



ON THE PREDICTION OF THE FREQUENCY RESPONSE OF A WOODEN PLATE FROM ITS MECHANICAL PARAMETERS

D. G. Badiane, R. Malvermi, S. Gonzalez, F. Antonacci, A. Sarti

Politecnico di Milano, Italy - Dipartimento di Elettronica, Informazione e Bioingegneria

Abstract

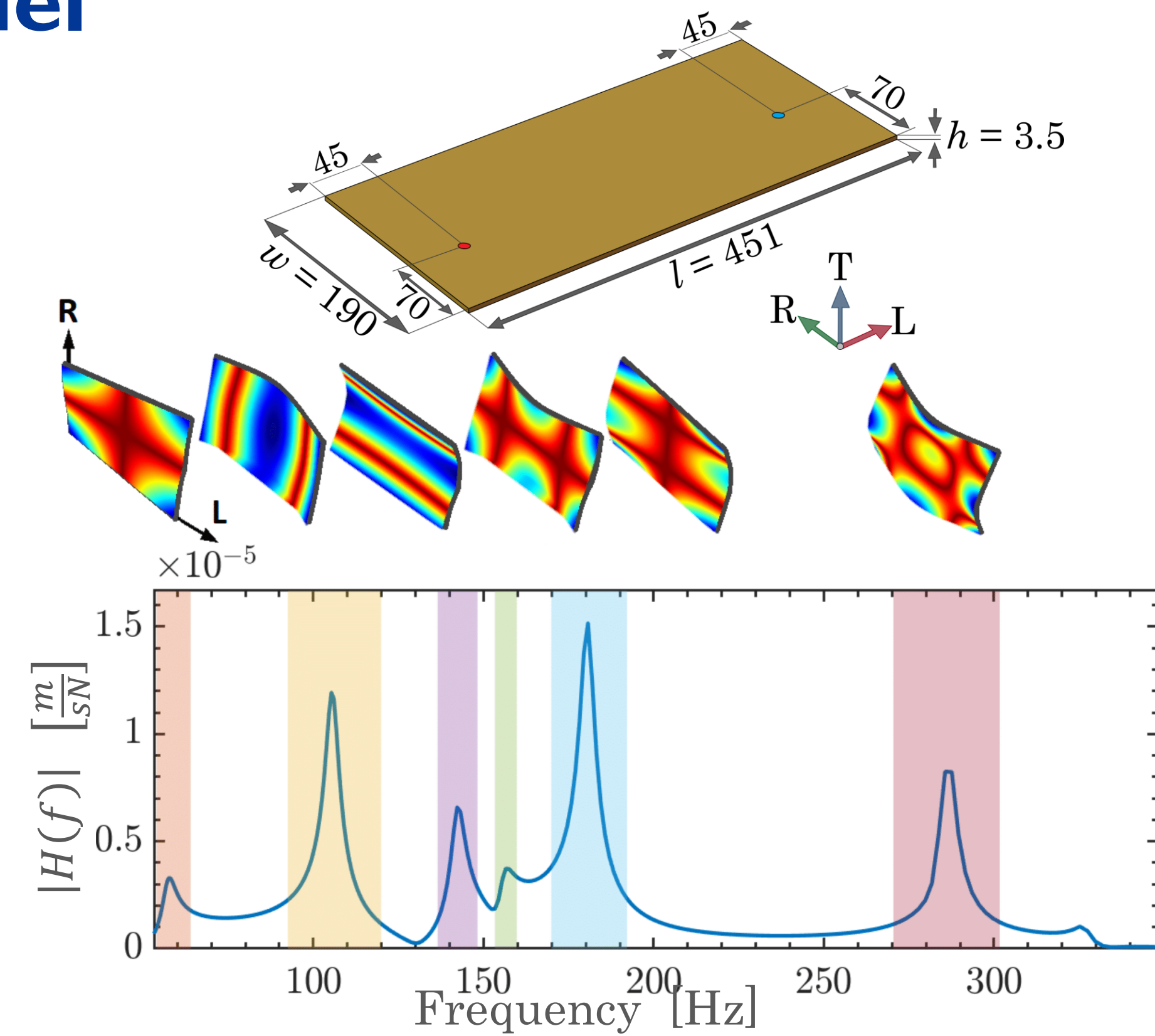
Inspired by deep learning applications in structural mechanics, we focus on how to train two predictors to model the relation between the vibrational response of a prescribed point of a wooden plate and its material properties. In particular, the eigenfrequencies of the plate are estimated via multilinear regression, whereas their amplitude is predicted by a feedforward neural network. We show that labeling the train set by mode numbers instead of by the order of appearance of the eigenfrequencies greatly improves the accuracy of the regression and that the coefficients of the multilinear regressor allow the definition of a linear relation between the first eigenfrequencies of the plate and its material properties.

