ON THE PREDICTION OF THE FREQUENCY RESPONSE OF A WOODEN PLATE FROM ITS MECHANICAL PARAMETERS

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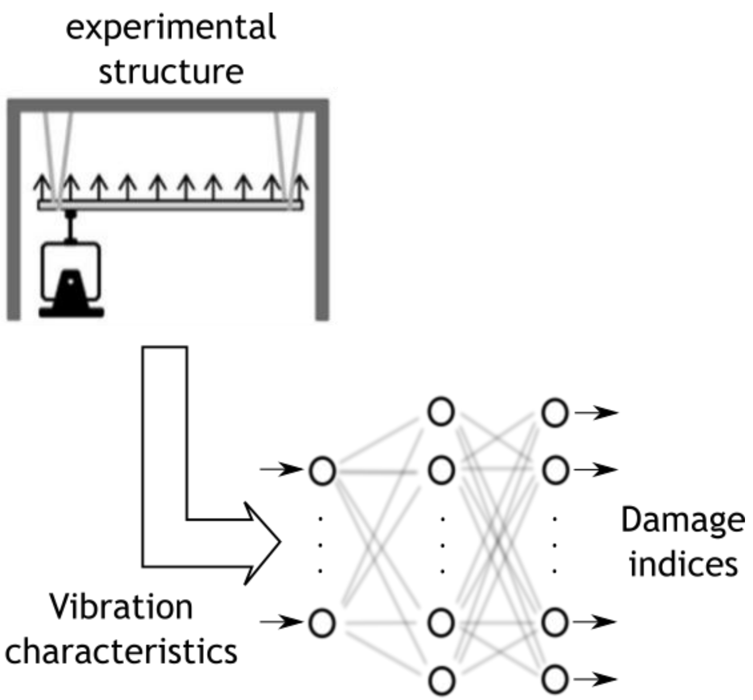
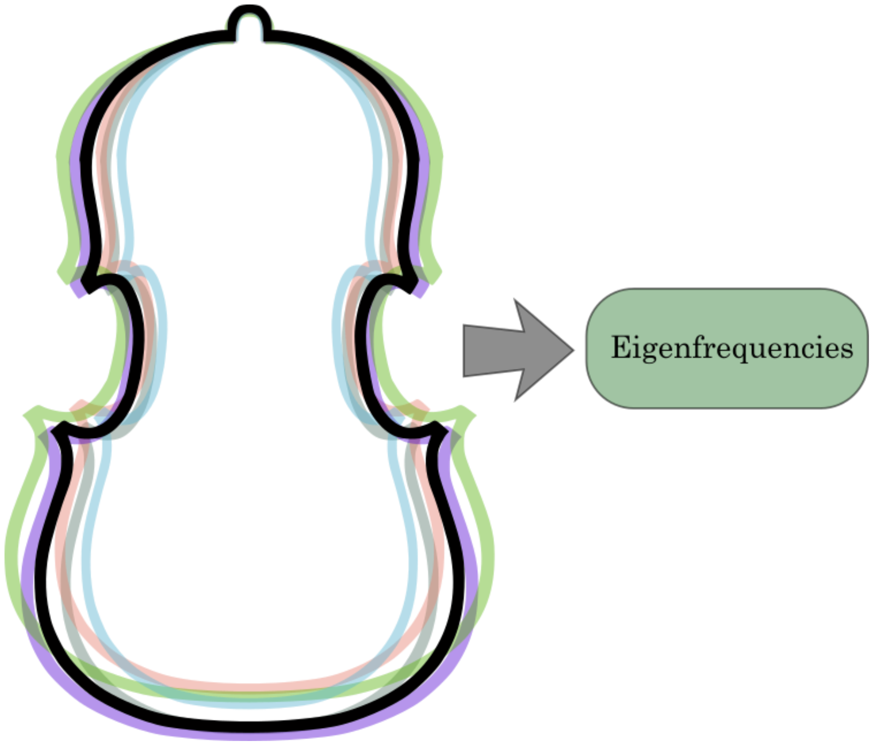
Context



Adapted from reference [2]



[1] F. E. Bock et al., "A review of the application of machine learning and data mining approaches in continuum materials mechanics", *Frontiers in Materials*, vol. 6, 2019.

[2] O. Avci et al., "A review of vibration-based damage detection in civil structures: from traditional methods to machine learning and deep learning applications", *Mechanical Systems and Signal Processing*, vol. 262, 2021

[3] S. Gonzalez, D. Salvi, D. Baeza, F. Antonacci, A. Sarti, "A data-driven approach to violin making", *Scientific reports*, vol. 11, no. 1, pp. 1-9, 2021.

