Provable Sample Complexity Guarantees for Learning of Continuous-Action Graphical Games with Nonparametric Utilities

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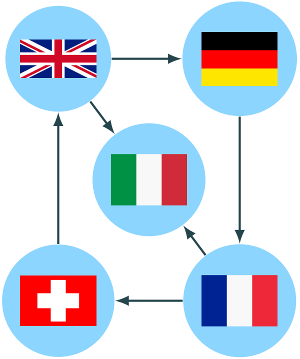
Graphical Games

Graphical Games | A representation of multiplayer game on a directed graph which captures and exploits locality or sparsity of direct influences.

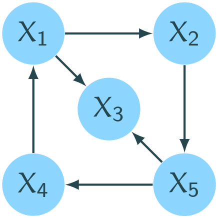
- Example: Trading between European countries
- Sparse graph: trading strategy of a country depends only on in-neighbors
- Selfish players
- **Problem:** Learn the directed structure



Graphical Games

Graphical Games | A representation of multiplayer game on a directed graph which captures and exploits locality or sparsity of direct influences.

- n players as vertices $X_1, \cdots, X_n$
- In-neighbors: $S_i = \{j \mid j \to i\}$, e.g., $S_3 = \{1, 5\}$
- Continuous action $x_i \in \mathbb{R}^k$, x_{-i} is collection of actions of all the players but i
- Pairwise decomposable payoff $u_i = \sum_{j \in S_i} u_{ij}(x_i, x_j)$
- $u_3 = u_{31}(x_3, x_1) + u_{35}(x_3, x_5)$



Nash Equilibria

Pure Strategy Nash Equilibria (PSNE)

A joint action $\mathbf{x}^* \in \prod_{i=1}^{n} \mathbb{R}^k$ where no player has any incentive to unilaterally deviate from the prescribed action $x_i^* \in \mathbb{R}^k$ given the joint action of its in-neighbors in the equilibrium.

$$\text{NE} = \{\mathbf{x}^* \mid x_i^* \in \arg\max_{x_i \in \mathbb{R}^k} u_i(x_i, x_{-i}^*), \forall i \in \{1 \cdots n\}\}$$

We allow small deviation from PSNE!

ϵ-PSNE

$$\text{NE}_\epsilon = \{\mathbf{x}^* \mid u_i(x_i^*, x_{-i}^*) \geqslant -\epsilon + \arg\max_{x_i \in \mathbb{R}^k} u_i(x_i, x_{-i}^*), \forall i \in \{1 \cdots n\}\}$$

Handling Non-parametric Payoffs

- Recall $u_i = \sum_{j \in S_i} u_{ij}(x_i, x_j)$ but they are still non-parametric
- We write $u_{ij}(x_i, x_j)$ as a weighted sum of uniformly bounded orthonormal basis functions (such as Fourier basis)

$$u_{ij}(x_i, x_j) = \sum_{k=0}^{\infty} \beta^*_{ijk} \psi_k(x_i, x_j) \ ,$$

where $\beta^*_{ijk} = \langle u_{ij}, \psi_k \rangle$

Handling Non-parametric Payoffs

- Still an infinite series for β^*_{ijk}
- We can truncate as long as coefficients β^*_{ijk} are convergent for $k \geqslant r+1$, e.g., Fourier basis
- We use $\overline{u}_i(x) = \sum_{j \in S_i} \overline{u}_{ij}(x_i, x_j)$ for estimation where $\overline{u}_{ij} = \sum_{k=0}^{r} \beta^*_{ijk} \psi_k(x_i, x_j)$

Problem Formulation

$x \longrightarrow$ **Blackbox** $\longrightarrow \tilde{u}_i(x)$

Blackbox

1. $\tilde{\psi}_k(x_i, x_j) = \psi_k(x_i, x_j) + \gamma_k(x_i, x_j)$
2. $\tilde{u}_i(x) = \sum_{j \in S_i} \sum_{k=0}^{\infty} \tilde{\psi}_k(x_i, x_j)$

- Get N noisy black-box measurements
- Estimation problem

$$\min_{\beta} \frac{1}{N} \sum_{x \in D} (\tilde{u}_i(x) - \bar{u}_i(x))^2 + \lambda \sum_{j \neq i} \sum_{k=0}^{r} |\beta_{ijk}|$$
$$\text{such that} \sum_{j \neq i} \sum_{k=0}^{r} |\beta_{ijk}| \leqslant C \,, \tag{1}$$

