen between position 16 and 25.