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# Multivariate Multiscale Cosine Similarity Entropy (Paper ID: 4332)

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# Introduction and background: Complexity

- Structural complexity is a key feature for characterizing the properties of nonlinear dynamic and chaotic systems.
- Traditionally, the loss of complexity is manifested as the decrease of randomness and irregularity, according to information theory (COSTA; PENG; GOLDBERGER, 2008).
- In line with the traditional Complexity Loss Theory, the highest degree of complexity indicates normal and healthy condition in physical and physiological systems (LIPSITZ; GOLDBERGER, 1992).
- New theory has been proposed that pathology exhibits an increase in self-correlated complexity, that is, structural complexity (CHANWIMALUEANG; MANDIC, 2017).

## Introduction and background: Complexity

- Analysis methods of complex dynamics:
  - Entropy (SHANNON, 1948);
  - Recurrence Plots (JR; ZBILUT, 1994);
  - Fractal Dimension (HIGUCHI, 1988);
  - Detrended Fluctuation Analysis (HAUSDORFF et al., 1997);
- Among existing approaches, entropy-based methods are the commonly investigated considering its following advantages (WANG; SI; LI, 2020; WANG; SI; LI, 2021):
  - independence on prior knowledge;
  - without requirement of pre-processing procedures;
  - easy implementation;

# Introduction and background: Entropy

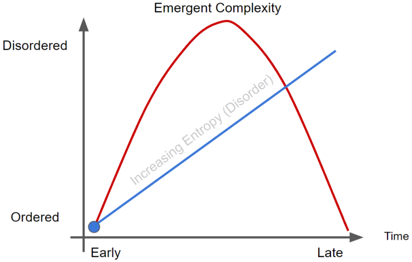


**Figure 1:** Complexity-Entropy curves (https://medium.com/swlh/what-is-entropy-an-exploration-of-life-time-and-immortality-85488e1eea36)

Traditional entropy measures, such as Sample Entropy and its enhanced versions, are designed to give a quantification of irregularity. However, several problems exist:

- shortage of sample points;
- reduced stability for multi-channel analysis;
- limited selection of embedding dimension;
- unknown optimal scales when examining long-term correlation.

# Aims and Objectives

- Extend the single channel Cosine Similarity Entropy (CHANWIMALUEANG; MANDIC, 2017) to cater for multichannel data, via Multivariate Multiscale Cosine Similarity Entropy (MMCSE).
- Explore the property of self-correlation and structural complexity at low scales based on MMCSE.
- Examine and compare the performance of the proposed Multivariate Multiscale Cosine Similarity Entropy (MMCSE) and the standard Multivariate Multiscale Sample Entropy (MMSE) (AHMED; MANDIC, 2011) on physical signals.

## Algorithm

1. Normalize the original multi-variate data sets by subtracting the median.
2. Perform the Coarse Graining process to obtain the scaled multi-channel time series $\{y_{k,i}^{(\tau)}\}_{j=1}^{N/\tau}$, according to

$$y_{k,i}^{(\tau)}(j) = \frac{1}{\tau} \sum_{i=j-\tau/2-1}^{j+\tau/2-1} x_k(i),\ 1 \leq j \leq \frac{N}{\tau},\ k = 1, 2, \ldots, P.$$

3. Form the Composite Delay Vectors (CDVs), $\mathrm{Y}_M(i)$, according to the embedding dimension, $M$, and time lag, $L$, in the form

$$\begin{aligned}\mathrm{Y}_M(i) = [&y_{1,i}, y_{1,i+l_1}, \ldots, y_{1,i+(m_1-1)l_1},\\ &y_{2,i}, y_{2,i+l_2}, \ldots, y_{2,i+(m_1-1)l_2},\\ &\vdots\\ &y_{p,i}, y_{p,i+l_p}, \ldots, y_{p,i+(m_1-1)l_p}]\end{aligned}$$

# Algorithm

4. Calculate the angular distance between pairwise CDVs $\mathrm{Y}_m(i)$ and $\mathrm{Y}_m(j)$ based on Cosine Similarity, that is,

$$d_m(i,j) = \frac{1}{\pi}\cos^{-1}\left(\frac{\mathbf{y}_m(i)\cdot\mathbf{y}_m(j)}{|\mathbf{y}_m(i)||\mathbf{y}_m(j)|}\right),\ i \neq j.$$

