ROBUST ADAPTIVE BEAMFORMING BASED ON POWER METHOD PROCESSING AND SPATIAL SPECTRUM MATCHING

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Introduction

Adaptive beamforming aims to extract the signal from a certain direction while suppressing interference and noise. Wireless communications, sonar and radar,…

Problems:

- Violation of the underlying assumptions on the environment, sources, or sensor array
- Small training sample size
- Presence of the desired signal in the training data
- Steering vector mismatches of the SOI
- Look direction errors, local scattering, array geometry distortions and array calibration errors
- Using the sample covariance matrix (SCM) instead of interference plus noise covariance (INC) matrix

Introduction

Solutions: Various adaptive beamformers have been developed

- Diagonal Loading
- Eigenspace-Based
- Worst-Case Optimization
- The uncertainty set-based algorithms
- Adaptive beamformings that remove the effect of the SOI component from the covariance matrix by reconstructing the INC matrix

$$\hat{\mathbf{R}}_{i+n} = \int_{\bar{\Theta}} \hat{P}(\theta)\,\mathbf{a}(\theta)\mathbf{a}^H(\theta)\,d\theta + \sigma_n^2\mathbf{I} \qquad (1)$$

- Reconstruct the array covariance matrix based on the theoretical definition

$$\hat{\mathbf{R}} = \hat{\mathbf{R}}_s + \hat{\mathbf{R}}_{i+n} = \sigma_1^2\mathbf{a}(\theta_1)\mathbf{a}^H(\theta_1) + \sum_{l=2}^{L}\sigma_l^2\mathbf{a}(\theta_l)\mathbf{a}^H(\theta_l) + \sigma_n^2\,\mathbf{I} \qquad (2)$$

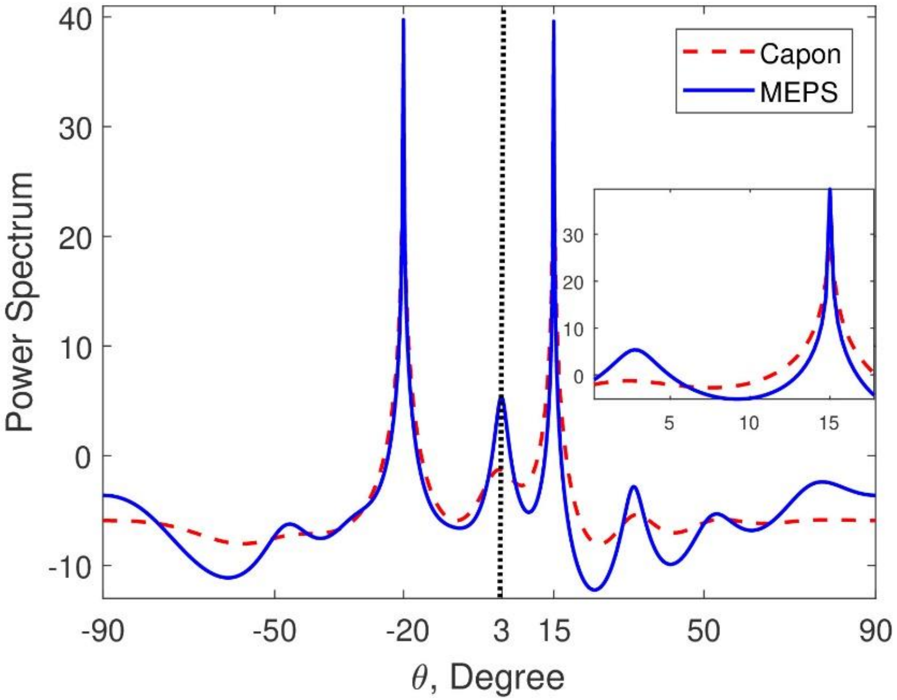
Maximum Entropy Power Spectrum (MEPS)

Use of the spatial spectrum distribution over all possible directions and coarse estimates of the angular regions where the desired signal and the interferers lie.