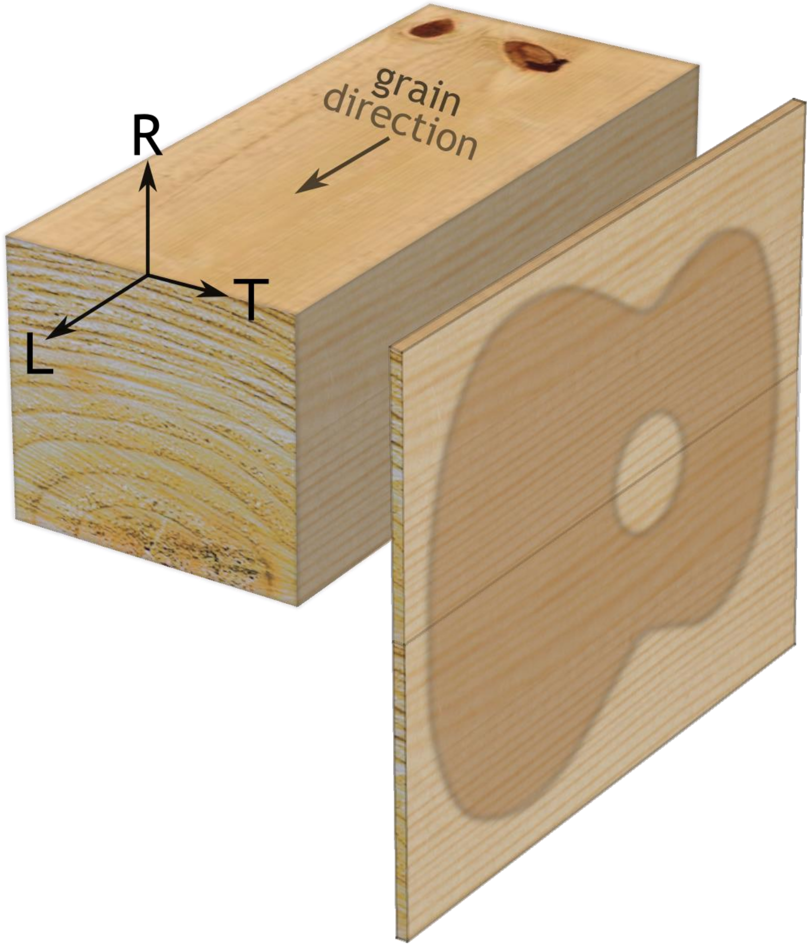
[4] Gonzalez, Sebastian, et al. "Eigenfrequency optimisation of free violin plates." *The Journal of the Acoustical Society of America* 149.3 (2021): 1400-1410.

Motivations

Need for *fast* and *accurate* characterization of wooden plates

Portable tool for wood characterization

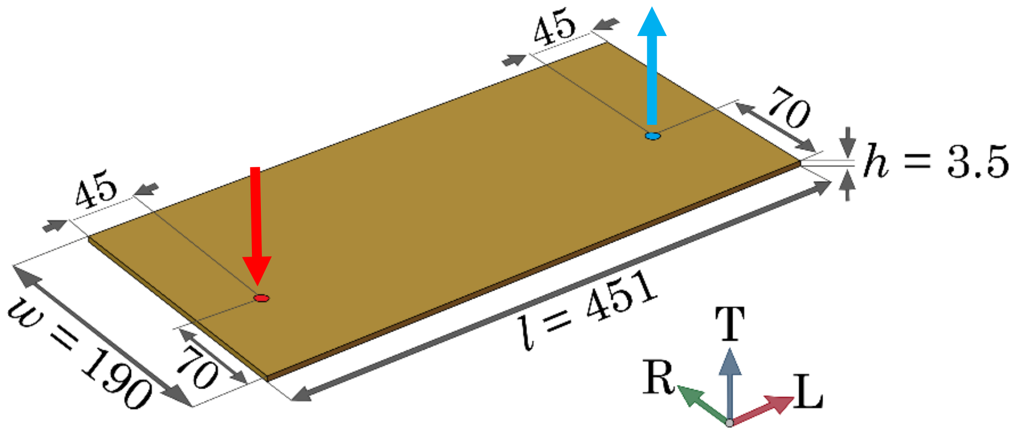
How to train the predictors?