Data Model

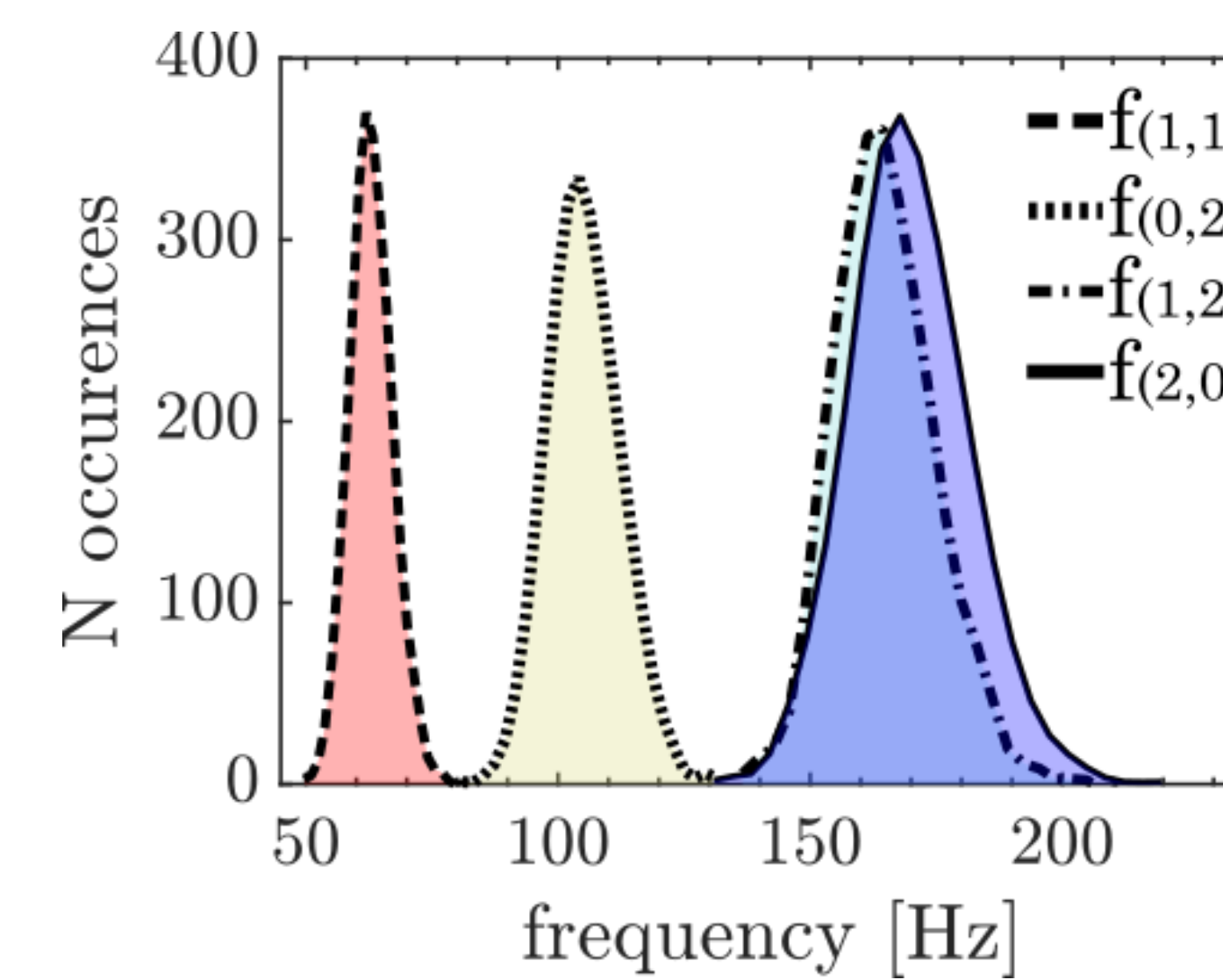
Finite Element modeling of rectangular wooden plates with fixed dimensions as a function of the material parameters