$$P_{meps} = \frac{1}{\varepsilon_p \left| \mathbf{a}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{b}_1 \right|^2} \qquad (3)$$

$$\mathbf{b}_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix} \quad , \qquad \varepsilon_p = \frac{1}{\mathbf{b}_1^T \hat{\mathbf{R}}^{-1} \mathbf{b}_1}$$

$$P_{Capon} = \frac{1}{\mathbf{a}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta)}$$



[1] S. Mohammadzadeh, V. H. Nascimento, R. C. de Lamare and O. Kukrer, "Maximum Entropy-Based Interference-Plus-Noise Covariance Matrix Reconstruction for Robust Adaptive Beamforming," in IEEE Signal Process. Lett., vol. 27, pp. 845-849, 2020.
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Proposed INC matrix reconstruction

Covariance matrix Reconstructed as

$$\hat{\mathbf{R}}_{i+n} \approx \sum_{i=1}^{Q} \frac{\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)}{\varepsilon_p \left|\mathbf{a}^H(\theta_i)\hat{\mathbf{R}}^{-1}\mathbf{b}_1\right|^2} \Delta\theta \qquad (4)$$

The summation is over the whole angular sector $\bar{\Theta}$ (requiring Q sampling points), $Q \gg M$

In the proposed method we only make summation over the small angular sector $\bar{\Theta}_l$ (thus requiring less sampling points) of the lth interference to obtain the lth interference covariance matrix as

$$\hat{\mathbf{C}}_l \approx \sum_{l_j=1}^{J_l} \frac{\mathbf{a}(\theta_{l_j})\mathbf{a}^H(\theta_{l_j})}{\varepsilon_p \left|\mathbf{a}^H(\theta_{l_j})\hat{\mathbf{R}}^{-1}\mathbf{b}_1\right|^2} \Delta\theta \qquad (5)$$

Where $\{\theta_{lj} \in \bar{\Theta}_{ld}\}_{j=1}^{J_l}$, $\bar{\Theta}_{ld}$ is a discretization of the angular sector $\bar{\Theta}_l$ with $J_l \ll Q$

Since only one interference signal is assumed in the uncertainty region ($\bar{\Theta}_l$), the principal eigenvector of the constructed matrix will be the SV we are looking for.

Proposed INC matrix reconstruction $\hat{\mathbf{R}}_{i+n}$

Interference Power & Steering Vector Estimation:

The principal eigenvalue and corresponding eigenvector of matrix $\mathbf{A}$ *or* $(\hat{\mathbf{C}}_l)$ can be computed by power method [3]. Utilizing this theorem, we achieve the largest eigenvalue and the principal eigenvector of the reconstructed interference matrices that retains as much as possible the power and SV of the interferences.

we obtain the *l*th interference covariance matrix term $\sigma_l^2 \mathbf{a}(\theta_l)\mathbf{a}^H(\theta_l) \quad, l = 2, \ldots, L$

Since each region $\bar{\Theta}_l$ is small, $\hat{\mathbf{C}}_l$ will be approximately a rank-one matrix, and thus the convergence in algorithm should be fast

Algorithm 1 Power Method

Initialization: $m_0 = 1$, $\mathbf{u}_0 = [1, 1, \cdots, 1]^T$, δ.
Compute $\mathbf{v}_0 = \mathbf{A}\mathbf{u}_0$
Compute $m_1 = \|\mathbf{v}_0\|_\infty$
Calculate error $= |m_0 - m_1|$
$k \longleftarrow 1$
While error $> \delta$ **do**;
  $\mathbf{v}_k = \mathbf{A}\mathbf{v}_{k-1}$,
  $m_k = \|\mathbf{v}_k\|_\infty$
  $\mathbf{v}_{k-1} = \frac{\mathbf{v}_k}{m_k}$
  error $= |m_{k-1} - m_k|$
  $m_{k-1} = m_k$
  $k = k + 1$
End

