A Novel Negative ℓ_1 Penalty Approach for Multiuser One-Bit Massive MIMO Downlink with PSK Signaling

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Motivation

- Employing one-bit DACs at the base station (BS) can greatly reduce the hardware cost and the energy consumption of a massive MIMO system.
- Nonlinear precoding scheme exhibits significantly better performance than linear precoding in the one-bit case.
  - ◇ The classical minimum mean square error (MMSE) criterion does not take advantage of symbol-level precoding.
  - ◇ Constructive interference (CI)-based approaches generally perform better than MMSE-based approaches.
- Massive MIMO and symbol-level precoding impose high requirement on the efficiency of the algorithms.
- State of the art CI-based algorithms:
  - ◇ Suffer from an error rate floor (MSM [2, 3])
  - ◇ Degrade in difficult cases (OPSU [4])
  - ◇ Suffer from a high computational complexity (P-BB [4])

Main Contribution

- Develop a negative ℓ_1 penalty (NL1P) approach that achieves a better tradeoff between the BER performance and the computational efficiency:
  - ◇ Propose an exact negative ℓ_1 penalty model for the original discrete model
  - ◇ Transform the penalty model into an equivalent min-max problem for which an efficient alternating optimization (AO) algorithm is designed

System Model

- A multiuser system with one N-antenna BS and K single-antenna users
- $\mathbf{s} = [s_1, s_2, \ldots, s_K]^\mathsf{T}$: the intended data symbol vector for the users, where each s_i is drawn from M-PSK constellation
- $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K]^\mathsf{T} \in \mathbb{C}^{K\times N}$: the flat-fading channel matrix between the BS and the users, with each element i.i.d. following $\mathcal{CN}\left(0, \frac{1}{N}\right)$
- $\mathbf{n} \in \mathbb{C}^K$: the additive white Gaussian noise, with each element i.i.d. following $\mathcal{CN}(0, 1)$
- $\mathbf{x}_T$: transmitted signal vector from the BS that satisfies the one-bit constraint, i.e., $\mathbf{x}_T \in \{\pm 1 \pm j\}^N$
- The received signal vector $\mathbf{y} \in \mathbb{C}^{K\times 1}$ can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x}_T + \mathbf{n}.$$

Problem Formulation

- $\hat{y}_k = \mathbf{h}_k^\mathsf{T}\mathbf{x}_T$: the noise-free received signal of user k
- $s_k^A = s_k e^{-j\frac{\pi}{M}}$, $s_k^B = s_k e^{j\frac{\pi}{M}}$: unit vectors parallel to the two decision boundaries of s_k
- Decompose $\hat{y}_k$ along s_k^A and s_k^B as

$$\hat{y}_k = \alpha_k^A s_k^A + \alpha_k^B s_k^B.$$

- An illustration of the CI formulation for 8-PSK:

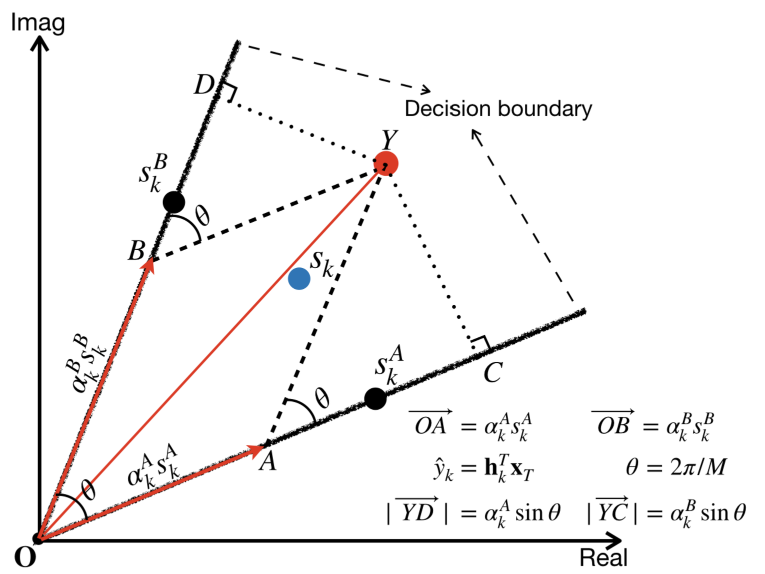


Figure 1: An illustration of the CI formulation.

