

## Problem Formulation

- This work addresses the source localization and association problem in a multipath propagation environment.
- The problem of source association (i.e., assigning a source to each detected path) is rarely considered by existing methods.

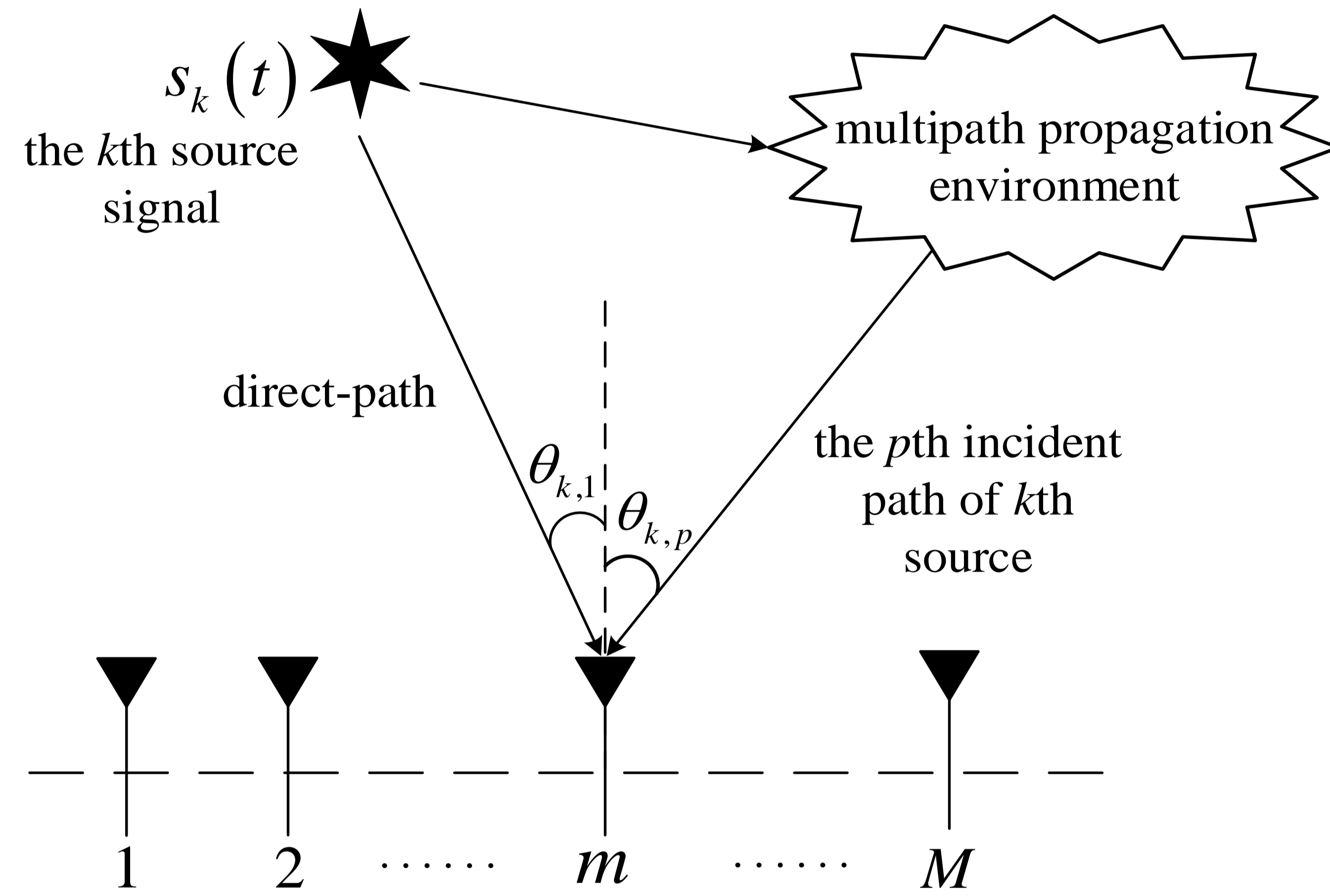


Fig.1 A multipath propagation scenario.

