



Electronics Research Institute
Sharif University of Technology



Multi-Head ReLU Implicit Neural Representation Networks

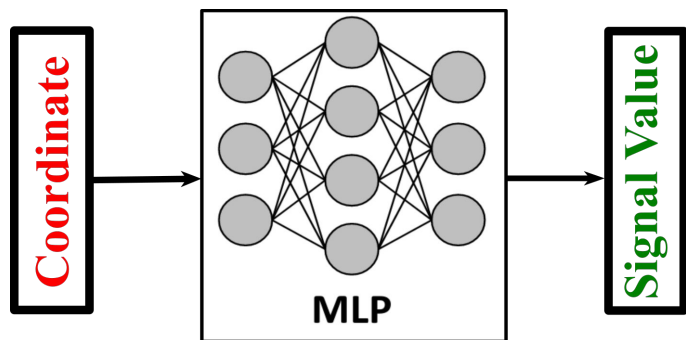
Authors: Arya Aftab, Alireza Morsali, Shahrokh Ghaemmaghami

Presented by: Arya Aftab

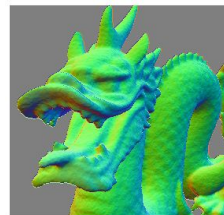
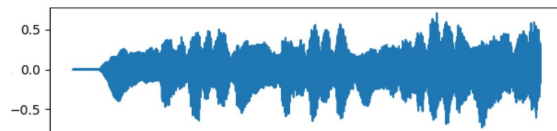


What is the implicit neural representation?

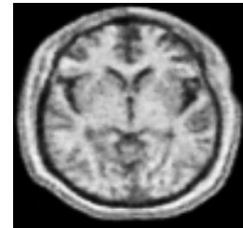
$$S = f(\theta, x)$$



Speech:

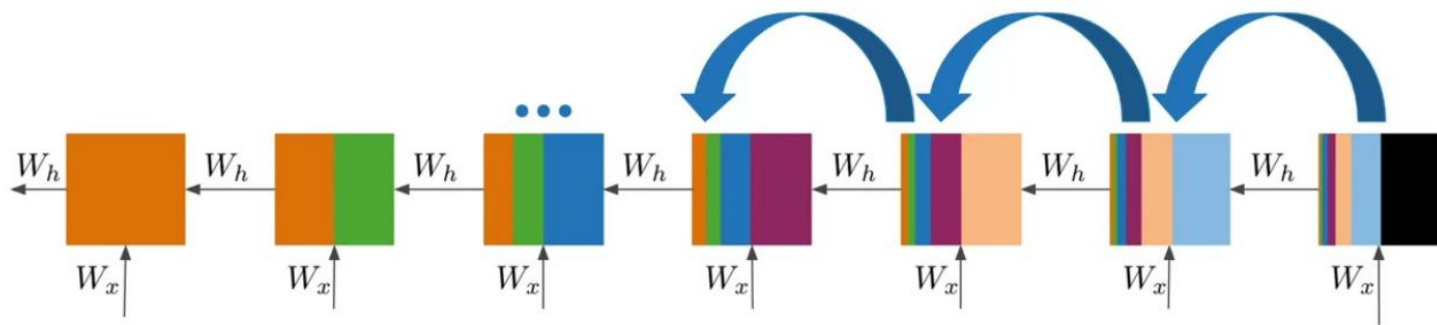
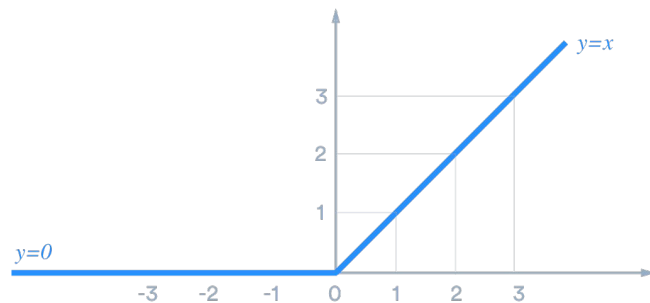