Datasets - generation

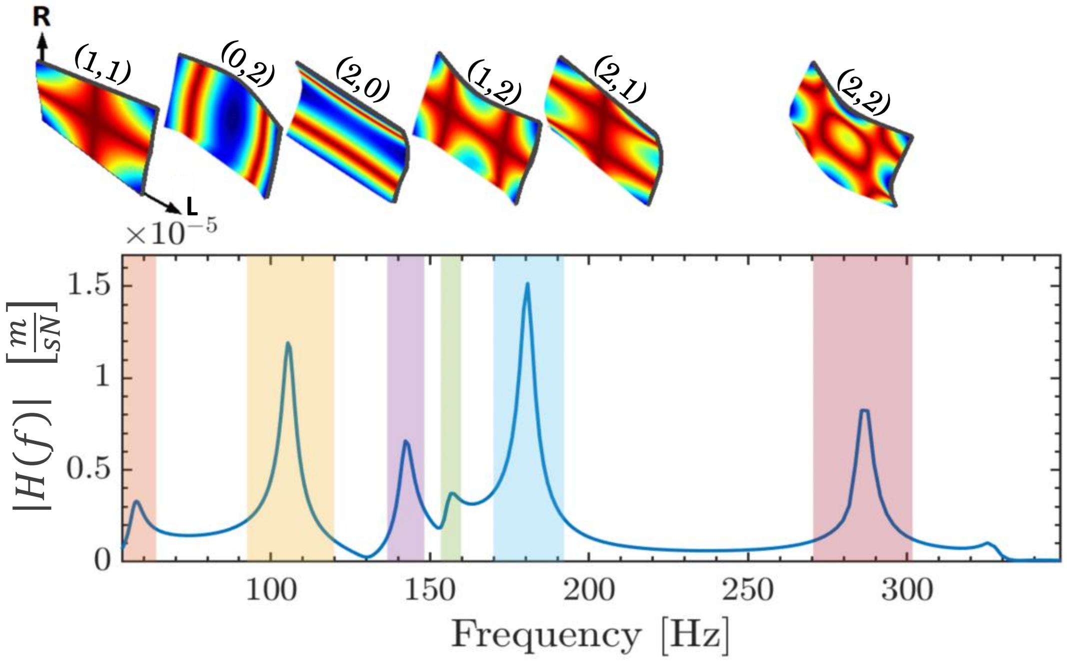
Model input:

Density, Young's moduli,
Shear moduli and Poisson ratios
(Sitka Spruce [5])



Model output:

Eigenfrequencies and corresponding amplitudes of a Frequency Response Function (FRF)



[5] Forest Products Laboratory (US), Wood Handbook: Wood as an Engineering Material, The Laboratory, 1987.

Datasets - generation

Model input:

Density, Young's moduli,
Shear moduli and Poisson ratios
(Sitka Spruce [5])

Model output:

Eigenfrequencies and corresponding amplitudes of a Frequency Response Function (FRF)