Consider an array system comprises  $M$  isotropic sensors that is impinged by distinct signals from  $K$  far-field narrowband sources. The  $M \times 1$  array output can be represented by

$$\mathbf{x}(t) = \sum_{k=1}^K \sum_{p=1}^{P_k} \mathbf{a}(\theta_{k,p}) c_{k,p} s_k(t) + \mathbf{n}(t) \quad (1)$$

$P_k$  path number corresponding to the  $k$ th source  
 $\mathbf{a}(\theta_{k,p})$  steering vector toward  $\theta_{k,p}$   
 $c_{k,p}$  attenuation coefficient  
 $\mathbf{n}(t)$  white Gaussian noise

## Contributions

- We propose a joint source localization and association (JSLA) algorithm in the presence of multipath propagation environment;
- The initialization of path detection set, additional decorrelation preprocessing, and the prior information pertaining to multipath propagation scenario such as multipath channel parameters are not required in JSLA;
- It has no specific restrictions on the array manifold.

## Methodology

### 1. Overcomplete Representation for Multiple Time Samples

We exploit the sparse characteristic of spatial signals and transform the location parameter estimation into sparse spectrum estimation. An overcomplete  $M \times N$  dictionary is constructed such that

$$\mathbf{D}_\Omega = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_n), \dots, \mathbf{a}(\theta_N)] \quad (2)$$

$\{\theta_n\}_{n=1}^N$  denote a sampling grid set with  $N$  being the number of potential path directions.

With  $\mathbf{D}_\Omega$ , (1) is then reformulated as

$$\mathbf{x}(t) = \mathbf{D}_\Omega \mathbf{r}(t) + \mathbf{n}(t) \quad (3)$$

$\mathbf{r}(t) \in \mathbb{C}^{N \times 1}$  denotes sparse (parameterized) coefficient vector with the  $n$ th element being

$$r_n(t) = \begin{cases} c_{k,p} s_k(t), & \theta_n = \theta_{k,p}, k = 1, 2, \dots, K, \\ & p = 1, 2, \dots, P_k; \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

Let  $\mathbf{X} = [\mathbf{x}(t_1), \dots, \mathbf{x}(t_L)] \in \mathbb{C}^{M \times L}$  and define  $\mathbf{R} \in \mathbb{C}^{N \times L}$  and  $\mathbf{N} \in \mathbb{C}^{M \times L}$  similarly, (3) can be extended as

$$\mathbf{X} = \mathbf{D}_\Omega \mathbf{R} + \mathbf{N} \quad (5)$$

### 2. Source Localization Based on Iterative Implementation with Semi-unitary Constraint

The proposed source localization technique aims to estimate the  $M \times N$  adaptive filter bank matrix  $\mathbf{W}$  and  $N \times L$  parameterized matrix  $\mathbf{R}$  via

$$\{\hat{\mathbf{R}}, \hat{\mathbf{X}}\} = \arg \min_{\mathbf{R}, \mathbf{X}} \|\mathbf{R} - \mathbf{W}^H \mathbf{X}\|_F^2 \quad \text{s.t. } \mathbf{W} \mathbf{W}^H = \mathbf{I}_M \quad (6)$$

We introduce an iterative optimization strategy to solve (6)

At the  $i$ -th iteration:

- 1) updating  $\mathbf{W}$ :  $\hat{\mathbf{W}}_{i+1} = \arg \min_{\mathbf{W}} \|\hat{\mathbf{R}}_i - \mathbf{W}^H \mathbf{X}\|_F^2 \quad \text{s.t. } \mathbf{W} \mathbf{W}^H = \mathbf{I}_M$
- 2) Dictionary learning:  $\hat{\mathbf{R}}_{i+1} = \hat{\mathbf{W}}_{i+1}^H \mathbf{X}$

A re-parameterization method is employed to address the non-convex problem.

### 3. Source Association via Subspace Technique

Performing eigen-value decomposition (EVD) on  $\mathbf{R}_x$  results in

$$\mathbf{R}_x = E \{ \mathbf{x}(t) \mathbf{x}^H(t) \} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H \quad (7)$$

Since the signal-subspace and the noise-subspace are orthogonal to each other, we have

$$\| \mathbf{U}_n^H \mathbf{A} \theta_k \mathbf{c}_k \|_2 = 0, \quad k = 1, 2, \dots, K \quad (8)$$

Non-full rank  $\rightarrow$  result in a peak for  $f_1(\theta) = \frac{\det(\mathbf{A}_\theta^H \mathbf{A}_\theta)}{\det(\mathbf{A}_\theta^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{A}_\theta)}$

$\Theta_g$ : the  $g$ th source association set.