Vision



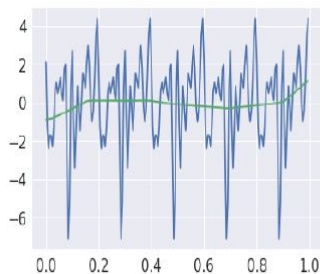
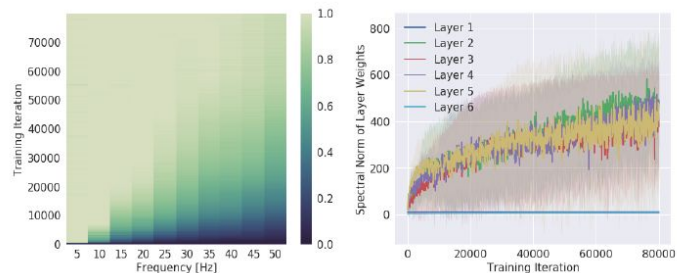
ReLU Networks

$$f(x) = \max(0, x)$$

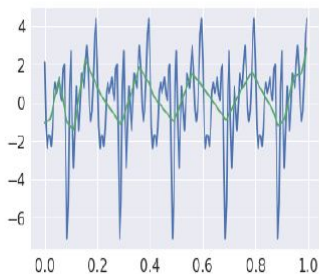


What is the problem with ReLU networks?

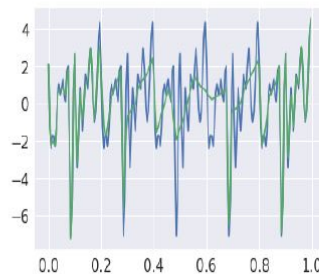
Spectral bias



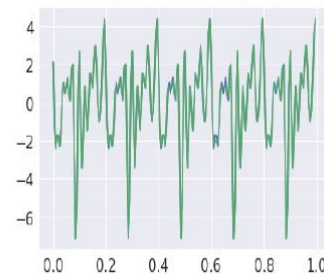
(a) Iteration 100



(b) Iteration 1000



(c) Iteration 10000



(d) Iteration 80000

M. Tancik, P. P. Srinivasan, B. Mildenhall, S. Fridovich-Keil, N. Raghavan, U. Singhal, R. Ramamoorthi, J. T. Barron, and R. Ng, "Fourier features let networks learn high frequency functions in low dimensional domains," in Conf. Neural Inf. Process. Syst. (NeurIPS), June 2020.

How to Solve this problem?

- **Positional encoding**

$$\gamma(p) = (\sin(2^0 \pi p), \cos(2^0 \pi p), \dots, \sin(2^{L-1} \pi p), \cos(2^{L-1} \pi p)) \quad \text{Eq. 1}$$

- **Fourier Features**

$$\gamma(\mathbf{v}) = [a_1 \cos(2\pi \mathbf{b}_1^T \mathbf{v}), a_1 \sin(2\pi \mathbf{b}_1^T \mathbf{v}), \dots, a_m \cos(2\pi \mathbf{b}_m^T \mathbf{v}), a_m \sin(2\pi \mathbf{b}_m^T \mathbf{v})]^T \quad \text{Eq. 2}$$

- **Sine activation function**

$$\gamma(\mathbf{v}) = \sin(W^T \mathbf{v} + b) \quad \text{Eq. 3}$$

Our approach

Localization

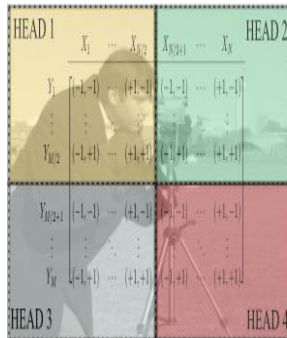
$$\Phi(x(r), y(c)) = \mathbf{I}[r, c] \quad (\text{Eq. 1})$$

$$\mathcal{D} = \left\{ \left((x(r), y(c)), \mathbf{I}[r, c] \right) \right\}_{r,c=1}^{N_x, N_y} \quad (\text{Eq. 2})$$

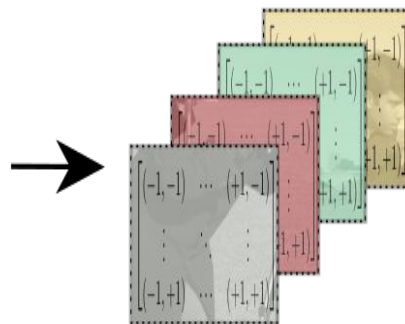
$$\mathbf{I}_{l,k}[\hat{r}, \hat{c}] = \mathbf{I}[\hat{N}_h(l-1) + \hat{r}, \hat{N}_w(k-1) + \hat{c}] \quad (\text{Eq. 3})$$

$$\phi_{l,k}(\hat{x}(\hat{r}), \hat{y}(\hat{c})) = \mathbf{I}_{l,k}[\hat{r}, \hat{c}] \quad (\text{Eq. 4})$$