G10: $x \sim x_0\,(1 + N(0,0.1))$

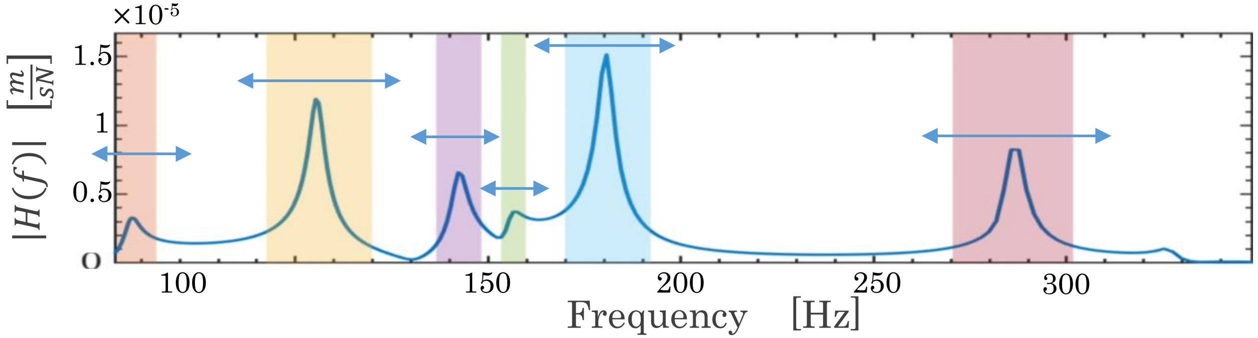
U50: $x \sim x_0\,(1 + U(-0.5,0.5))$

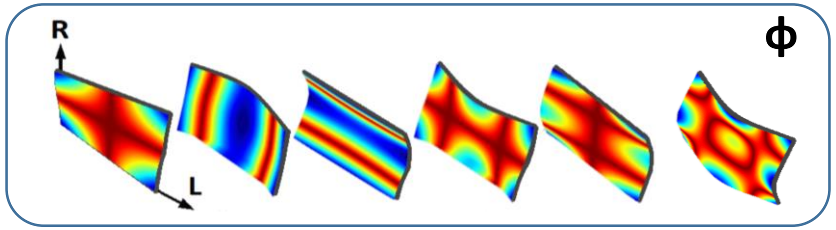
U75: $x \sim x_0\,(1 + U(-0.75,0.75))$

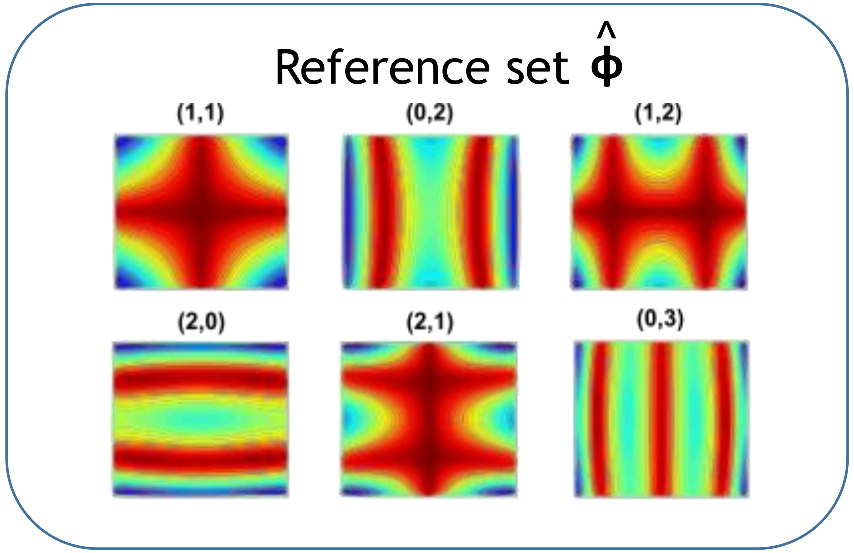
[5] Forest Products Laboratory (US), Wood Handbook: Wood as an Engineering Material, The Laboratory, 1987.

Modal shapes identification

variable order of appearance of modes in the FRF[6] (i.e. mode shifts)







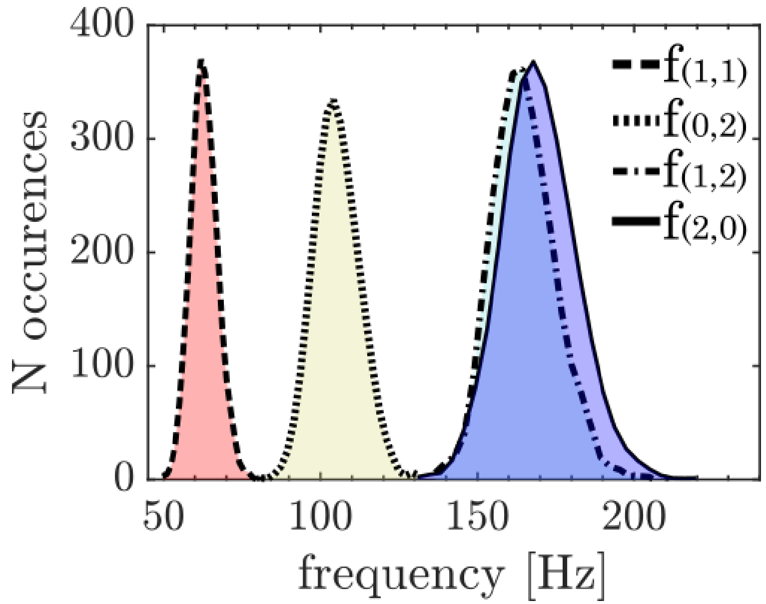
$$\mathrm{NCC}(\hat{\mathbf{\Phi}}, \mathbf{\Phi}) = \frac{\hat{\mathbf{\Phi}}^T \mathbf{\Phi}}{\|\hat{\mathbf{\Phi}}\|_2 \|\mathbf{\Phi}\|_2}$$

Modal shapes identification

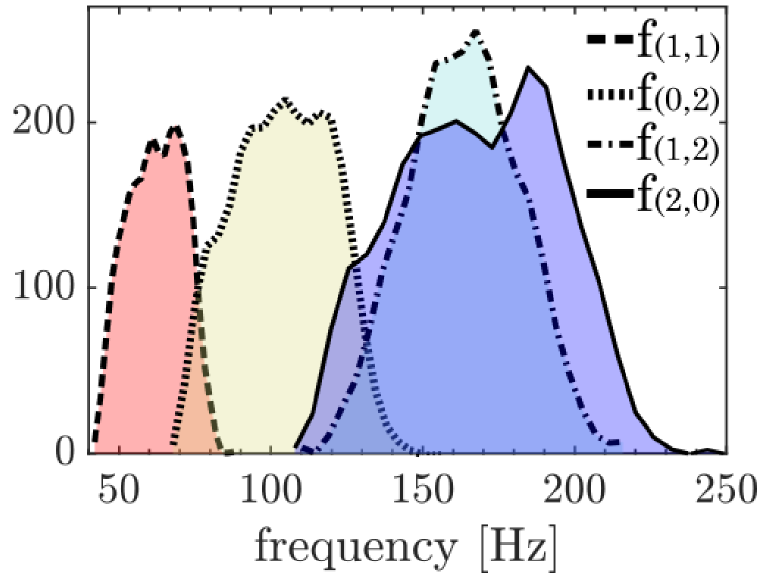
[6] Peter Persson et al. , “Improved low-frequency performance of cross-laminated timber floor panels by informed material selection,” Applied Acoustics, 2021.