Model Input: Density, Young's moduli, Shear moduli and Poisson ratios randomly sampled around Sitka Spruce nominal values^[1]

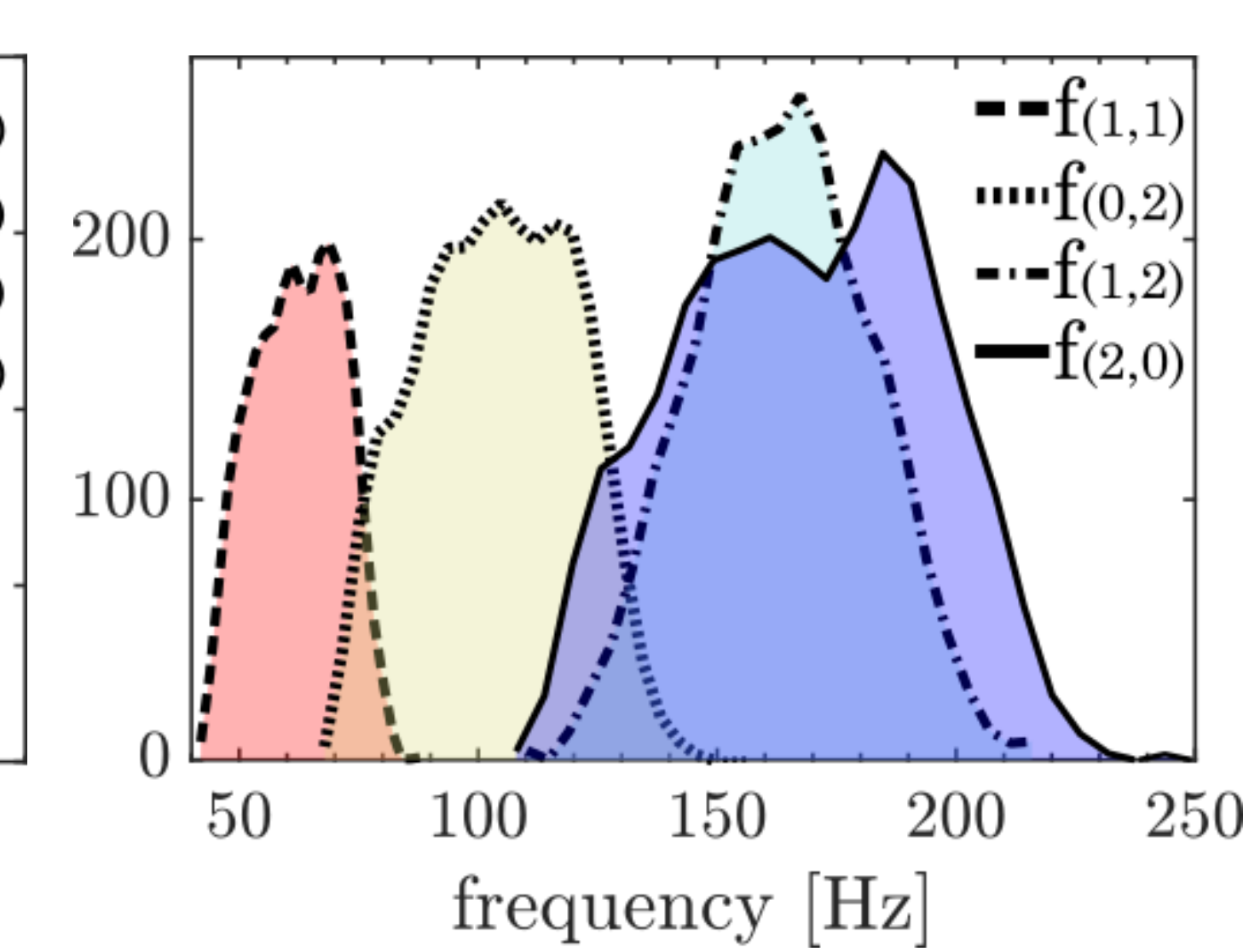
Model Output: Eigenfrequencies and mode amplitudes of a Frequency Response Function (FRF) evaluated at prescribed points of the plate