5. Compute the number of similar patterns defined as similar pairs, $B^r_M(i)$, that satisfy the criterion $d_M(i,j) \leq r$.
6. Compute the local probability of $B^r_M(i)$ by $C^r_M(i) = \frac{B^r_M(i)}{N-n-1}$, where $n = max(M) * max(L)$.
7. Compute the global probability of $B^r_M(i)$ as $\Phi^r_M = \frac{\sum_{i=1}^{N-n} C^r_M(i)}{N-n}$.
8. Multivariate Multiscale Cosine Similarity Entropy is defined as

$$MMCSE(M,L,r,N) = -[\Phi^r_M \log_2 \Phi^r_M + (1-\Phi^r_M)\log_2(1-\Phi^r_M)].$$

# Selection of Tolerance, *r*

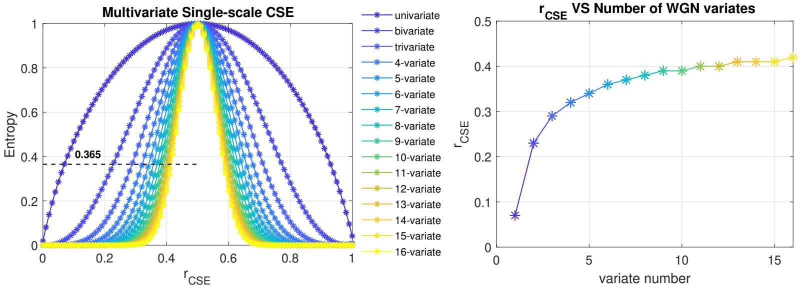


**Figure 2:** Behaviour of the Multivariate Single-scale CSE on the estimation of White Gaussian Noise (WGN), as a function of the tolerance, *r*.

- The right graph gives the tolerance, *r*, as a function of the number of variates that $r_{cse} = -0.4(p^{-0.71}) + 0.47$, where $p$ denotes the number of variates.
- The selection of tolerance, *r*, in CSE is independent of the amplitude or the the variance of the raw data.

# Selection of Embedding Dimension, $m$

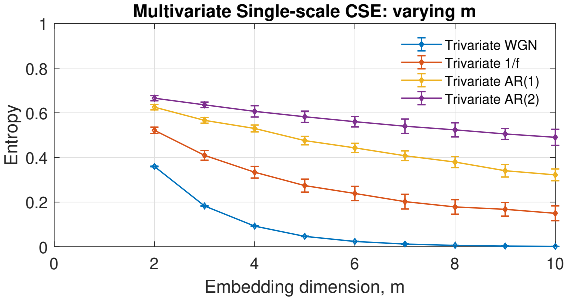


**Figure 3:** Multivariate Single-scale CSE as a function of the embedding dimension, m.

The angular distance based Cosine Similarity Entropy exhibits an order of magnitude higher temporal resolution in complexity estimation compared to standard Sample Entropy.

Synthetic Signals:

- White Gaussian Noise (WGN)
- 1/f noise
- AR(1): $x(t) = 0.9x(t-1) + \varepsilon(t)$
- AR(2): $x(t) = 0.85x(t-1) + 0.1x(t-2) + \varepsilon(t)$, where $\varepsilon \sim \mathcal{N}(0, 1)$

# Effect of Data Length, $N$, on performance

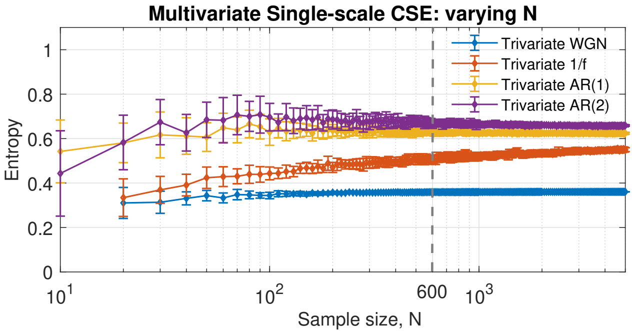


Figure 4: Multivariate Single-scale CSE as a function of the number of sample points, $N$, evaluated for the same four synthetic models. The default parameters are set as $m = 2$, $r = 0.287$, and $l = 1$.