Datasets – after postprocessing

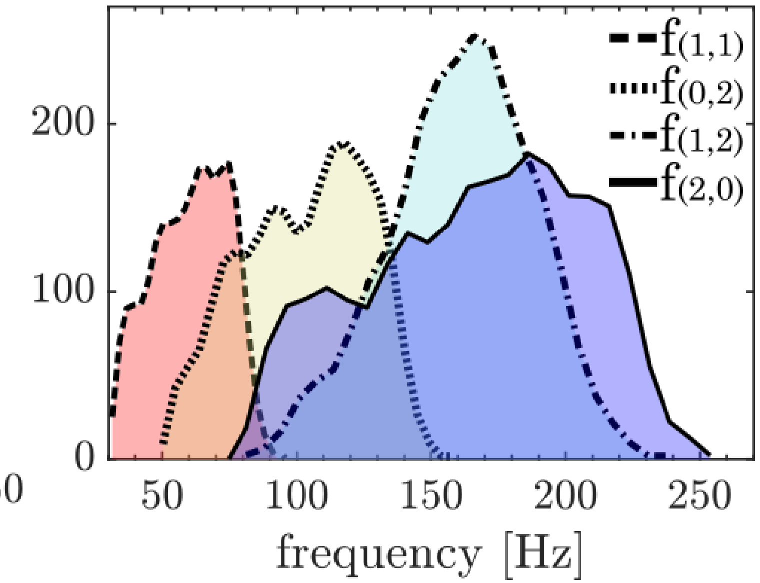
G10: $x \sim x_0\,(1 + N(0{,}0.1))$



U50: $x \sim x_0\,(1 + U(-0.5{,}0.5))$



U75: $x \sim x_0\,(1 + U(-0.75{,}0.75))$



Prediction - frequency

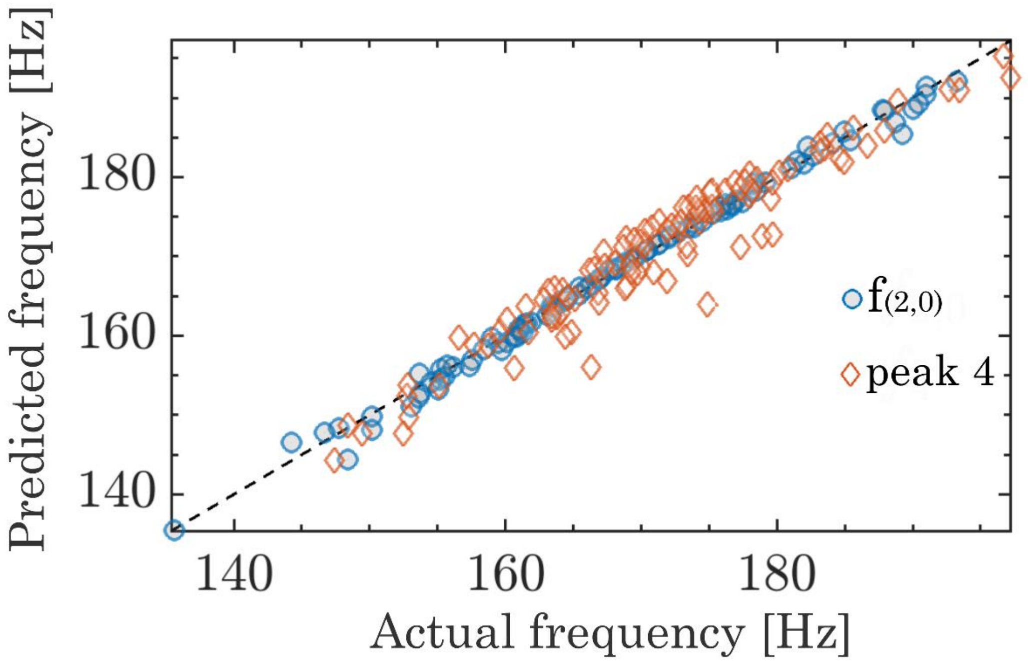
Multiple Linear Regression (MLR) for modeling the material-frequency relation

$$\overline{R^2} = \frac{1}{4}\sum_{i=1}^{4} R_{m_i}^2 \qquad \boldsymbol{m} = \{(1,1),\ (0,2),\ (1,2),\ (2,0)\}$$