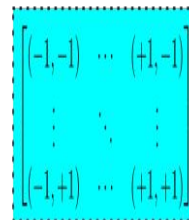
Whole Input Space



Head Input Space

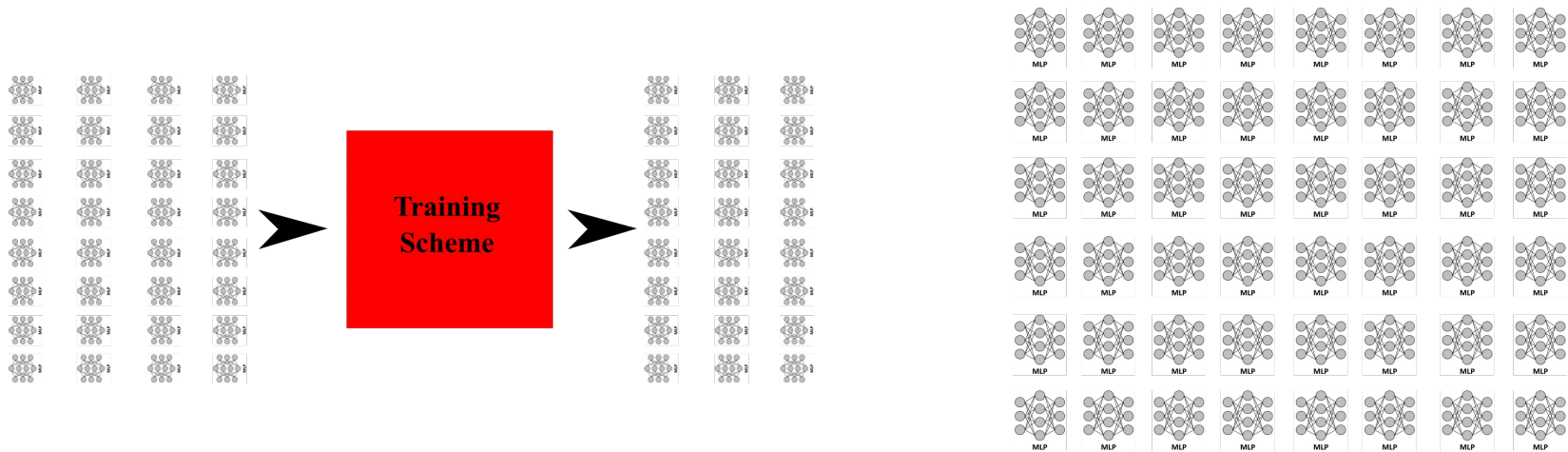


Joint Input Space



Challenge in our approach

- High number of small networks
- Difficulty in training the whole structure

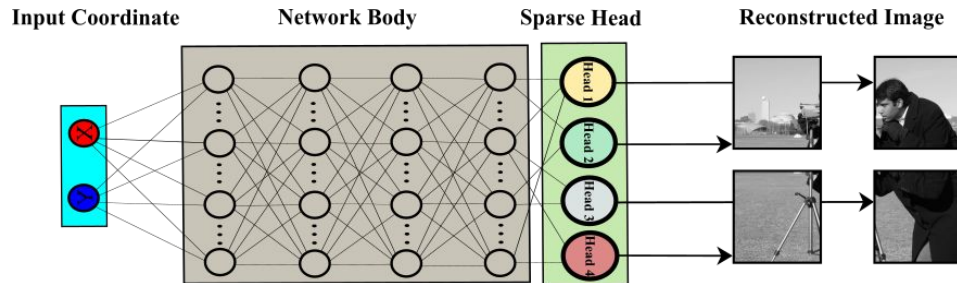


How to solve this Challenge?

