

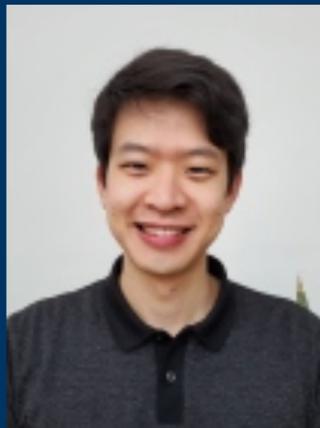
# Mixture Model Auto-Encoders: Deep Clustering through Dictionary Learning

IEEE ICASSP 2022 Paper #4887



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Brigham and Women's Hospital



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# Introduction

## The Difficulty of Clustering Natural Signals

4

4

9

9

# Introduction

## The Difficulty of Clustering Natural Signals

A handwritten digit '4' in a standard, slightly slanted font.A distorted handwritten digit '4' that is tilted and has a jagged, pixelated appearance.A distorted handwritten digit '9' that is tilted and has a jagged, pixelated appearance.A handwritten digit '9' in a standard, slightly slanted font.

- Unsupervised: No labels

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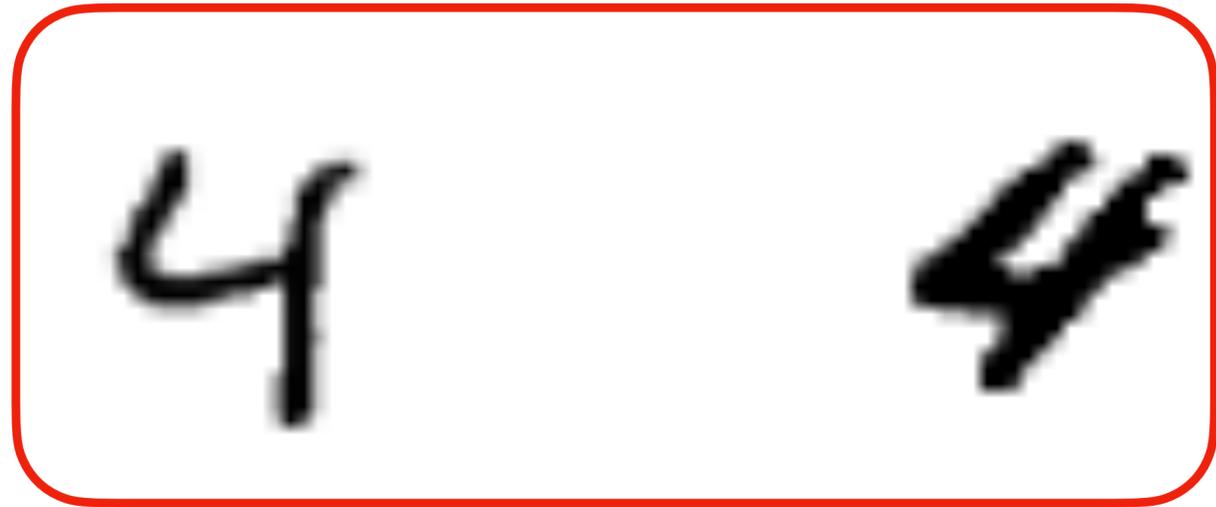
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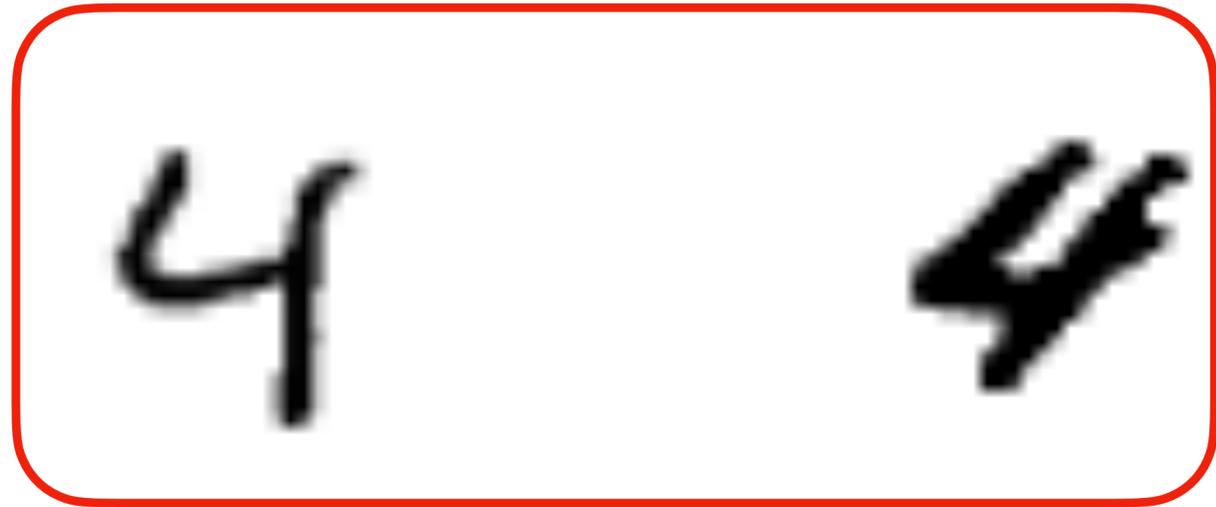
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**Model-Based  
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- Simple models with principled effects (e.g. sparsity, low rank, dictionary learning)
- Incorporate prior knowledge and learns few parameters
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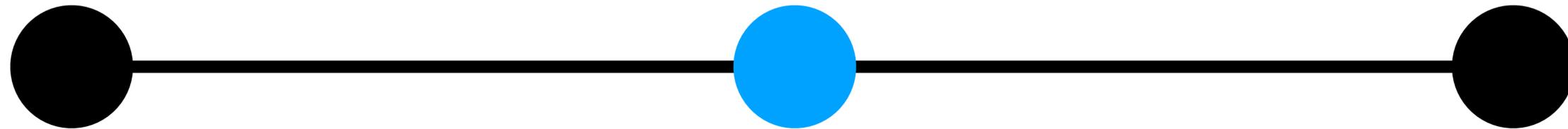
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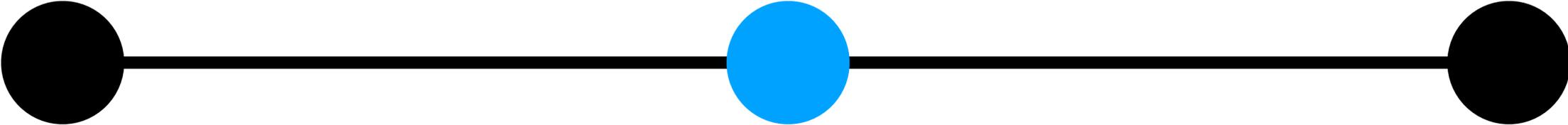
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## Model-Based Deep Learning [1]