[3] William Ford, Numerical linear algebra with applications: Using MATLAB, Academic Press, 2014

Proposed INC matrix reconstruction

Noise power Estimation:

In the noise region $\bar{\Theta}_n$, which is the complement of $\bar{\Theta}_l \cup \bar{\Theta}_s$, we employ the maximum entropy power spectrum distribution in (1) to calculate the noise power as the average

$$\hat{\sigma}_n^2 = \sum_{t=1}^{T} \frac{1}{\varepsilon_p \left| \mathbf{a}^H(\theta_t)\hat{\mathbf{R}}^{-1}\mathbf{b}_1 \right|^2} \qquad (6)$$

where θ_t is a discrete sample point in $\bar{\Theta}_n$, T is the number of sample points.

Then, the INC matrix is reconstructed as :

$$\hat{\mathbf{R}}_{i+n} = \sum_{l=2}^{L} \hat{\sigma}_l^2 \hat{\mathbf{a}}(\theta_l)\hat{\mathbf{a}}^H(\theta_l) + \hat{\sigma}_n^2 \mathbf{I} \qquad (7)$$

Desired Signal Steering Vector estimation

Signal plus Noise covariance matrix Estimation $\hat{\mathbf{R}}_{s+n}$:

Assumptions:

- Θ_s is assumed to be distinguishable from the location of the interference signal.
- $\hat{\mathbf{P}}_{s+n}(\theta)$ is the calculated power spectrum by the signal plus noise covariance matrix $\hat{\mathbf{R}}_{s+n}$
- $\hat{\mathbf{P}}(\theta)$ is the calculated power spectrum by the sample covariance matrix $\hat{\mathbf{R}}$

The spectrum matching processing is utilized for signal-plus noise covariance matrix reconstruction as

$$\begin{aligned} &\min_{\hat{\mathbf{R}}_{s+n}} \left\| \hat{\mathbf{P}}_{s+n}(\theta) - \hat{\mathbf{P}}(\theta) \right\|_2 \\ &s.t. \left\| \hat{\mathbf{P}}_{s+n}(\theta) - \hat{\sigma}_n^2 \right\|_2 < \xi \\ &\hat{\mathbf{R}}_{s+n} \in \mathbf{S}_+^M \end{aligned} \qquad (8)$$

Objectives:

(i) Minimize the difference between $\hat{\mathbf{P}}_{s+n}(\theta)$ and $\hat{\mathbf{P}}(\theta)$ in the angular sector of Θ_s.
(ii) Constrain the difference between the average noise power $\hat{\sigma}_n^2$ and the spatial spectrum of $\hat{\mathbf{P}}_{s+n}(\theta)$ in the angular sector.

Desired Signal Steering Vector estimation

$$\min_{\hat{\mathbf{R}}_{s+n}} \left(\int_{\Theta_s} \left| \frac{1}{\varepsilon_p \left| \mathbf{a}^H(\theta)\hat{\mathbf{R}}_{s+n}^{-1}\mathbf{b}_1 \right|^2} - \frac{1}{\mathbf{a}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta)} \right| d\theta \right)^{1/2}$$

$$s.t. \int_{\bar{\Theta}} \left| \frac{1}{\varepsilon_p \left| \mathbf{a}^H(\theta)\hat{\mathbf{R}}_{s+n}^{-1}\mathbf{b}_1 \right|^2} - \hat{\sigma}_n^2 \right| d\theta)^{1/2} < \xi \qquad (9)$$

