

A NEW COPRIME-ARRAY-BASED CONFIGURATION WITH AUGMENTED DEGREES OF FREEDOM AND REDUCED MUTUAL COUPLING

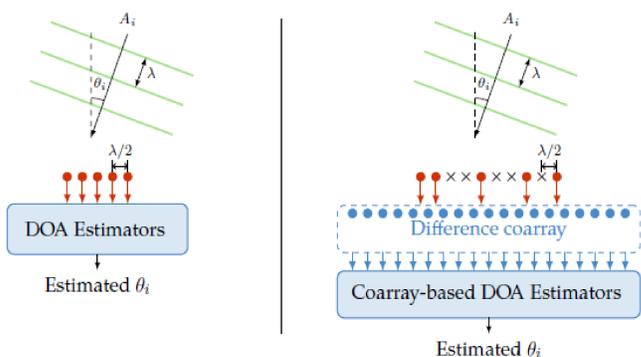
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ABSTRACT

In this paper, a new type of coprime-array-based structure, named AtCADiS, is proposed to achieve increased degrees of freedom (DoFs) and reduced mutual coupling. The AtCADiS is constructed via two steps. First, we shift the leftmost sensor of tailored coprime array with displaced subarrays (tCADiS) to the right by N . Second, we increase the number of sensors of tCADiS by $\lfloor M/2 \rfloor$ that are appropriately placed to connect the positive and negative lags of difference coarray of tCADiS and further improved its uDoFs remarkably. The simulations show that AtCADiS can achieve higher number of uDoFs than the existing coprime array-based structures by using the same number of physical sensors, which leads to stronger resolution capability and higher direction of arrival (DoA) estimation accuracy.

Introduction

- In DoA estimation, the classical uniform linear arrays ULAs, where the sensors are uniformly placed, is the most frequently used configuration. A ULA of M elements can resolve at most $M - 1$ independent signals. As a result, more physical sensors are needed if there are M or more independent sources to be identified, which leads to an increased system complexity and cost.
- To overcome the limitation of ULA, coprime array (CA) is proposed to enhance the capability of source identification by using a given number of sensors. Compared to the classical ULAs, CA can resolve more sources than sensors.



Array Design Factors

(A)

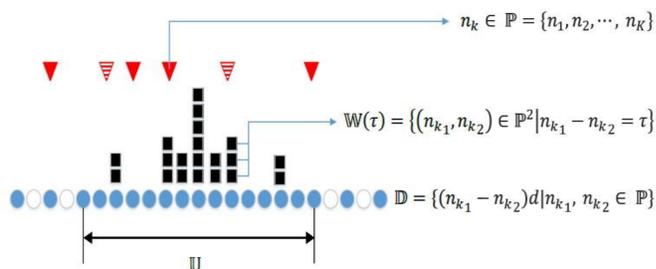
The amount of mutual coupling between sensors, which has an adverse effect on the DoA estimation performance. It is found that the amount of mutual coupling is directly proportional to the number of sensor pairs with small separations which are represented by the weight function of the difference coarray. The weight function $w(\tau)$ indicates the number of virtual sensors of difference coarray sharing the same value τ .

(B)

The number of uniform degrees of freedom (uDoFs) that represents the number of consecutive lags resulting from the difference in locations between different physical sensors.

(C)

The closed-form expression for the array sensor locations and the achievable uDoFs.



A sparse array \mathbb{P} and its difference coarray \mathbb{D} . Triangles (or blue bullets) indicate physical (or virtual) sensors respectively, while white bullets denote holes, and a square represents a sensor pair producing a τ .

The Main Contributions

Construct a new array with:

- Higher number of uniform DoFs.
- Comparable mutual coupling leakage compared with the other arrays.
- Closed-form expressions for the sensor locations and the achievable uDoFs.

Array Structure

The location indices of physical sensors of the proposed AtCADiS are given by the following set:

$$\mathbb{P}_{AtCADiS} = \begin{cases} \mathbb{P}_1 \cup \mathbb{P}_2 \cup \mathbb{A}_1 & \text{if } M \leq 4, \\ \mathbb{P}_1 \cup \mathbb{P}_2 \cup \mathbb{A}_2 & \text{if } M > 4 \text{ and } M \text{ is even,} \\ \mathbb{P}_1 \cup \mathbb{P}_2 \cup \mathbb{A}_3 & \text{if } M > 4 \text{ and } M \text{ is odd,} \end{cases}$$

where

$$\mathbb{P}_1 = \{n'M | 1 \leq n' \leq N - 1\},$$

$$\mathbb{P}_2 = \{mN + M(N - 1) + (N + M) | 0 \leq m \leq 2M - 2 - \lfloor \frac{M}{2} \rfloor\},$$

$$\mathbb{A}_1 = \{((a + 1)N - aM | 0 \leq a \leq \lfloor \frac{M}{2} \rfloor - 1\} \cup \{a_1\}\},$$

$$\mathbb{A}_2 = \{aN | 1 \leq a \leq \lfloor \frac{M}{2} \rfloor - 2\} \cup \{a_1, a_2\}\},$$

$$\mathbb{A}_3 = \{aN | 1 \leq a \leq \lfloor \frac{M}{2} \rfloor - 1\} \cup \{a_1, a_3\}\}.$$