$g = 1, 2, \dots, G$ ,  $G$  is the number of possible combinations.

$\mathbf{z}_g$ : corresponding number index vector.

$z_{g,h}$ : the number of sources that have  $h$  propagation paths in the  $g$ th association set with  $h = 1, 2, \dots, D_K - K + 1$ .

The SA can be achieved by selecting one that minimizes  $F_{\Theta_w}(\theta) = \sum_{\theta \in \Theta_w} f_1^{-1}(\theta)$  among all the  $G$  combinations.

## Simulation and experiment results

### Radar System and Scenario Parameter

A 16-sensor uniform linear array (ULA) with half-wavelength element spacing is employed.

sensor number: 16	number of snapshots: 128	Monte Carlo trials: 500
$\theta_1 = [-25^\circ, -16^\circ]^T$ , $\theta_2 = 8^\circ$ , and $\theta_3 = [-10^\circ, 25^\circ]^T$		
$c_1 = [1, 0.8 \exp(j1.8\pi)]^T$ , $c_2 = 1$ , and $c_3 = [1, 0.7 \exp(j1.2\pi)]^T$		

### 1. Simulation Results

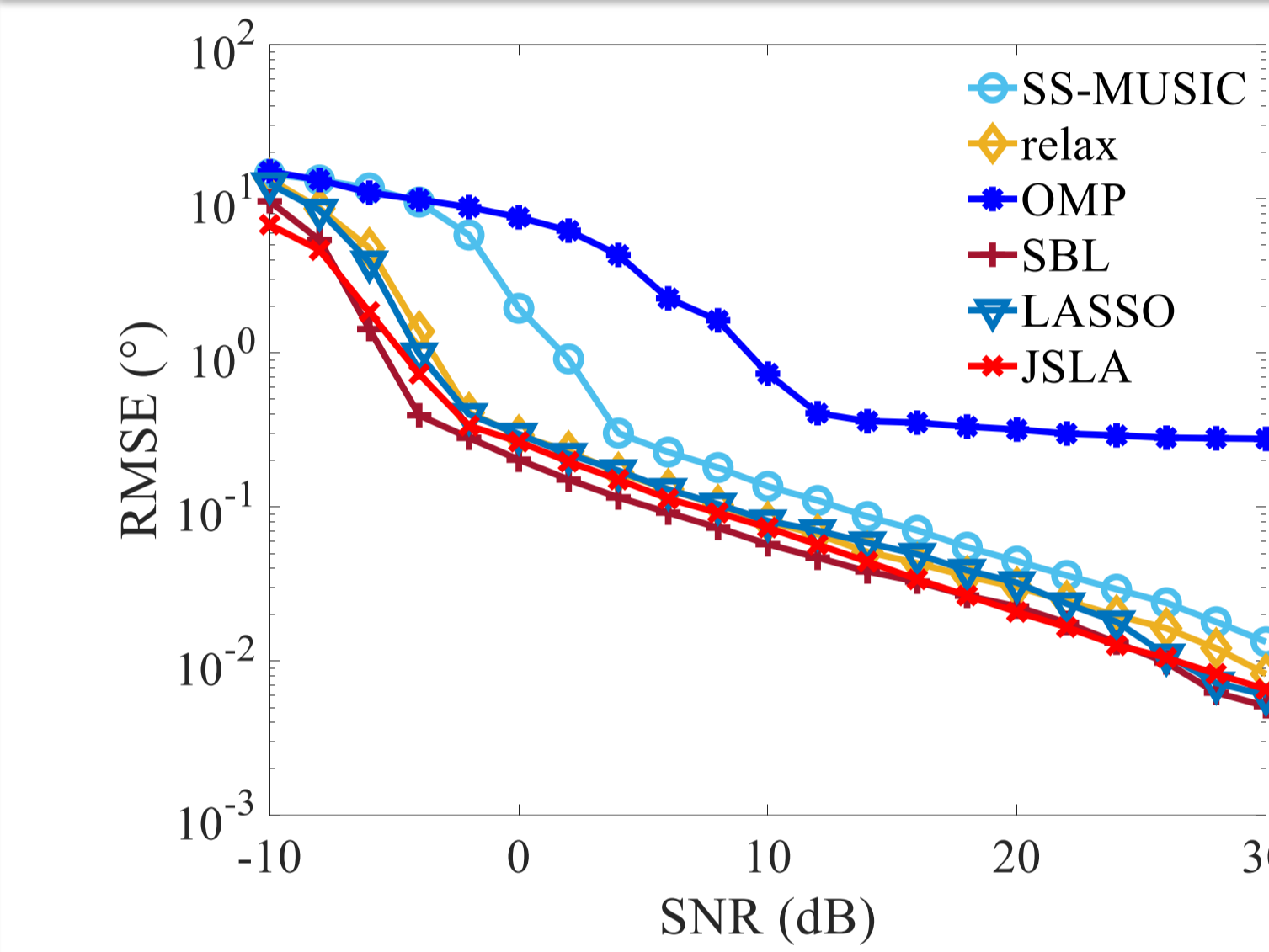


Fig. 2 Variation of RMSE in terms of DOA estimate.