- $\min\{\alpha_k^A, \alpha_k^B\}$ characterizes the distance between $\hat{y}_k$ and the decision boundary of s_k.
- The symbol scaling model for one-bit precoding [3]:

$$\begin{aligned} \max_{\mathbf{x}_T}\ \min_{k\in\{1,2,\ldots,K\}}\ & \{\alpha_k^A, \alpha_k^B\} \\ \text{s.t.}\ \ & \mathbf{h}_k^\mathsf{T}\mathbf{x}_T = \alpha_k^A s_k^A + \alpha_k^B s_k^B, \quad k = 1, 2, \ldots, K, && \text{(1a)} \\ & \mathbf{x}_T(i) \in \{\pm 1 \pm j\}, \quad i = 1, 2, \ldots, N. && \text{(1b)} \end{aligned}$$

- An equivalent form in the real space:

$$\min_{\mathbf{x}\in\{-1,1\}^{2N}}\ \max_{l\in\{1,2,\ldots,2K\}}\ \mathbf{a}_l^\mathsf{T}\mathbf{x}, \qquad \text{(P)}$$

where

- ◇ $\mathbf{x} = [\mathcal{R}(\mathbf{x}_T)^\mathsf{T}, \mathcal{I}(\mathbf{x}_T)^\mathsf{T})]^\mathsf{T}$
- ◇ $\mathbf{a}_l$, $l = 1, 2, \ldots, 2K$, are problem-dependent vectors.

NL1P Approach

- Basic idea: using the penalty approach and the homotopy technique to solve the discrete model (P):
  - ◇ Transform (P) into an equivalent continuous penalty model
  - ◇ Gradually increase the penalty parameter and solve the corresponding penalty model by taking care of its special structure

NL1P Approach (Cont.)

- Negative ℓ_1 penalty model: penalize the discrete one-bit constraint into the objective with a negative ℓ_1-norm term, and relax the constraint to its convex hull:

$$\min_{\mathbf{x}\in[-1,1]^{2N}}\ \max_{l\in\{1,2,\ldots,2K\}}\ \mathbf{a}_l^\mathsf{T}\mathbf{x} - \lambda\|\mathbf{x}\|_1. \qquad (\mathrm{P}_\lambda)$$

  - ◇ Exactness: when $\lambda > \max_l \|\mathbf{a}_l\|_\infty$, (P_λ) is equivalent to (P) both globally and locally.
- Min-max reformulation of (P_λ):

$$\min_{\mathbf{x}\in[-1,1]^{2N}}\ \max_{\mathbf{y}\in\Delta}\ \mathbf{y}^\mathsf{T}\mathbf{A}\mathbf{x} - \lambda\|\mathbf{x}\|_1, \qquad (\widehat{\mathrm{P}}_\lambda)$$

where $\Delta = \{\mathbf{y} \in \mathbb{R}^{2K} \mid \mathbf{1}^\mathsf{T}\mathbf{y} = 1,\ \mathbf{y} \geq \mathbf{0}\}$, $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_{2K}]^\mathsf{T}$.

- AO algorithm for solving $(\widehat{\mathrm{P}}_\lambda)$: update $\mathbf{x}$ and $\mathbf{y}$ iteratively as follows until some stopping criterion is satisfied:

$$\begin{aligned} \mathbf{x}_{k+1} &\in \arg\min_{\mathbf{x}\in[-1,1]^n}\ \mathbf{y}_k^\mathsf{T}\mathbf{A}\mathbf{x} - \lambda\|\mathbf{x}\|_1 + \frac{\tau_k}{2}\|\mathbf{x} - \mathbf{x}_k\|^2 && \text{(2a)} \\ \mathbf{y}_{k+1} &= \mathrm{Proj}_\Delta(\mathbf{y}_k + \rho_k\mathbf{A}\mathbf{x}_{k+1} - \rho_k c_k \mathbf{y}_k), && \text{(2b)} \end{aligned}$$

where $\rho_k \geq 0, \tau_k \geq 0$, and $c_k \geq 0$ are the algorithm parameters.

  - ◇ (2a) admits a closed-form solution as

$$\mathbf{x}_{k+1}(i) = \mathrm{sgn}(a_k^i)\min\left\{|a_k^i| + \frac{\lambda}{\tau_k}, 1\right\},\ i = 1, 2, \ldots, 2N,$$