- Multi-layer architecture derived from interpretable signal processing model
- Can use model to inject prior knowledge into deep architecture (fewer params)
- Can leverage deep learning technology for scalable training

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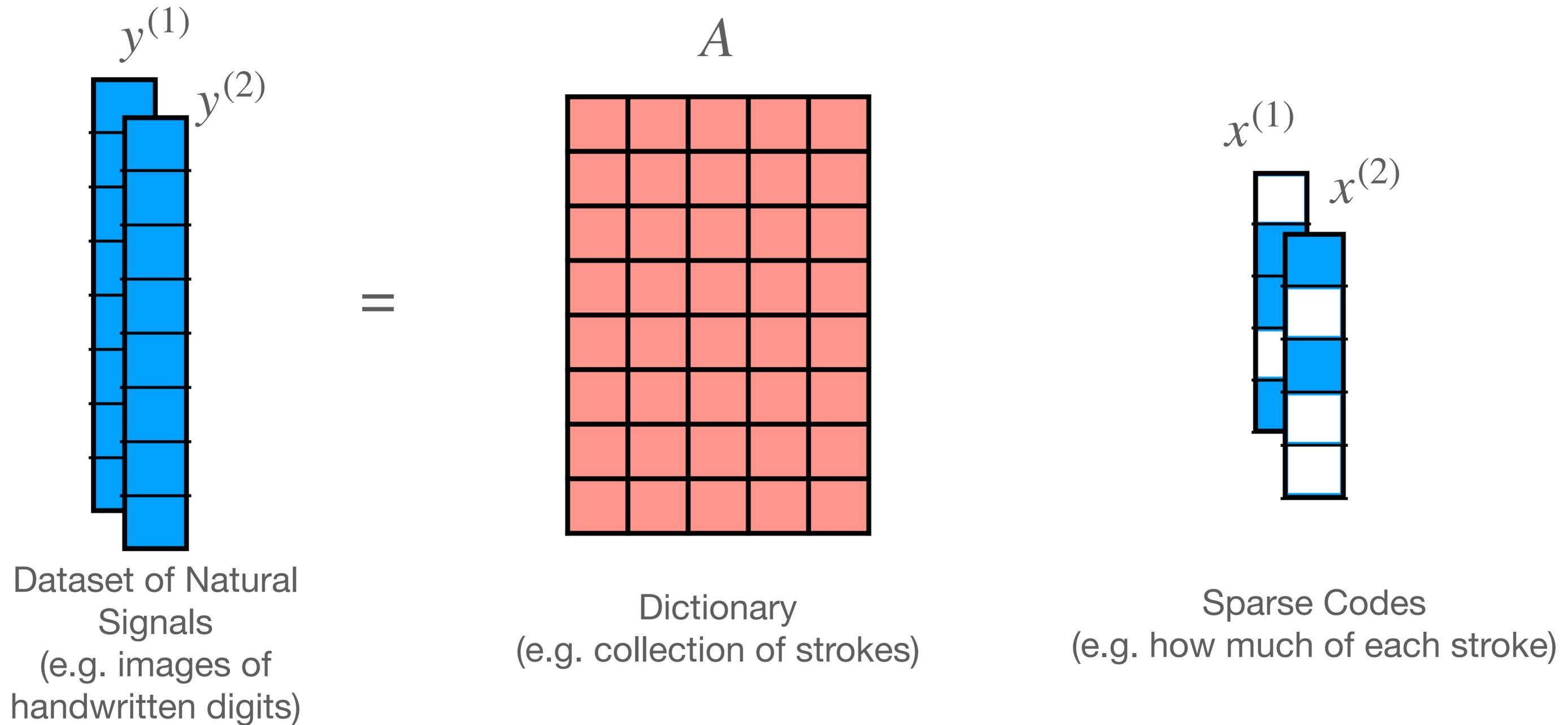
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Consequences of  
signal processing  
model!

