VARIABLE SPAN TRADE-OFF FILTER FOR SOUND ZONE CONTROL WITH KERNEL INTERPOLATION WEIGHTING

Jesper Brunnström[*]

Shoichi Koyama[†]

Marc Moonen[*]

[*] STADIUS Center for Dynamical Systems
Department of Electrical Engineering (ESAT)
KU Leuven

[†] Graduate School of Information Science and Technology
The University of Tokyo

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Problem statement

Reproduce sound in the bright zone with minimal sound produced in the dark zones

Optimize over continuous sound zones instead of only at discrete control points

Advantageous with fewer control points available, such as when microphones at the control points are left in place during operation.

Cost function: minimize pressure error over continuous sound zones

$$\mathcal{J} = \mathcal{J}_b + \mu \mathcal{J}_d$$

$$\mathcal{J}_b = \underbrace{\frac{1}{|\Omega_b|}}_{\text{Size of zone}} \int_{\Omega_b} |\underbrace{p(\boldsymbol{r})}_{\text{Sound pressure}} - \underbrace{d(\boldsymbol{r})}_{\text{Desired pressure}}|^2 \, d\underbrace{\boldsymbol{r}}_{\text{Position in space}}$$

$$\mathcal{J}_d = \frac{1}{\sum_{d \in \mathcal{D}} |\Omega_d|} \sum_{d \in \mathcal{D}} \int_{\Omega_d} |p(\boldsymbol{r})|^2 \, d\boldsymbol{r}$$

Sound field interpolation

Estimate unobservable sound pressure function with kernel interpolation

Kernel function determines function space from which the interpolating function $p(\boldsymbol{r})$ is chosen

$$\kappa(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{\theta}) = j_0\left(\sqrt{\boldsymbol{\xi}^\top \boldsymbol{\xi}}\right)$$

$$\boldsymbol{\xi} = \mathrm{j}\rho\boldsymbol{\theta} - \frac{\omega}{c}(\boldsymbol{r} - \boldsymbol{r}')$$

Restricts $p(\boldsymbol{r})$ to functions satisying the homogenous Helmholtz equation. Has a directional weighting, if prior knowledge of arrival direction is available.

Closed form expression for interpolated sound pressure

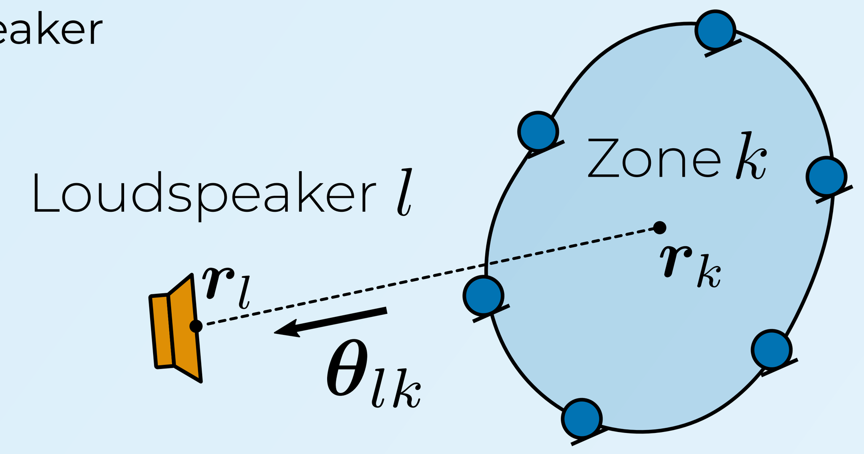
$$p(\boldsymbol{r}) = \boldsymbol{z}_k^\top(\boldsymbol{r}, \boldsymbol{\theta}) \underbrace{\boldsymbol{p}_k}_{\text{Sound pressure at control points}}$$

$$\boldsymbol{z}_k(\boldsymbol{r}, \boldsymbol{\theta}) = (\boldsymbol{K}_k(\boldsymbol{\theta}) + \lambda \boldsymbol{I})^{-\top} \boldsymbol{\kappa}_k(\boldsymbol{r}, \boldsymbol{\theta})$$

Interpolate for each loudspeaker-zone pair separately

Arrival direction can be assumed to be direction from center of zone to loudspeaker

$$\boldsymbol{\theta}_{lk} = \frac{\boldsymbol{r}_l - \boldsymbol{r}_k}{\|\boldsymbol{r}_l - \boldsymbol{r}_k\|}$$



Control filter

Control filter minimizing pressure error

$$\boldsymbol{w} = (\boldsymbol{R}_b + \mu \boldsymbol{R}_d)^{-1} \boldsymbol{r}_b$$

Bright zone spatial covariance matrix

Dark zone matrix defined similarly

$$\boldsymbol{R}_b = \frac{1}{|\Omega_b|} \boldsymbol{H}_b^{\mathsf{H}} \boldsymbol{A}_b \boldsymbol{H}_b \qquad \boldsymbol{H}_k = \mathrm{blkdiag}\{\boldsymbol{h}_{kl}\}_{l \in \mathcal{L}}$$