$$\hat{\mathbf{R}}_{s+n} \in \mathbf{S}_+^M$$

To simplify the calculation we choose a finite number of angles

$$\nu_j \in \Theta_s \; (j = 1, 2, \ldots, G)$$

$$\phi_i \in \bar{\Theta} \, (i = 1, 2, \ldots, Q)$$

Desired Signal Steering Vector estimation

$$\min_{\hat{\mathbf{R}}_{s+n}} \left\| \begin{pmatrix} \frac{1}{\varepsilon_p \left| \mathbf{a}^H(\nu_1)\hat{\mathbf{R}}_{s+n}^{-1}\mathbf{b}_1 \right|^2} \\ \vdots \\ \frac{1}{\varepsilon_p \left| \mathbf{a}^H(\nu_G)\hat{\mathbf{R}}_{s+n}^{-1}\mathbf{b}_1 \right|^2} \end{pmatrix} - \begin{pmatrix} \frac{1}{\mathbf{a}^H(\nu_1)\hat{\mathbf{R}}\,\mathbf{a}(\nu_1)} \\ \vdots \\ \frac{1}{\mathbf{a}^H(\nu_G)\hat{\mathbf{R}}\,\mathbf{a}(\nu_G)} \end{pmatrix} \right\|_2$$

$$s.t. \left\| \begin{pmatrix} \frac{1}{\varepsilon_p \left| \mathbf{a}^H(\phi_1)\hat{\mathbf{R}}_{s+n}^{-1}\mathbf{b}_1 \right|^2} \\ \vdots \\ \frac{1}{\varepsilon_p \left| \mathbf{a}^H(\phi_Q)\hat{\mathbf{R}}_{s+n}^{-1}\mathbf{b}_1 \right|^2} \end{pmatrix} - \mathbf{1}.\hat{\sigma}_n^2 \right\|_2 < \xi \qquad (10)$$

$$\hat{\mathbf{R}}_{s+n} \in \mathbf{S}_+^M$$

However, this problem is a nonconvex optimization problem since the objective function and the inequality constraint function are nonconvex functions.

Desired Signal Steering Vector estimation

$$\min_{\mathbf{D}_s} \left\| \mathrm{diag}(\mathbf{V}_s^H \mathbf{D}_s \mathbf{V}_s) - \mathrm{diag}(\mathbf{V}_s^H \hat{\mathbf{R}}^{-1} \mathbf{V}_s) \right\|_2$$

$$s.t. \left\| \mathrm{diag}(\mathbf{Y}_s^H \mathbf{D}_s \mathbf{Y}_s) - \mathbf{1}.(1/\hat{\sigma}_n^2) \right\|_2 < \xi \qquad (11)$$

$$\mathbf{D}_s \in \mathbf{S}_+^M$$

Where $\mathbf{V}_s = \left[\mathbf{a}(\nu_1), \mathbf{a}(\nu_2), \ldots, \mathbf{a}(\nu_G)\right]$ $\quad \mathbf{Y}_s = \left[\mathbf{a}(\phi_1), \mathbf{a}(\phi_2), \ldots, \mathbf{a}(\phi_Q)\right]$

The solution is $\mathbf{D}_s = \hat{\mathbf{R}}_{s+n}^{-1}$

To decrease affect of the noise components $\hat{\mathbf{R}}_s = \hat{\mathbf{R}}_{s+n}^{-1} - \hat{\sigma}_n^2 \mathbf{I}$ (12)

According to Algorithm 1 (Power Method):

$\hat{\sigma}_1^2$ and $\hat{\mathbf{a}}(\theta_1)$ are achieved.

$$\mathbf{w}_{pro} = \frac{\hat{\mathbf{R}}_{i+n}^{-1} \hat{\mathbf{a}}(\theta_1)}{\hat{\mathbf{a}}^H(\theta_1) \hat{\mathbf{R}}_{i+n}^{-1} \hat{\mathbf{a}}(\theta_1)} \qquad (13)$$