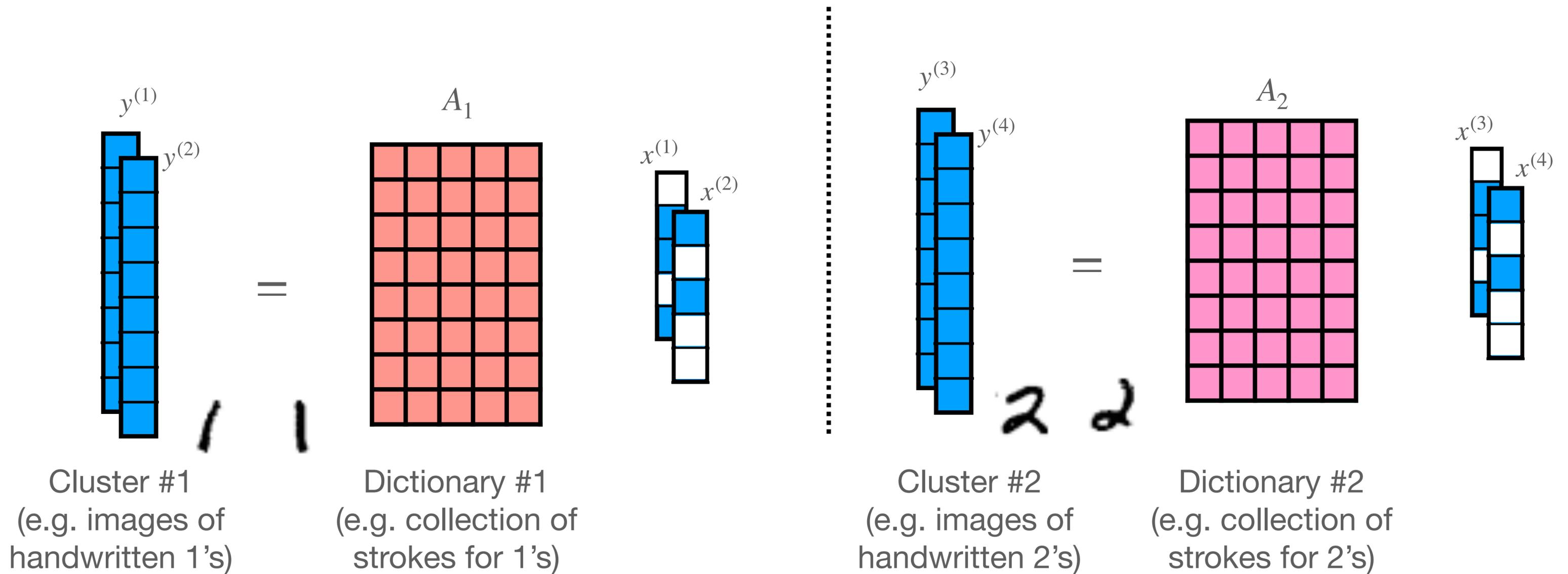
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## Intro to Dictionary Learning