The angular-based CSE requires much less sample points than the amplitude-based SampEn, and this property is further highlighted and improved in the multivariate case.

# Complexity profiles on synthetic data

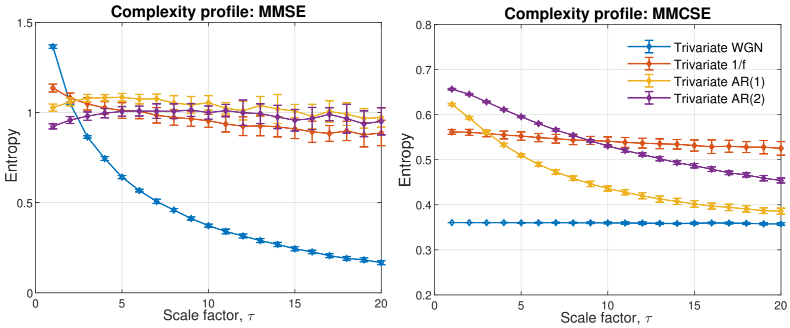


Figure 5: Complexity profiles of the MMSE and MMCSE algorithm.

The proposed Multivariate Multiscale Cosine Similarity Entropy yields stable estimates at high temporal scales, thus making it possible to examine structural complexity of physical processes with long-range correlations.

# Performance of standard MMSE on gait dynamics

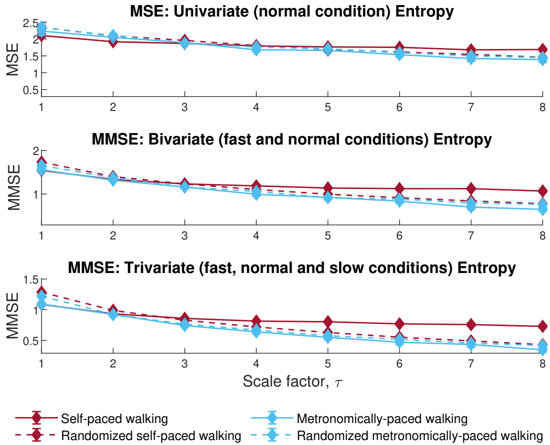


Figure 6: Multivariate Multiscale Sample Entropy.

- Univariate MSE yields a non-significant discrimination between the two conditions.
- Multi-variate cases were able to separate the conditions only at large scales, whereas the constrained condition wrongly suggested lower complexity than the uncorrelated randomized signals.

# Performance of proposed MMCSE on gait dynamics

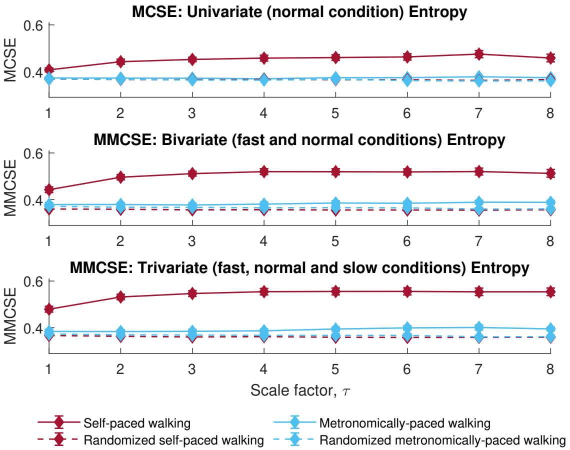


Figure 7: Multivariate Multiscale Cosine Similarity Entropy.

- The constrained condition exhibits less structure than the unconstrained condition.
- The Multivariate-MCSE achieved a good structure separation of all real signals from their randomized versions, which guarantees physically meaningful estimation for quantifying structural complexity real world signals.

# Conclusion

- This work has extended the univariate Cosine Similarity Entropy (CSE) method to the multivariate case, to provide efficient quantification of structural complexity of real world data.
- The proposed MMCSE method has been shown to exhibit a valid estimation of the structure and long-term correlation present in multichannel signals, with higher stability at large scales and an order of magnitude lower requirement on data length compared to the existing MMSE methods.
- The performance of MMCSE has been examined on four synthetic benchmark signals as well as on real world stride dynamical signals, with MMCSE showing an improved separation of constrained walking conditions from unconstrained conditions and uncorrelated randomized time series.

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