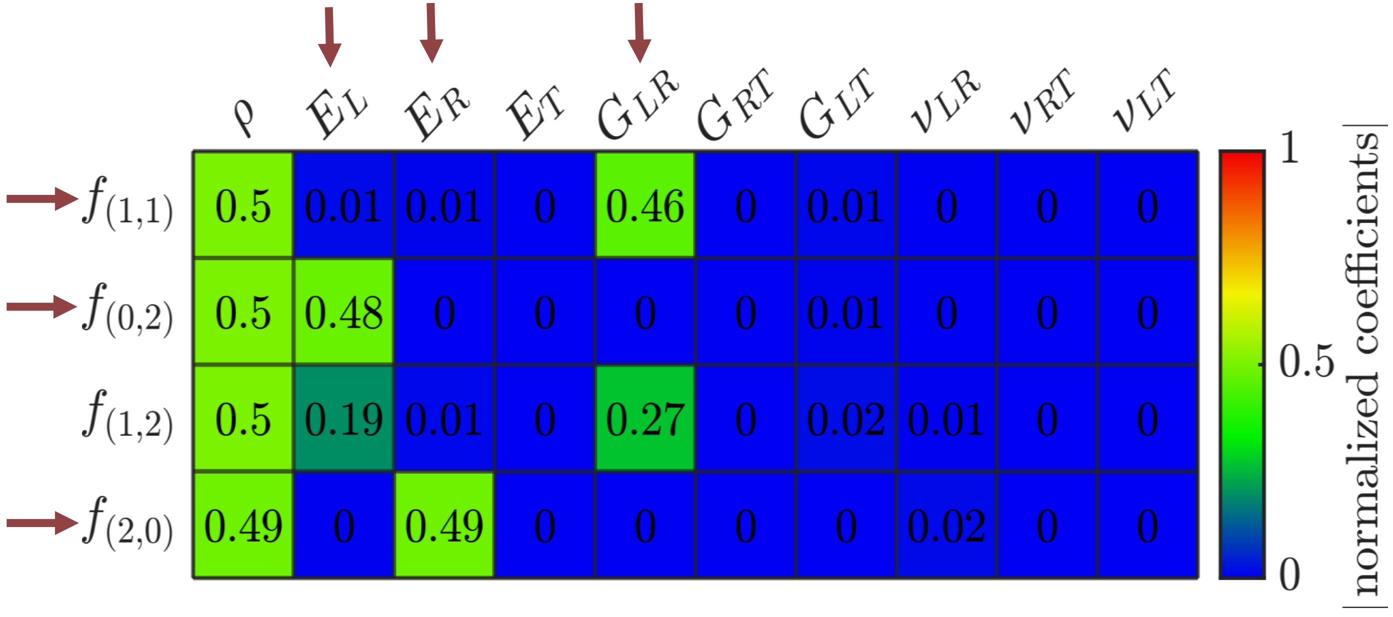
	G10	U50	U75
$\overline{R^2}$	0.993	0.990	0.983

Postprocessing: before vs. after

	R^2	F-stat	p-val
(2,0)	0.994	3228	0.000
Peak 4	0.921	369	0.000



MLR equations



$$E_L = a_1 + 10^6\left(b_1\rho + c_1 f_{(0,2)}\right)$$
$$E_R = a_2 + 10^6\left(b_2\rho + c_2 f_{(2,0)}\right)$$
$$G_{LR} = a_3 + 10^6\left(b_3\rho + c_3 f_{(1,1)}\right)$$

$$\vec{a} = [-29,\ -2.15,\ -1.85]\ \text{GPa}$$
$$\vec{b} = [35.6,\ 2.71,\ 2.35]\ \frac{m^2}{s^2}$$
$$\vec{c} = [274,\ 12.9,\ 29.3]\ \frac{\text{Kg}}{\text{m}\cdot\text{s}}$$