Room transfer functions from loudspeaker l to zone k

$$\boldsymbol{A}_b = \int_{\Omega_b} \boldsymbol{z}_b^*(\boldsymbol{r}) \boldsymbol{z}_b^\top(\boldsymbol{r}) \, d\boldsymbol{r} \qquad \boldsymbol{z}_b(\boldsymbol{r}) = \mathrm{col}\{\boldsymbol{z}(\boldsymbol{r}, \boldsymbol{\theta}_{lb})\}_{l \in \mathcal{L}}$$

$$\boldsymbol{r}_b = \frac{1}{|\Omega_b|} \boldsymbol{H}_b^{\mathsf{H}} \tilde{\boldsymbol{A}}_b \tilde{\boldsymbol{h}}$$

$\tilde{\boldsymbol{h}}$ Room transfer functions from virtual sources to bright zone

Generalized eigenvalue decomposition with a rank R approximation gives variable span trade-off filter (VAST)

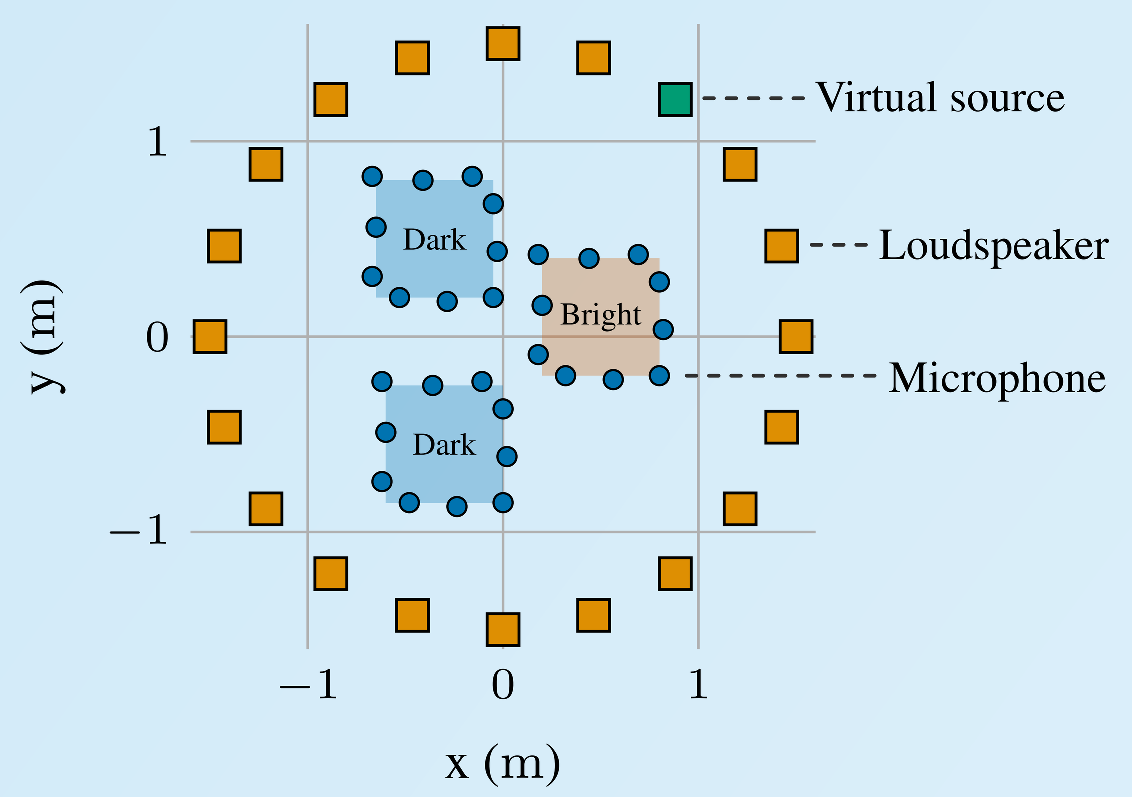
$$\boldsymbol{w} = \boldsymbol{U}_R (\boldsymbol{\Lambda}_R + \mu \boldsymbol{I})^{-1} \boldsymbol{U}_R^{\mathsf{H}} \boldsymbol{r}_b$$

$$\boldsymbol{U}^{\mathsf{H}} \boldsymbol{R}_b \boldsymbol{U} = \boldsymbol{\Lambda}$$

$$\boldsymbol{U}^{\mathsf{H}} \boldsymbol{R}_d \boldsymbol{U} = \boldsymbol{I}$$

Choice of rank trades off between low distortion in the bright zone and high acoustic contrast

Simulation results



VAST
Pointwise optimization

VAST-KI
With kernel interpolation $\rho = 0$

VAST-DKI
With directional kernel interpolation $\rho = 3$