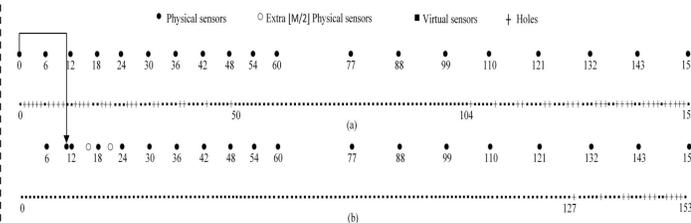
Here $a_1 = (2M - 2 - \lfloor (M/2) \rfloor)N + M(N - 1) + 2N$, $a_2 = (\lfloor \frac{M}{2} \rfloor - 2)N + k(N - M)$ with $k = 1, 2$ and $a_3 = (\lfloor \frac{M}{2} \rfloor - 1)N + (N - M)$.

Lemma 1: The Maximum Number of uDoFs

M	The maximum number of uDoF
2	$4MN - 1$
3	$4MN + 2M - 1$
4	$4MN + M + 1 - 2(N - M - 1)$
Even > 4	$3N(M + 1) + 6M - (N + 1)$
Odd > 4	$3N(M + 1) + 4M - 1$

A concrete Example

(a) An example of tCADiS structure when $(M, N) = (6, 11)$ and $L = 19$. (b) An example of AtCADiS structure when $(M, N) = (6, 11)$ and $L = 22$.



Simulation Results

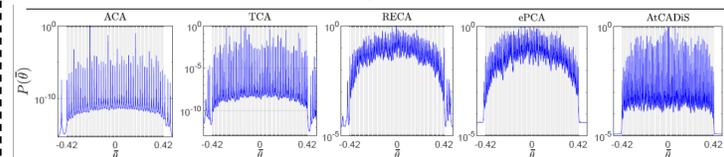
1 Mutual Coupling

A comparison of mutual coupling leakage L_c with $c_0 = 1$, $c_1 = 0.3e^{\frac{j\pi}{3}}$ and $c_i = c_1 e^{\frac{-j(i-1)\pi}{8}}/i$ for $2 \leq i \leq B$ with $B = 100$.

Array configurations	ACA (M, N)	TCA (M, N)	RECA (M, N)	ePCA (M, N)	AtCADiS (M, N)
$L = 8$ uDoFs, $L_c(0.3)$	(2, 5) 23, 0.2883	—	—	—	(2, 5) 39, 0.2661
$L = 12$ uDoFs, $L_c(0.3)$	(4, 5) 47, 0.2282	(5, 6) 69, 0.1687	(3, 7) 81, 0.1864	(4, 5) 83, 0.1823	(3, 7) 89, 0.1828
$L = 30$ uDoFs, $L_c(0.3)$	(8, 15) 255, 0.1461	(11, 15) 351, 0.1053	(7, 17) 401, 0.1123	(9, 13) 417, 0.1072	(8, 15) 437, 0.1119

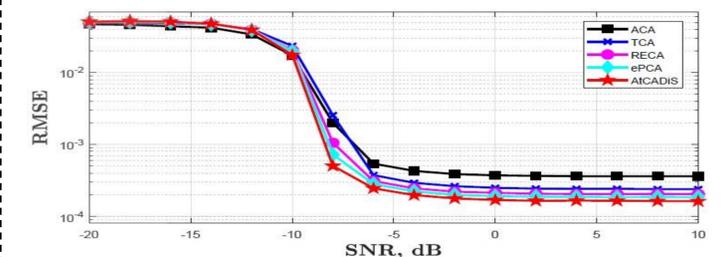
2 MUSIC Spectra

Comparison among ACA, TCA, RECA, ePCA, AtCADiS and their MUSIC spectra $P(\hat{\theta})$ when $L = 18$ physical sensors, $Q = 75$ signals are distributed uniformly over $\hat{\theta} = [-0.42, 0.42]$, $\text{SNR} = 0$ dB and $T = 1000$ snapshots.



3 RMSE Performance

Comparison among ACA, TCA, RECA, ePCA, and AtCADiS when $L = 18$ physical sensors, $Q = 35$ signals are distributed uniformly over $\hat{\theta} = [-0.42, 0.42]$ and $T = 1000$ snapshots.



Conclusion

In this paper, a coprime-array-based structure, called AtCADiS is proposed. Our motivations not only contribute to obtain the highest uDoFs, but also achieve comparable mutual coupling.

References

- P. Pal and P. P. Vaidyanathan, 2011 *Digital signal processing and signal processing education meeting (DSP/SPE)*, 2011.
- S. Qin Y. D. Zhang, and M. G. Amin, *IEEE Trans. Sig. Proc.*, 2015.
- W. Zheng, X. Zhang, Y. Wang, *IEEE Trans. Veh. Technol.*, 2019.
- W. Zheng, X. Zhang, Y. Wang, J. Shen, and B. Champagne, *IEEE Trans. Signal Process.*, 2020.
- A. Raza, W. Liu, and Q. Shen, *IEEE Trans. Signal Process.*, 2019.
- X. M. Wang and X. Wang, *IEEE Trans. Signal Process.*, 2019.
- C.-L. Liu and P. P. Vaidyanathan, *IEEE Trans. Signal Process.*, 2016.