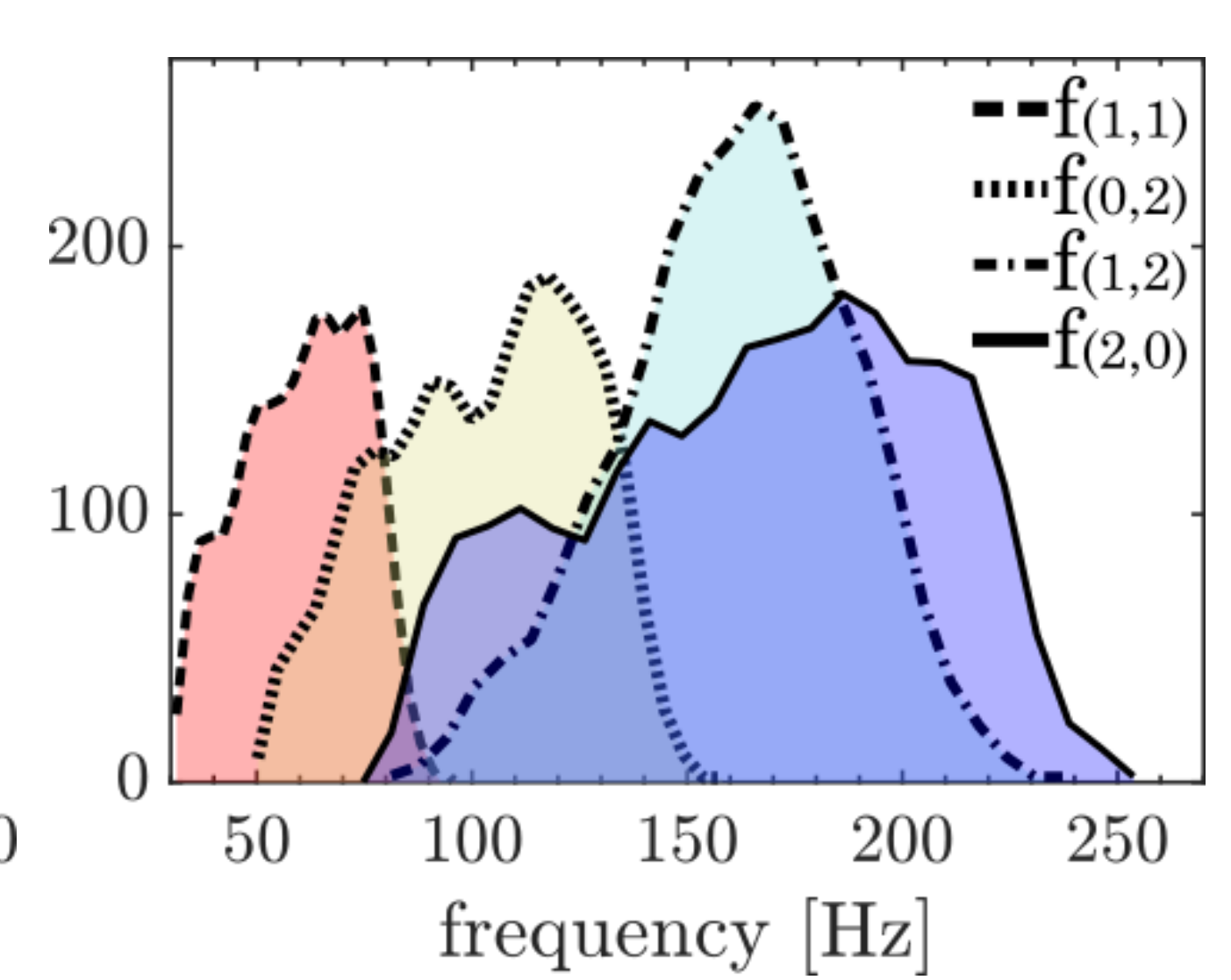
Modes Ordering



$$\underline{G10}: x \sim x_0 (1 + N(0,0.1))$$



$$\underline{U50}: x \sim x_0 (1 + U(-0.5,0.5))$$



$$\underline{U75}: x \sim x_0 (1 + U(-0.75,0.75))$$

Elastic constants distributions

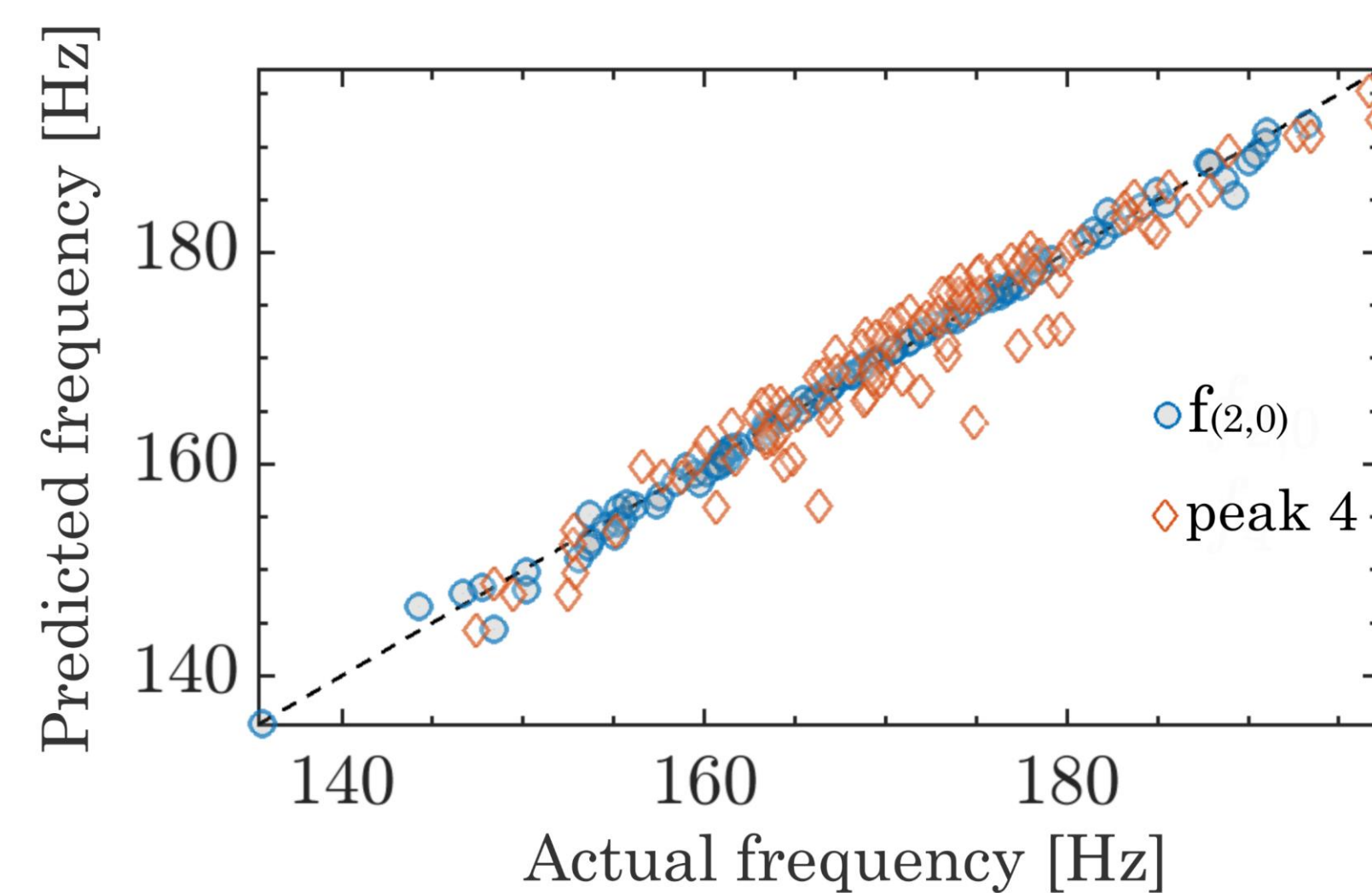
Results

Frequency Prediction

Multi-Linear Regression (MLR) used for the material-frequency relation

Regression tried with the frequency dataset ordered by

- Appearance of peaks inside the plate response
- Mode labeling (requires identification of the modes, first)



Worst case scenario:
mode (2,0) vs. fourth peak

$R^2 = 0.921$ (no labeling)

$R^2 = 0.994$ (with labeling)

Mechanical Parameters Estimation

MLR Equations

$$E_L = a_1 + 10^6 (b_1 \rho + c_1 f_{(0,2)})$$

$$E_R = a_2 + 10^6 (b_2 \rho + c_2 f_{(2,0)})$$

$$G_{LR} = a_3 + 10^6 (b_3 \rho + c_3 f_{(1,1)})$$

$$\vec{a} \text{ [MPa]}, \vec{b} \left[\frac{m^2}{s^2} \right], \vec{c} \left[\frac{Kg}{m \cdot s} \right] \text{ from MLR}$$

$$MAPE = \frac{1}{I} \sum_{i=1}^I \left| \frac{f - \hat{f}}{f} \right|$$

Caldersmith's Equations^[2]