Summary of Proposed Method

Algorithm 2 Proposed PMP-SSM Adaptive Beamforming

Input: Array received data vector $\{\mathbf{x}(k)\}_{k=1}^{K}$,
Initialize: $m_0 = 1$, $\mathbf{u}_0 = [1, 1, \cdots, 1]^T$, δ;
Compute $\hat{\mathbf{R}} = (1/K)\sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k)$;
For $l = 2 : L$
  Construct $\hat{\mathbf{C}}_l$ using (5),
  Apply power method algorithm in (1) to $\hat{\mathbf{C}}_l$,
  Obtain $\hat{\sigma}_l^2$ and $\hat{\mathbf{a}}_l$,
End For
Compute estimated noise power using (6);
Construct INC matrix as $\hat{\mathbf{R}}_{\mathrm{i+n}} = \sum_{l=2}^{L} \hat{\sigma}_l^2 \hat{\mathbf{a}}_l \hat{\mathbf{a}}_l^{\mathrm{H}} + \hat{\sigma}_n^2 \mathbf{I}$.
Compute $\mathbf{D}_s = \hat{\mathbf{R}}_{\mathrm{s+n}}^{-1}$ using (11);
Estimate the desired signal covariance matrix, $\hat{\mathbf{R}}_{\mathrm{s}}$ utilizing (12),
Apply power method theorem in (1) to $\hat{\mathbf{R}}_{\mathrm{s}}$,
Obtain $\hat{\sigma}_1^2$ and $\hat{\mathbf{a}}_1$.
Design proposed beamformer using (13)
Output: Proposed beamforming weight vector $\mathbf{w}_{\mathrm{prop}}$.

Experiment Results

We evaluate INCPMP-SSM in the presence of look direction and model mismatches due to the sensor displacement errors. In this example, it is assumed that the DS and the interferers are uniformly distributed in $[-5^{\circ}, 5^{\circ}]$ while the difference between the actual and assumed SV is modeled as array geometry errors, assuming the sensor position is drawn uniformly from $[-0.05, 0.05]$ wavelength and the DoA of the DS and the actual sensor position changes from run to run while remaining constant over samples.