- Share network body between small networks
- Consider all heads as a single layer
- Use a sparse layer instead of a dense layer for the heads

$$\phi_{l,k}(\hat{x}, \hat{y}) = \tau_{l,k}(\psi(\hat{x}, \hat{y})) \quad (\text{Eq. 1})$$

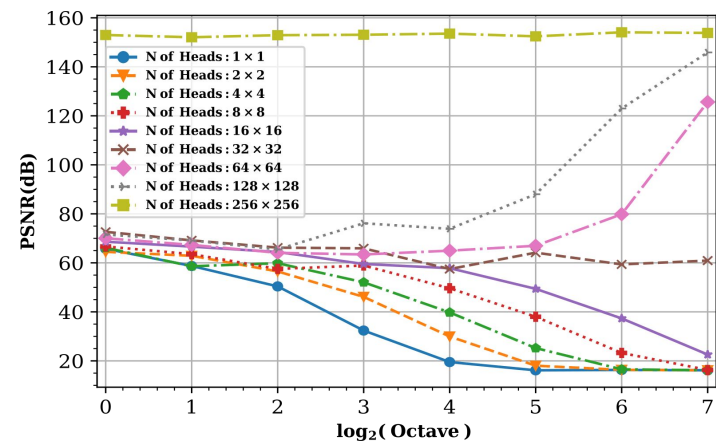
$$\min_{\tau_{l,k}(\cdot), \psi(\cdot)} \sum_{\hat{r}=1}^{\hat{N}_h} \sum_{\hat{c}=1}^{\hat{N}_w} \sum_{l=1}^{H_x} \sum_{k=1}^{H_y} \left(\tau_{l,k}(\psi(\hat{x}(\hat{r}), \hat{y}(\hat{c}))) - \mathbf{I}_{l,k}[\hat{r}, \hat{c}] \right)^2 \quad (\text{Eq. 2})$$



