# Closed-form single source direction-of-arrival estimator using first-order relative harmonic coefficients


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## Objectives

❶ All the existing relative harmonic coefficients (RHC) based direction-of-arrival (DOA) estimators suffer from resolution limitations, as they require searching over the DOA grid.

❷ This paper utilizes the **first-order RHC to propose a closed-form DOA estimator by deriving a directional vector, which points towards to the desired source direction**, thus circumventing the exhaustive search over directional space, while achieving equivalent localization accuracy.

## Spherical Harmonics Decomposition

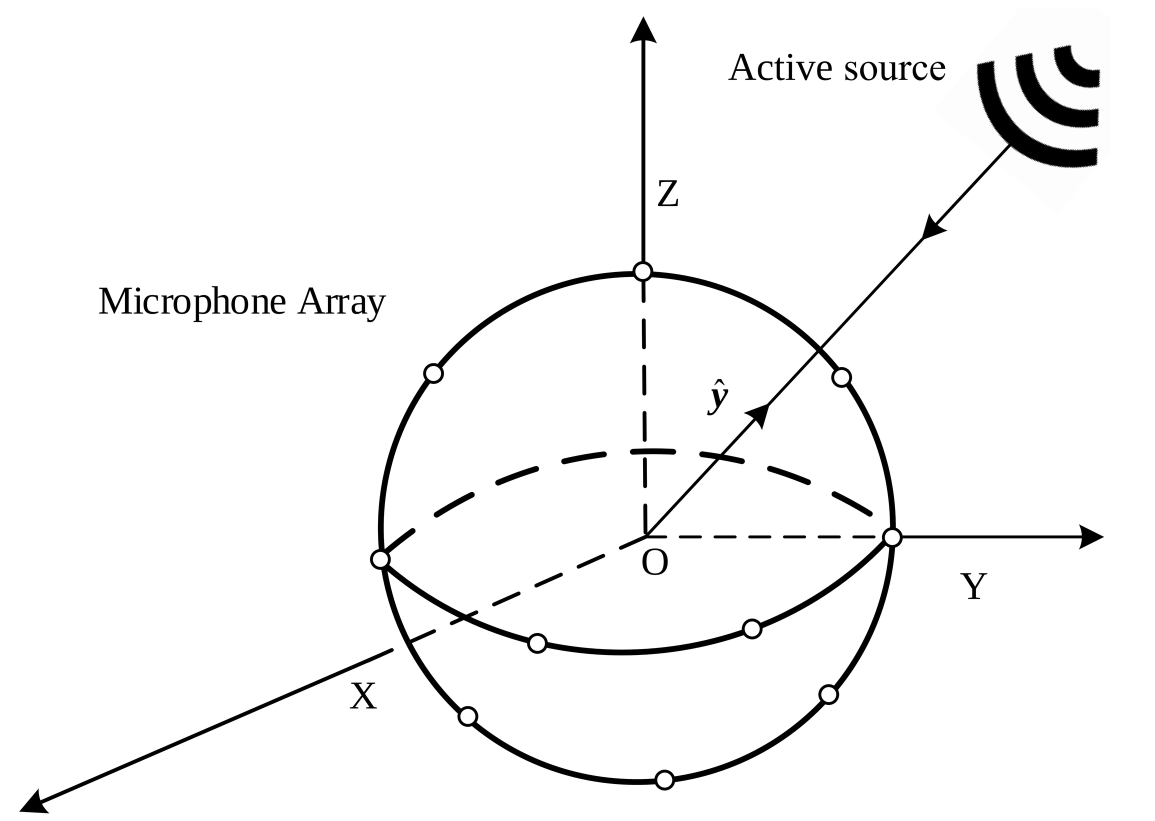


Figure 1: DOA estimation using a spherical microphone array.