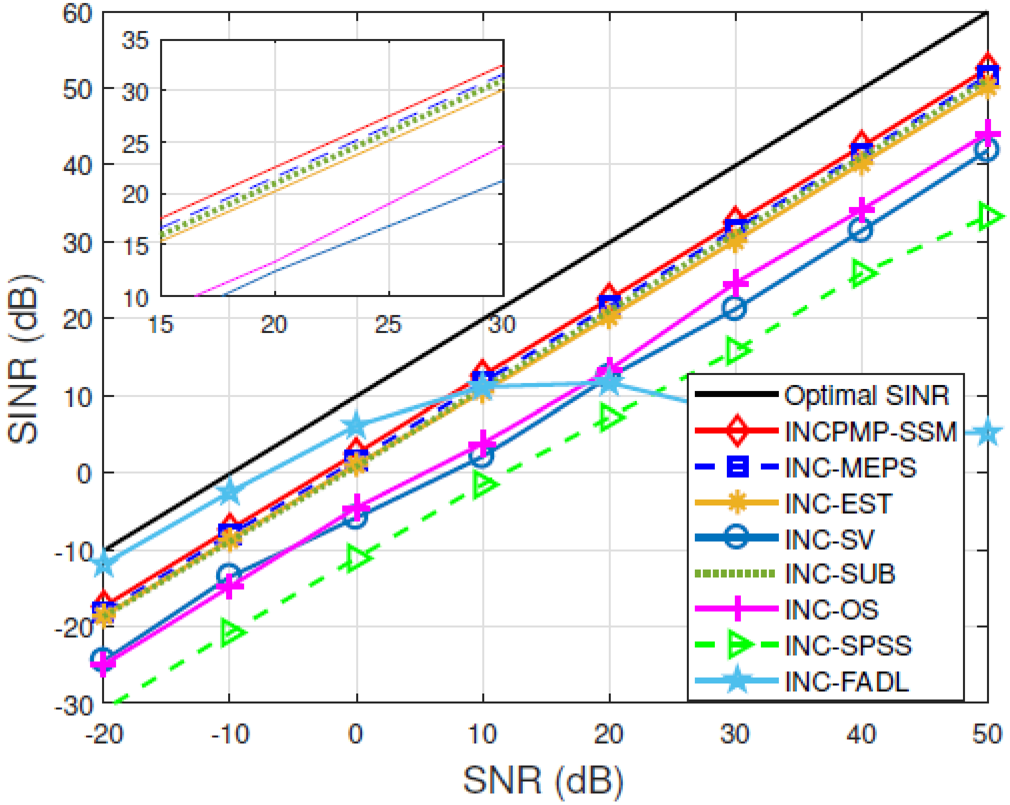

Fig. 1: Output SINR versus input SNR

Experiment Results

Fig. 2: All tested beamformers are evaluated as the number of snapshots is increased at SNR=10 dB

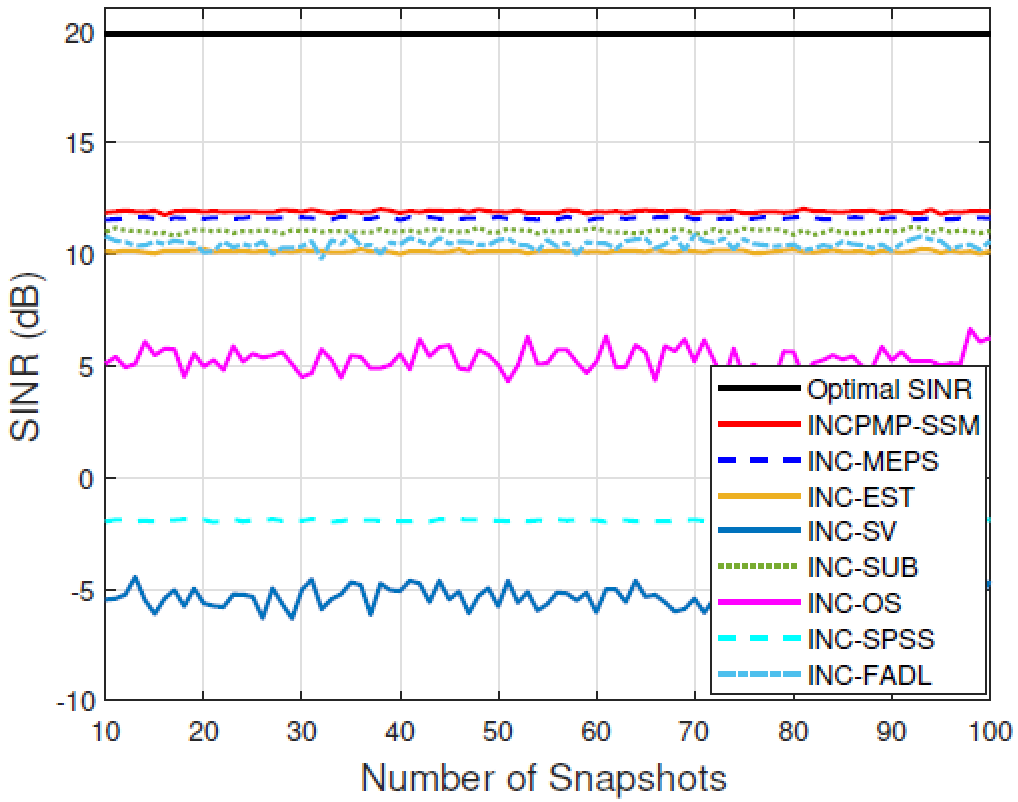

Fig. 2: Output SINR versus number of snapshots

Conclusion

- An efficient and accurate estimation of the INC matrix and DS steering vector has been proposed.
- The actual power and SV of the interferences and desired signal are estimated by the power method.
- The proposed technique avoids the eigenvalue decomposition (EVD) required to reconstruct the INC matrix.
- An algorithm based on the matching spectrum is devised to reconstruct the DS covariance matrix and estimate the SV of the SOI.
- The knowledge of the presumed steering vector is not essential.

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