Results

Spectral bias

Train with Perlin noise



a: Octave=1

b: Octave=2

c: Octave=4

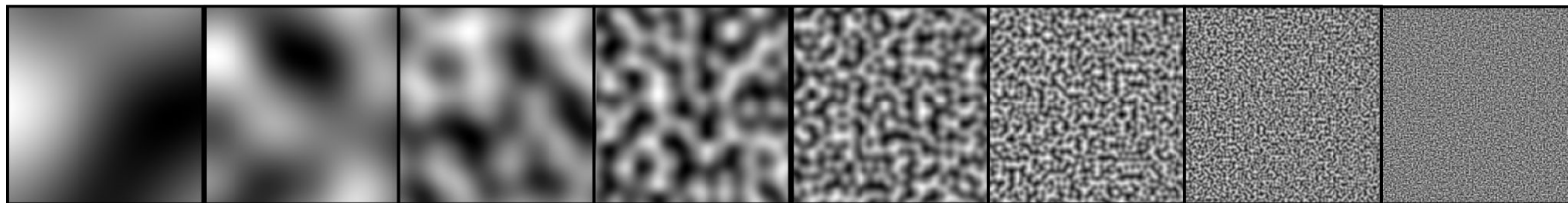
d: Octave=8

e: Octave=16

f: Octave=32

g: Octave=64

h: Octave=128

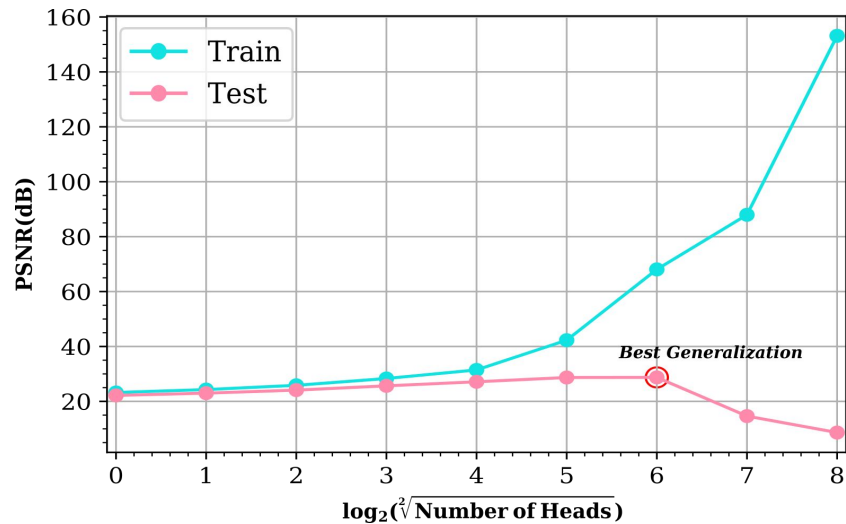


Results

Generalization ability

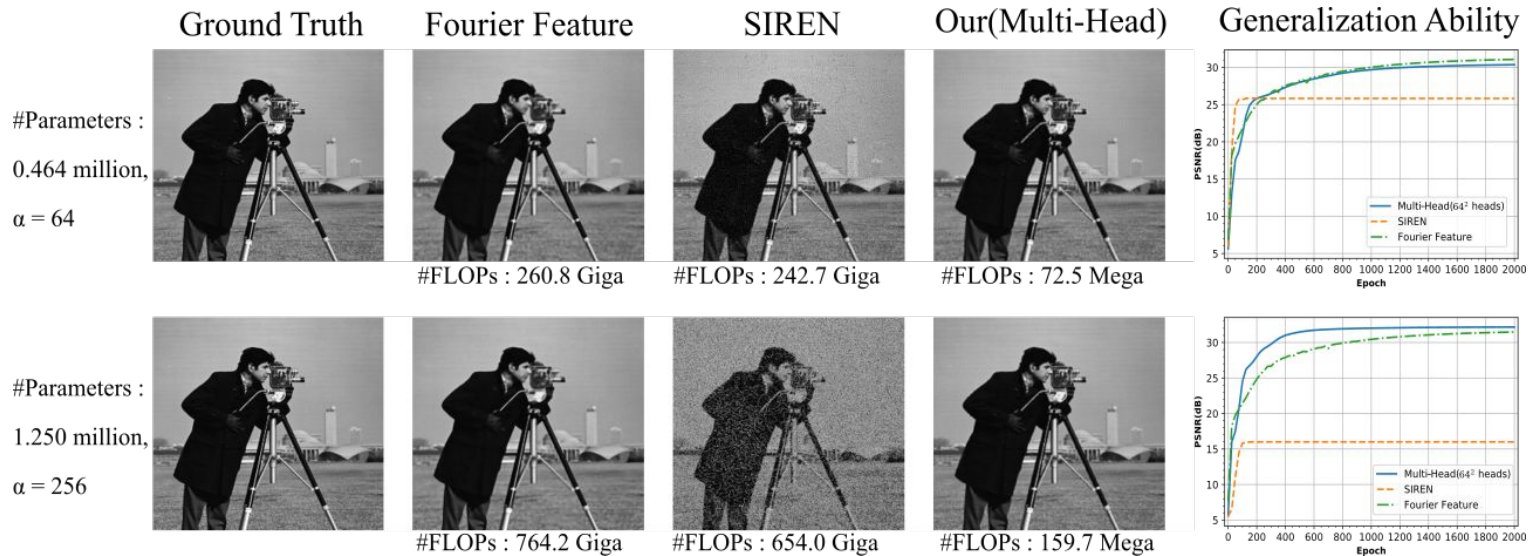
Train with down-sampled image: 256*256

Evaluate with original image: 512*512



Results

Comparison



Conclusions

- We proposed a novel structure that tackle spectral bias.
- The proposed structure has a much lower computational cost, as compared to other methods.
- Due to the shrinking input space with the increasing number of heads, training of the proposed structure takes less time than the time needed in existing methods.
- Experimental results show that the performance of our model is comparable to that of state-of-the-art methods.

References

- 1-M. Tancik, P. P. Srinivasan, B. Mildenhall, S. Fridovich-Keil, N. Raghavan, U. Singhal, R. Ramamoorthi, J. T. Barron, and R. Ng, “Fourier features let networks learn high frequency functions in low dimensional domains,” in Conf. Neural Inf. Process. Syst. (NeurIPS), June 2020.
- 2-V. Sitzmann, J. Martel, A. Bergman, D. Lindell, and G. Wetzstein, “Implicit neural representations with periodic activation functions,” in Conf. Neural Inf. Process. Syst. (NeurIPS), June 2020
- 3-B. Mildenhall, P. P. Srinivasan, M. Tancik, J. T. Barron, R. Ramamoorthi, and R. Ng, “NERF: Representing scenes as neural radiance fields for view synthesis,” in European Conf. Comput. Vis. (ECCV), Aug. 2020, pp. 405–421.
- 4-R. Arora, A. Basu, P. Mianjy, and A. Mukherjee, “Understanding deep neural networks with rectified linear units,” arXiv preprint arXiv:1611.01491, 2016.
- 5-N. Rahaman, A. Baratin, D. Arpit, F. Draxler, M. Lin, F. Hamprecht, Y. Bengio, and A. Courville, “On the spectral bias of neural networks,” in Int. Conf. Mach. Learn. (ICML). PMLR, July 2019, pp. 5301–5310.