Decomposed into the spherical harmonics domain,

$$P(\boldsymbol{x}_j, k) = \Sigma_{n=0}^{N} \Sigma_{m=-n}^{n} \alpha_{nm}(k)\, j_n(kr)\, Y_{nm}(\theta_j, \phi_j) \quad (1)$$

❶ $\alpha_{nm}(\cdot)$: spherical harmonic coefficient

❷ $N = \lceil kr \rceil$: truncated order of soundfield

❸ $j_n(\cdot)$: spherical Bessel function of the first kind

❹ $Y_{nm}(\cdot)$: spherical harmonic function

## Relative Harmonic Coefficients (RHC)

We define the RHC of order $n$ and mode $m$ as:

$$\beta_{nm}(k) = \frac{\alpha_{nm}(k)}{\alpha_{00}(k)}. \quad (2)$$

Analytically, the feature can be expressed as:

$$\beta_{nm}(k) = 2\sqrt{\pi} i^n Y^*_{nm}(\vartheta_s, \varphi_s). \quad (3)$$

For the first-order microphone array, our feature vector is,

$$[2\sqrt{\pi} i Y^*_{1,-1}(\vartheta_s, \varphi_s), 2\sqrt{\pi} i Y^*_{1,0}(\vartheta_s, \varphi_s), 2\sqrt{\pi} i Y^*_{1,1}(\vartheta_s, \varphi_s)]^T, \quad (4)$$

Several advantages:

❶ It is easily estimated under noisy conditions.

❷ It is source-signal invariant, frequency-independent and solely dependent on the source position (e.g., DOA).

❸ It can be used to derive a closed-form DOA estimator (see below).

## A Directional Vector

Explicitly, the first-order RHC in (4) is,

$$\begin{aligned}
\beta_{1,-1} &= i\sqrt{3/2}\sin(\vartheta_s) e^{i\varphi_s} \\
&= -\sqrt{3/2}\sin\vartheta_s \sin\varphi_s + i\sqrt{3/2}\sin\vartheta_s \cos\varphi_s \\
\beta_{1,0} &= i\sqrt{3}\cos(\vartheta_s) \\
\beta_{1,1} &= -i\sqrt{3/2}\sin(\vartheta_s) e^{-i\varphi_s} \\
&= -\sqrt{3/2}\sin\vartheta_s \sin\varphi_s - i\sqrt{3/2}\sin\vartheta_s \cos\varphi_s .
\end{aligned} \quad (5)$$

*Theorem*: Denote the estimated first-order RHC as $\bar{\beta}_{1,-1}$, $\bar{\beta}_{1,0}$, and $\bar{\beta}_{1,1}$. Derive the *direction vector*:

$$\bar{\boldsymbol{I}} = \begin{bmatrix} \mathrm{Im}\{\bar{\beta}_{1,-1} - \bar{\beta}_{1,1}\} \\ \mathrm{Re}\{\bar{\beta}_{1,-1} + \bar{\beta}_{1,1}\} \\ \mathrm{Im}\{\bar{\beta}_{1,0}\} \end{bmatrix} \otimes \begin{bmatrix} \sqrt{1/6} \\ -\sqrt{1/6} \\ \sqrt{1/3} \end{bmatrix} \quad (6)$$

*Conclusion*: **If the estimated first-order RHC coefficients are equal to their analytical values, the directional vector $\bar{\boldsymbol{I}}$ points towards the source direction, i.e.,** $(\vartheta_s, \varphi_s)$. (Please see the proof in the paper)

## Directional vector estimation procedure

The estimations of the directional vector comprise four steps:

❶ Measure the soundfield due to an unknown single sound source, and then transform the time-domain multichannel recordings into the short-time Fourier transform domain.

❷ Decompose the multichannel STFT coefficients into the spherical harmonics domain and estimate the first-order spherical harmonic coefficients.

❸ Extract the first-order RHC and then apply frequency-smoothing over a wide frequency band, as the RHC may slightly differ from the frequency-independence property.

❹ Substitute the smoothed first-order RHC into the closed-form operation in (6), and normalize, as the estimated direction vector may deviate from the unit-norm property, and finally obtain a practical direction vector.

## Algorithm properties

❶ *Usability with other microphone arrays*: the algorithm is independent of the specific microphone constellation provided the array facilitates first-order spherical harmonics decomposition.

❷ *Computational-efficiency*: (i) we applied time averaging and frequency smoothing, respectively, thus circumventing the need to localize the source for each time-frequency bin; and (ii) closed-form solution circumvents the tedious grid search.