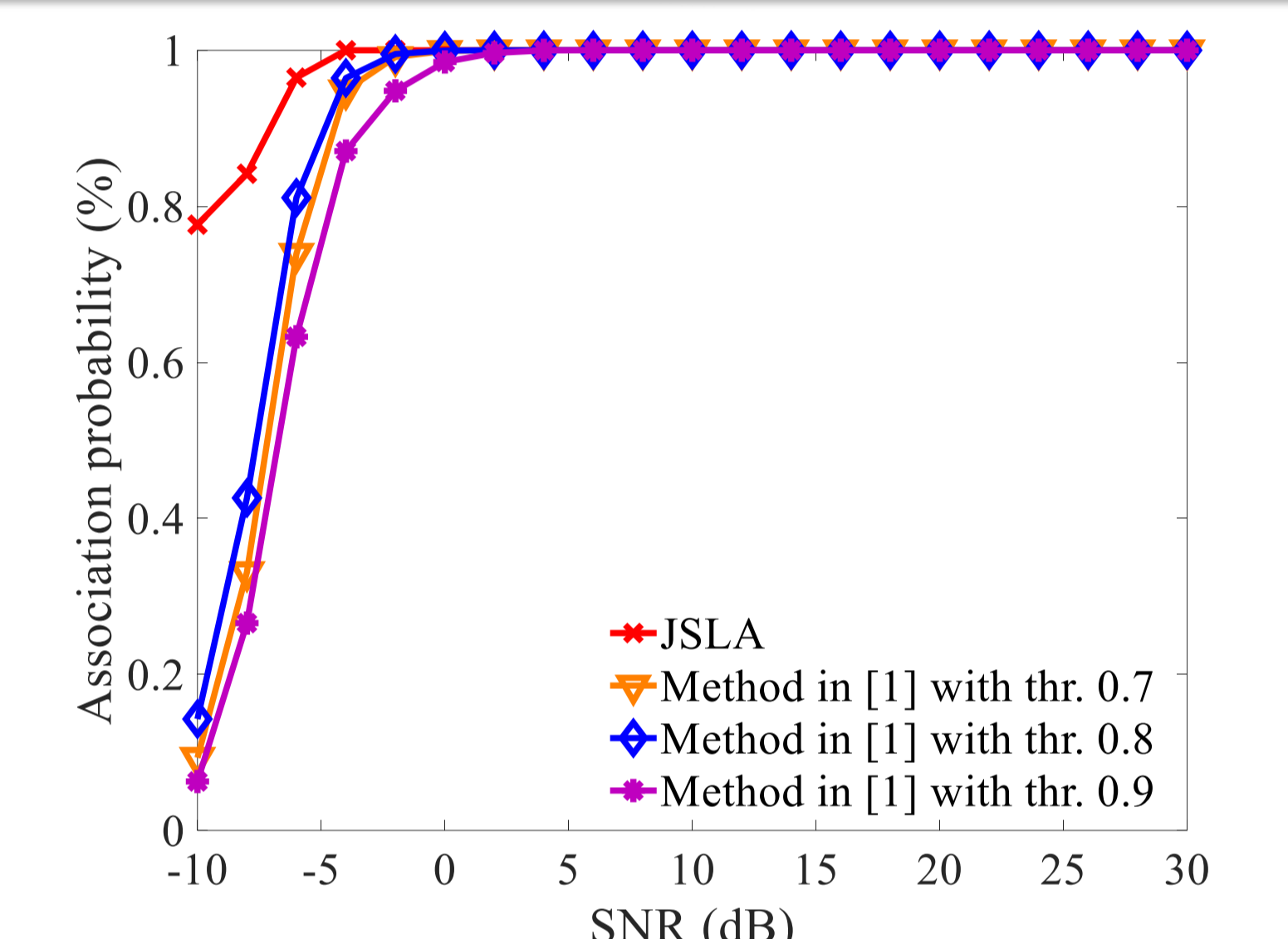


Fig. 3 The correct association probability versus SNR.

### 2. Measured Data Validation

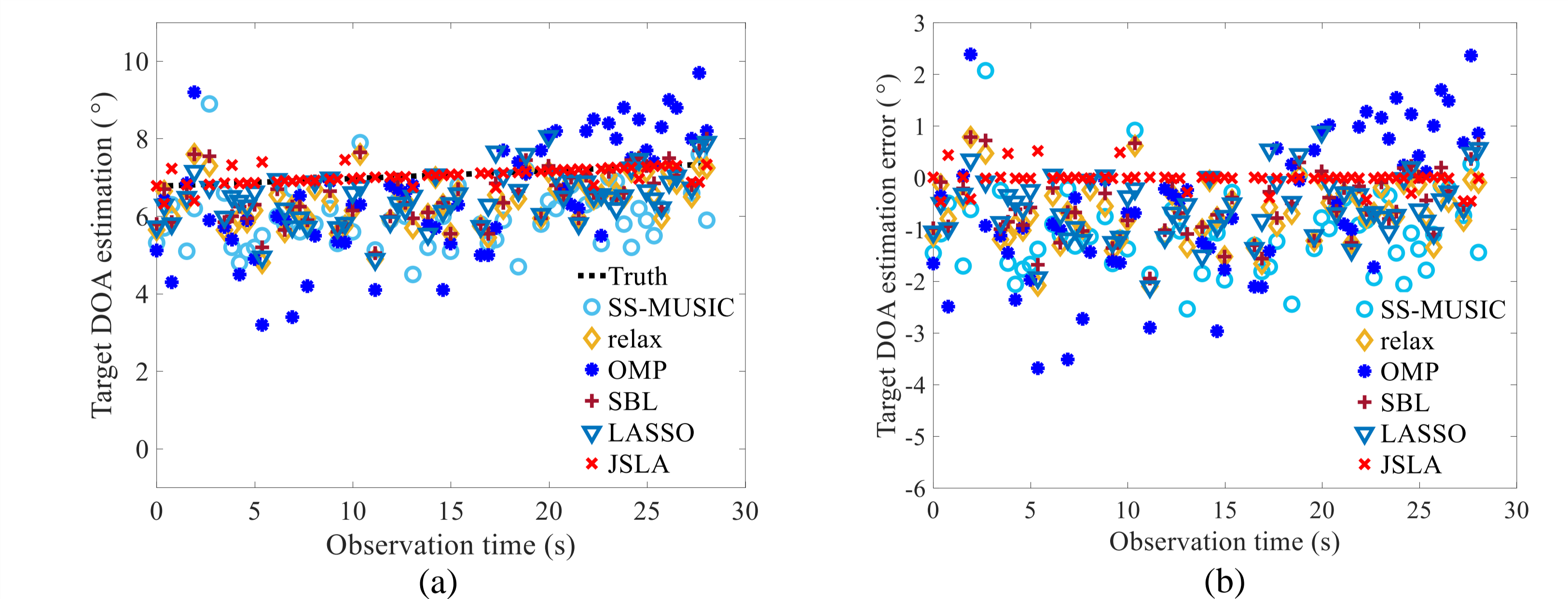


Fig. 4 Target location parameter estimate result versus the observation time. (a) DOA estimate produced by different methods. (b) estimate error of target DOA produced by different methods.

These results indicate that JSLA outperforms the baseline methods in terms of estimation accuracy and robustness.

## References

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