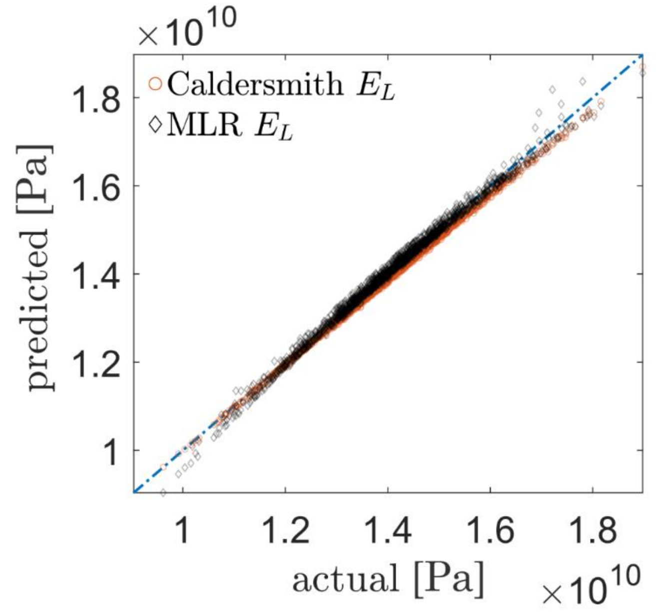
from MLR coefficients

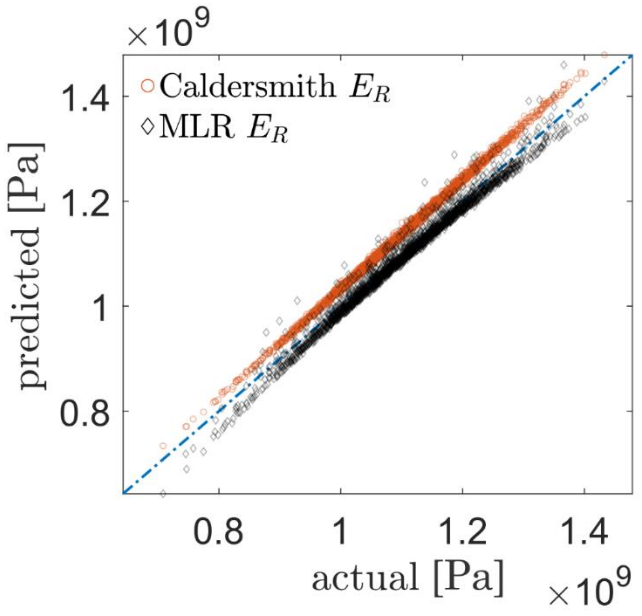
MLR equations - test

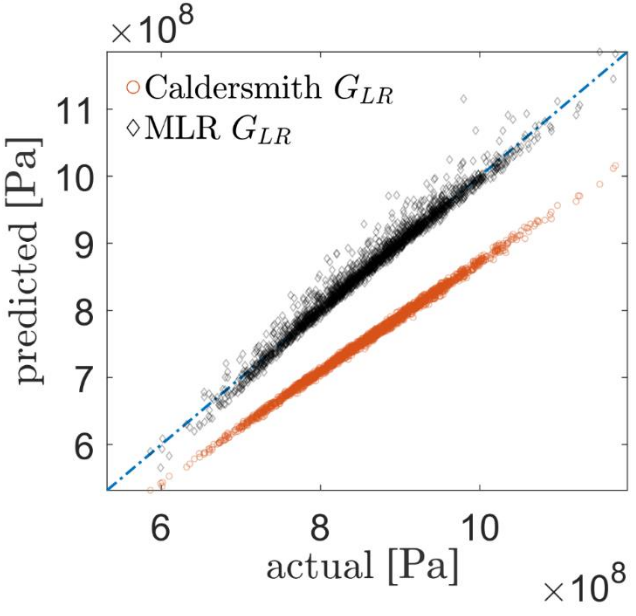
Caldersmith's Equations[7]: $$E_L = \frac{0.096\,\rho\,l^4 f_{(0,2)}^2}{h^2}\;,\; E_R = \frac{0.096\,\rho\,w^4 f_{(2,0)}^2}{h^2}\;,\; G_{LR} = \frac{0.822\,\rho\,l^2 w^2 f_{(1,1)}^2}{h^2}\;.$$

$$\mathrm{MAPE} = \frac{1}{I}\sum_{i=1}^{I}\left|\frac{f-\hat{f}}{f}\right|$$

MAPE [%]	E_L	E_R	G_{LR}
MLR	0.35	1.99	9.22
Caldersmith	0.88	3.38	11.77







[7] G. W. Caldersmith, ''Vibrations of orthotropic rectangular plates'', Acta Acustica, 1984.

Prediction - amplitude

Multi-layer Feedforward Neural Network (MFNN)[8] used for the material-amplitude relation

$$\overline{R^2} = \frac{1}{4}\sum_{i=1}^{4} R^2_{m_i} \qquad \boldsymbol{m} = \{(1,1),\ (0,2),\ (1,2),\ (2,0)\}$$

G10

N Neurons \ N Layers	1	2	4
2	0.803	0.683	0.379
4	0.989	0.999	0.995
8	0.999	0.999	0.999
16	0.999	0.999	0.999
32	0.999	0.999	0.999
64	0.999	0.999	0.992

U75

N Neurons \ N Layers	1	2	4
2	0.87	0.875	0.826
4	0.926	0.928	0.93
8	0.946	0.985	0.975
16	0.978	0.983	0.973
32	0.981	0.895	0.821
64	0.975	0.742	0.581