- We show that if $\beta_{ijk} \neq 0$, then j is in-neighbor of i

Main Result

Our result - Recovering Graphical Games

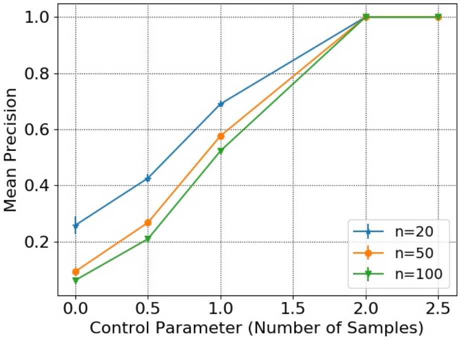
- Let $d = \max |S_i|$
- Estimation problem learns the exact structure of graphical game with high probability provided that:

$$N \geqslant \Omega(r^3 d^3 \log(rn))$$

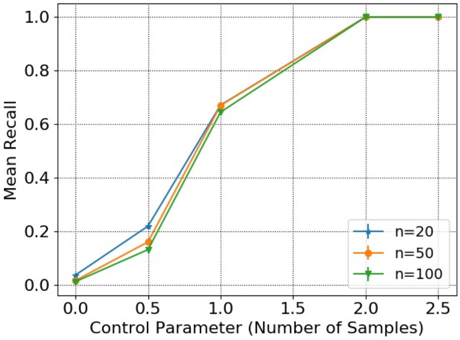
- The estimated game has $\widehat{\mathrm{NE}} \subseteq \mathrm{NE}_\epsilon$ and $\mathrm{NE} \subseteq \widehat{\mathrm{NE}}_\epsilon$

Use of primal-dual witness technique to construct a solution which is unique and recovers correct in-neighbors with bounded error!

Validation



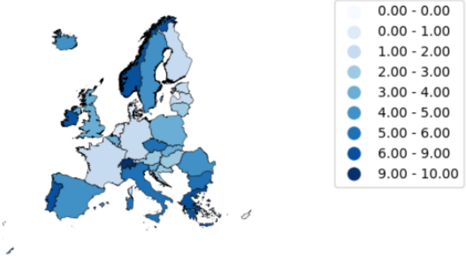
(a)



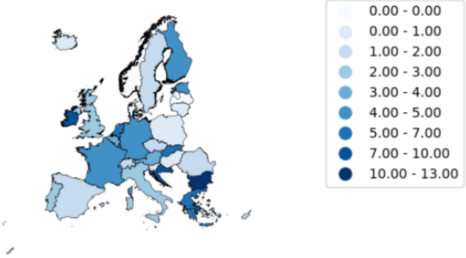
(b)

Figure: Precision and recall with number of samples. N varies with $10^{C_p}\log(rn)$ where $r = 2$

Real World Application



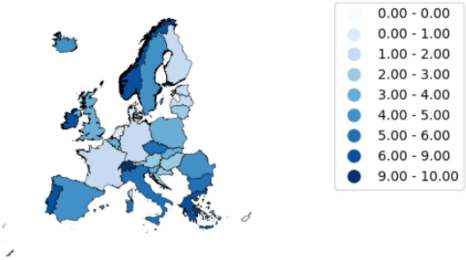
(a) 1973-1995



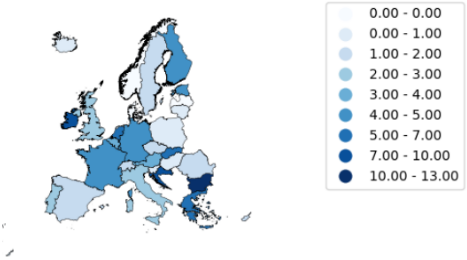
(b) 1996-2019

- Potato trade data among European countries between $1973 - 2019$
- Action: Production volume, Payoff: Trade price

Real World Application



(a) 1973-1995



(b) 1996-2019

- The influence of a country is measured by the numbers of its out-neighbors
- Observation: Countries in Eurozone(formed in 1999) have gained influence during $1996 - 2019$

Thank You!