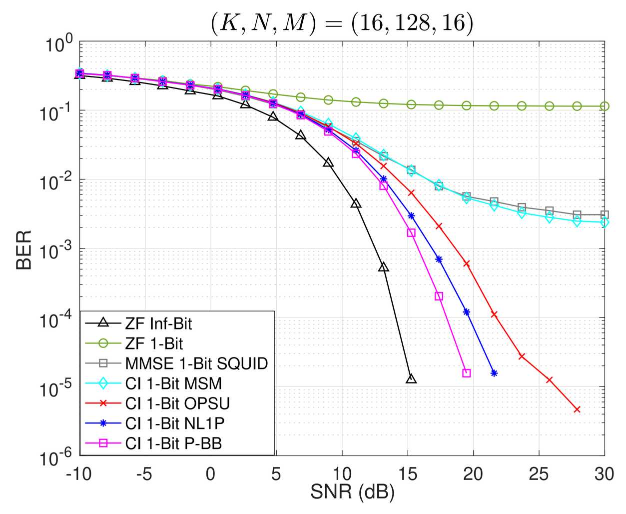
  where $a_k^i = \mathbf{x}_k(i) - \frac{\mathbf{A}_i^\mathsf{T}\mathbf{y}_k}{\tau_k}$ and $\mathbf{A}_i$ is the i-th column of $\mathbf{A}$.
  - ◇ Convergence property: With properly selected parameters, every limit point $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ of $\{(\mathbf{x}_k, \mathbf{y}_k)\}$ is a stationary point of $(\widehat{\mathrm{P}}_\lambda)$. Moreover, if $\lambda > \max_l \|\mathbf{a}_l\|_\infty$, $\hat{\mathbf{x}}$ is a local minimizer of (P_λ) and satisfies the one-bit constraint.
- The homotopy framework for solving (P): initialize λ with a small value at the beginning, then gradually increase it and trace the solution path of the corresponding penalty problems, until λ is sufficiently large and a one-bit solution is obtained.
- The pseudocodes of the proposed NL1P approach are given in Algorithm 1.

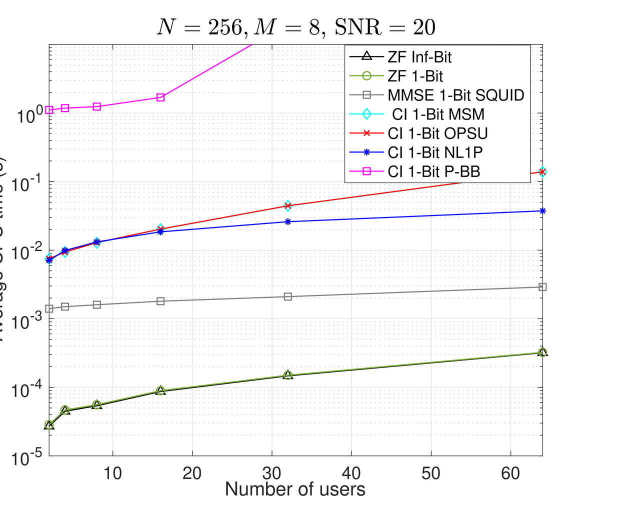
Algorithm 1 Proposed NL1P Approach for Solving Problem (P)

Step 1 Input $\lambda^{(0)}, \delta > 1, \mathbf{x}^{(0)}$; set $t = 1$.
Step 2 Apply the AO algorithm to solve problem (P_λ) with parameter $\lambda = \lambda^{(t-1)}$ and initial point $\mathbf{x}^{(t-1)}$; let the solution be $\mathbf{x}^{(t)}$.
Step 3 Stop if $\mathbf{x}^{(t)}$ satisfies the one-bit constraint; otherwise, set $\lambda^{(t)} = \delta\lambda^{(t-1)}$ and $t = t + 1$, go to Step 2.

Simulation Results

- Compare the proposed Algorithm 1 with
  - ◇ Linear precoders: quantized and unquantized ZF precoders
  - ◇ MMSE-based precoder: SQUID [1]
  - ◇ CI-based precoders: MSM [2, 3], OPSU [4], and P-BB [4]
- Parameters setting:
  - ◇ Algorithm 1: $\lambda^{(0)} = \frac{0.001M}{8}$, $\delta = 5$, and $\mathbf{x}^{(0)} = \mathbf{0}$
  - ◇ the AO algorithm: $\rho_k = \rho = \frac{0.2}{\|\mathbf{A}\|_2}$, $c_k = \frac{0.01}{\rho k^{0.05}}$, $\tau_k = \frac{2\log_2 N+1}{10}\mathrm{mean}(|\mathbf{A}|)k^{0.1}$, and $\mathbf{y}_0 = \frac{1}{2K}\mathbf{1}$
- Stopping criterion for the AO algorithm: stop when the iteration number is more than 500 or when the distance between successive iterates is less than 10^{-3}.
- Average over 10^3 channel realization





Left: BER performance; Right: CPU Time.

- The CI-based approaches generally perform much better than the MMSE-based SQUID approach and the quantized ZF approach.
- The proposed NL1P approach achieves a better tradeoff between the BER performance and the computational efficiency than the state-of-the-art CI-based algorithms.
  - ◇ Compared to OPSU [4], NL1P performs better with a lower computational cost.
  - ◇ Compared to P-BB [4], NL1P is much more computationally efficient with a little performance loss.

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