Hyperparameters tuning

G10: (8 x 1) MFNN — $\overline{R^2} = 0.999$

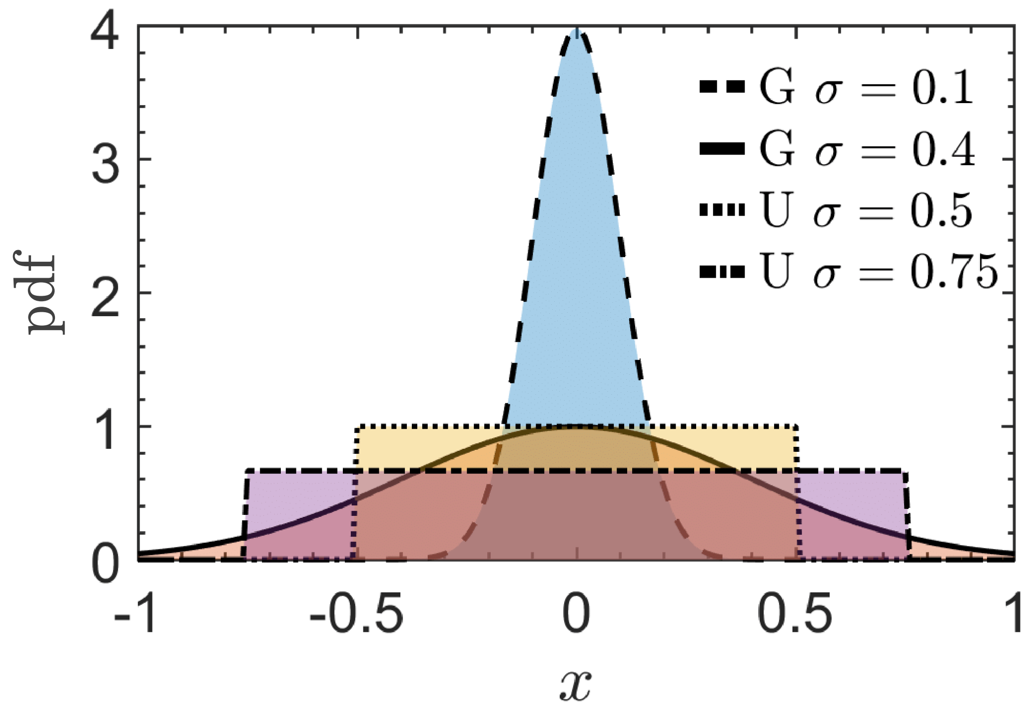
U50: (14 x 1) MFNN — $\overline{R^2} = 0.991$

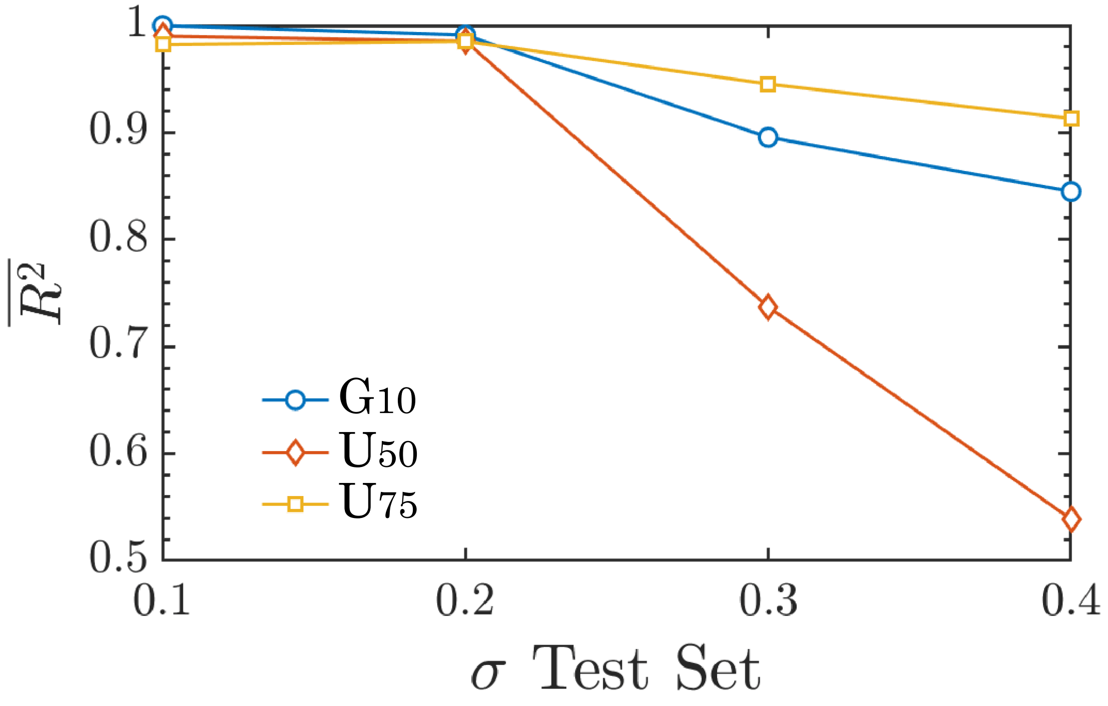
U75: (8 x 2) MFNN — $\overline{R^2} = 0.985$

[8] D. Svozil, V. Kvasnicka, and J. Pospichal, "Introduction to multi-layer feed-forward neural networks," Chemometrics and intelligent laboratory systems, 1997

Prediction - amplitude - test

Four Gaussian test sets with increasing std (i.e. 0.1 to 0.4 with a step of 0.1)





$$\overline{R^2} = \frac{1}{4}\sum_{i=1}^{4} R^2_{m_i} \qquad \boldsymbol{m} = \{(1,1),\ (0,2),\ (1,2),\ (2,0)\}$$

Conclusions

- Labeling the dataset by mode numbers is an effective postprocessing procedure

- MLR to derive a novel set of equations to estimate the material properties of the plate

- Modeling frequency and amplitude opens the door to the development of optimization procedures for material characterization

THANK YOU FOR YOUR ATTENTION