$$E_L = \frac{0.096 \rho l^4 f_{(0,2)}^2}{h^2}$$

$$E_R = \frac{0.096 \rho w^4 f_{(2,0)}^2}{h^2}$$

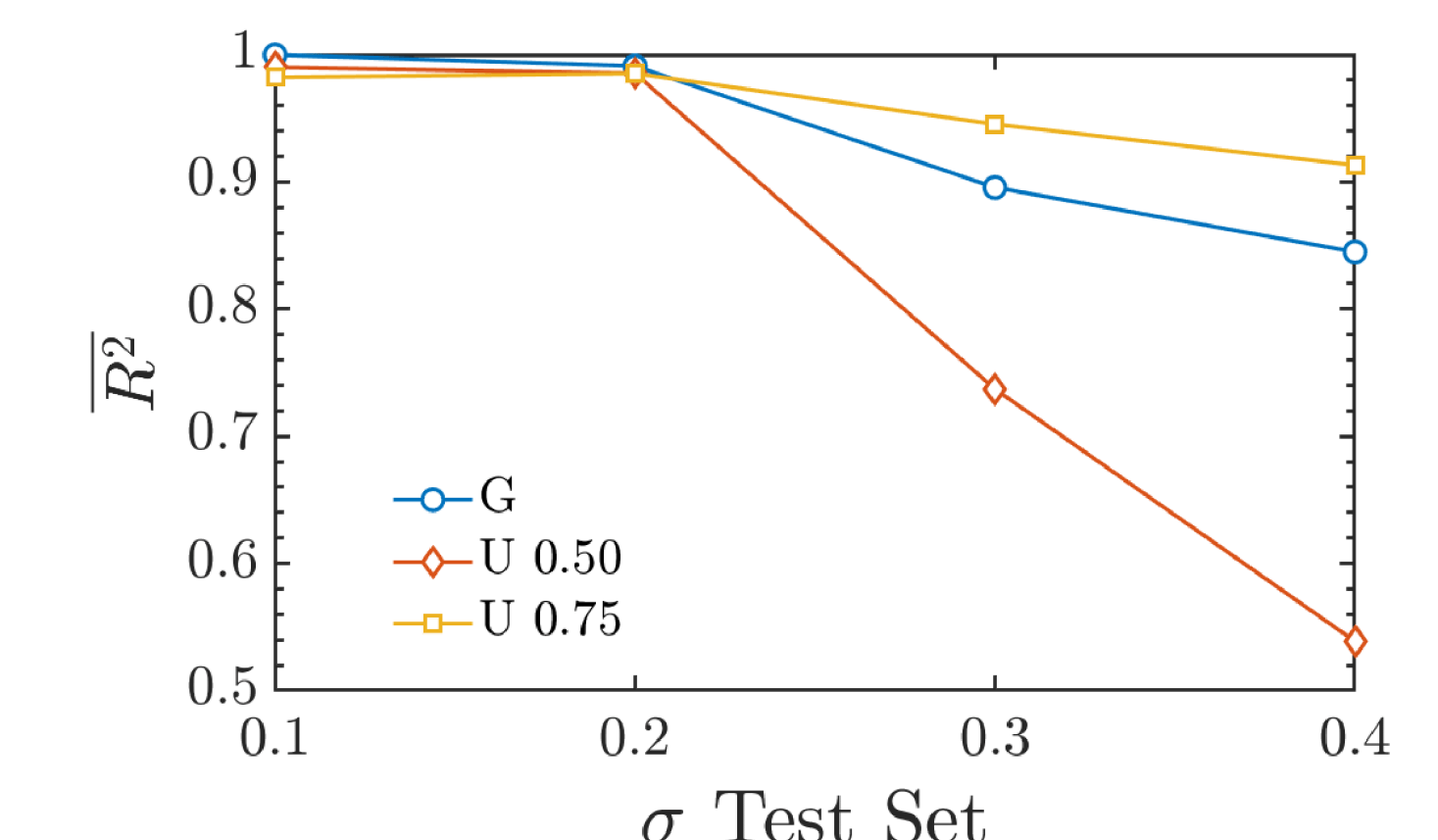
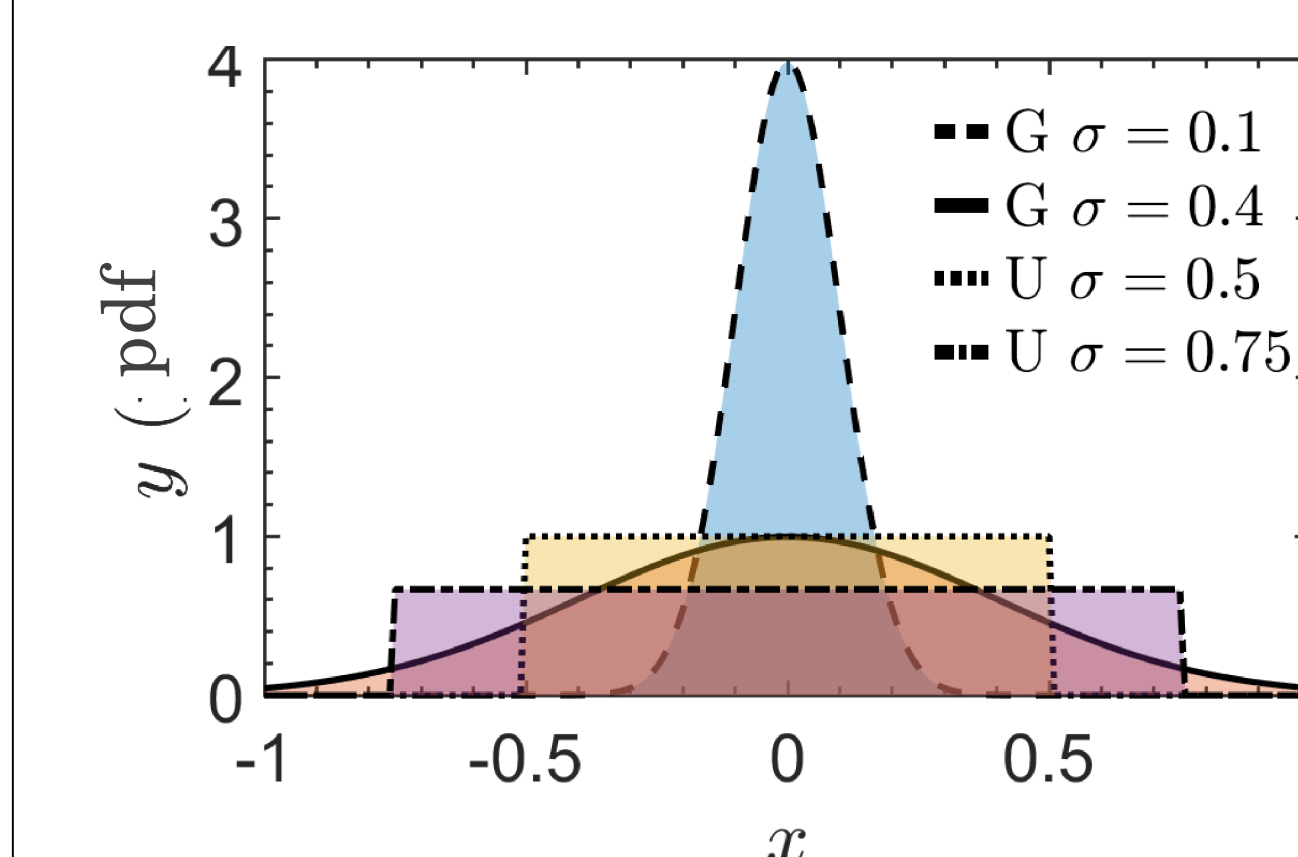
$$G_{LR} = \frac{0.822 \rho l^2 w^2 f_{(1,1)}^2}{h^2}$$