Table 2: Various reverberation levels (SNR = 10 dB).

| SR/MAEE | Reverberation time ($T_{60}$) | | |
|---|---|---|---|
| Methods | 150 ms | 350 ms | 550 ms |
| Decoupled | 100%/1.05° | 99%/3.56° | 88%/4.69° |
| Proposed | 100%/0.97° | 99%/3.26° | 93%/4.62° |

## Experimental results

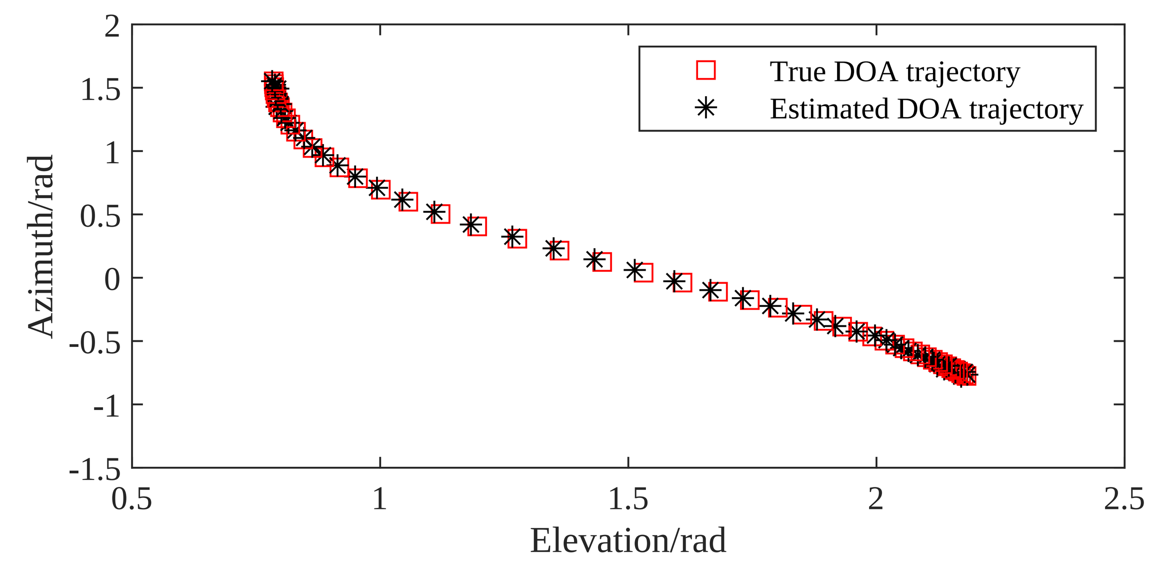


Figure 2: DOA tracking using the proposed algorithm.

Table 1: Localization accuracy for various SNR levels.

| SR/MAEE | Localization methods | | |
|---|---|---|---|
| SNR level | Intensity | Decoupled | Proposed |
| 5 dB | 82%/3.54° | 100%/0.35° | 100%/0.33° |
| 15 dB | 98%/1.21° | 100%/0.17° | 100%/0.12° |
| 25 dB | 100%/0.33° | 100%/0.15° | 100%/0.07° |

❶ *Baselines*: (i) RHC-based decoupled DOA estimator [1] and (ii) the intensity-based method [2].

❷ *Metrics*: (i) success-ratio (SR/%) over the $M_{\text{tot}} = 100$ cases and (ii) average mean absolute estimated error (MAEE) over the successful cases.

❸ *Low-complexity*: average time cost by the proposed method is 2.9 ms, while the decoupled approach takes 578 ms. See Figure 2 for tracking performance and more results in the paper.

## Conclusion

❶ Proposed a closed-form DOA estimator using the first-order relative harmonic coefficients.

❷ Our algorithm achieves better localization accuracy and reduced complexity as compared with the baseline approaches.

## Reference


❶ Y. Hu, et al. "Decoupled DOA Estimators using relative harmonic coefficients," in 2020 28th EUSIPCO, 246-250.

❷ D. P. Jarrett, et al. "3D source localization in the spherical harmonic domain using a pseudointensity vector," in 2010 18th EUSIPCO, pp. 442-446.