MAPE [%]	MLR	Caldersmith
E_L	0.35	0.88
E_R	1.99	3.38
G_{LR}	9.22	11.77

Amplitude Prediction

Multi-layer Feedforward Neural Network (MFNN)^[3] used for the material-amplitude relation

- Architecture (n° neurons x n° hidden layers):
 $\underline{G10}$ (8 x 1), $\underline{U50}$ (14 x 1), $\underline{U75}$ (8 x 2)
- Testing done using Gaussian test sets with varying std (10% - 40%)



$$\overline{R^2} = \frac{1}{4} \sum_{i=1}^4 R_{m_i}^2, \quad \mathbf{m} = \{(1,1) \quad (0,2) \quad (2,1) \quad (2,0)\}$$

References

- [1] Forest Products Laboratory (US), Wood Handbook: Wood as an Engineering Material, The Laboratory, 1987.
- [2] Graham Caldersmith and Freeman Elizabeth, "Wood properties from sample plate measurements I", Journal of Catgut Acoustic, vol. 1, no. II, pp. 8-12, 1990.
- [3] Daniel Svozil, Vladimir Kvasnicka, and Jiri Pospichal, "Introduction to multi-layer feed-forward neural networks," Chemometrics and intelligent laboratory systems, vol. 39, no. 1, pp. 43-62, 1997