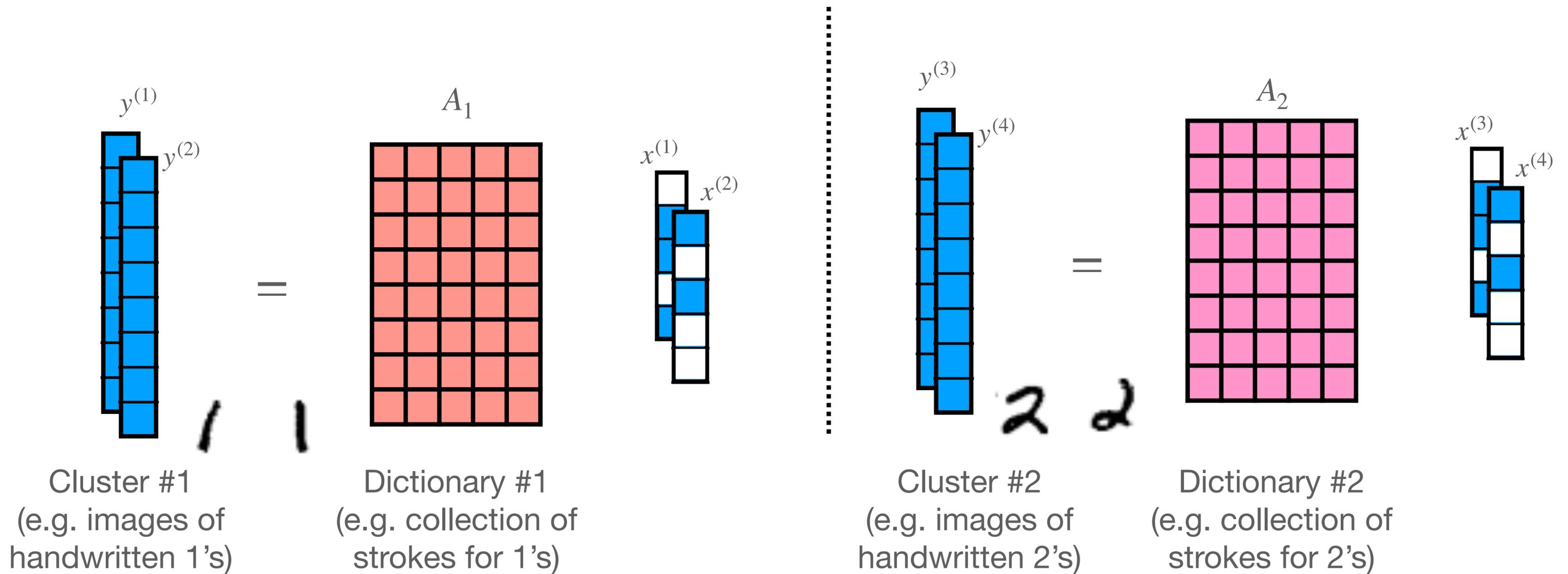
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Key Insight: Each cluster of images is generated by a *different* dictionary.

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## Mixture of Dictionary Learning Models

Assume  $K$  total clusters.

(Latent) cluster identity:  $z \sim \text{Categorical}(\pi_1, \pi_2, \dots, \pi_K)$

(Latent) sparse code:  $x \sim \text{Laplace}(\lambda) \propto \exp(-\lambda ||x||_1)$

(Observed) data:  $y | x, z = k \sim \mathcal{N}(A_k x, I) \propto \exp(-||y - A_k x||_2^2)$

Goal: Learn parameters/dictionaries:  $A_1, A_2, \dots, A_K$

Infer latent variables (for each  $y$ ):  $z, x$

$$\sum_{k=1}^K \pi_k = 1$$

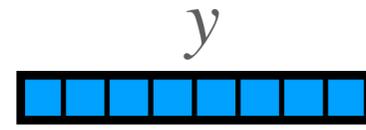
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# MixMate Architecture

## Diagram of Inference

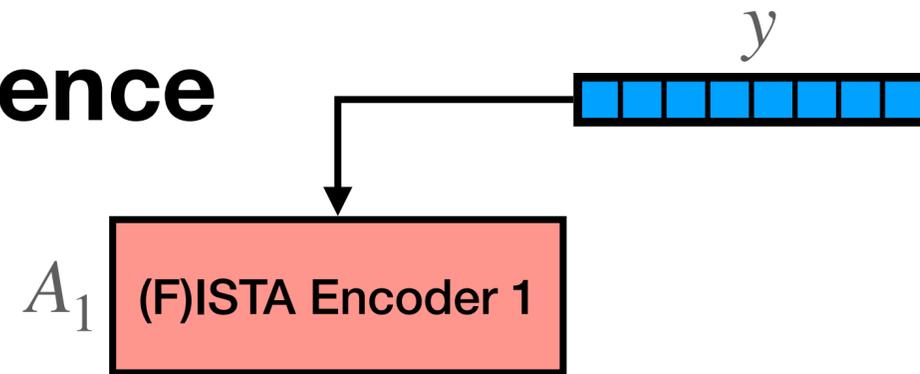
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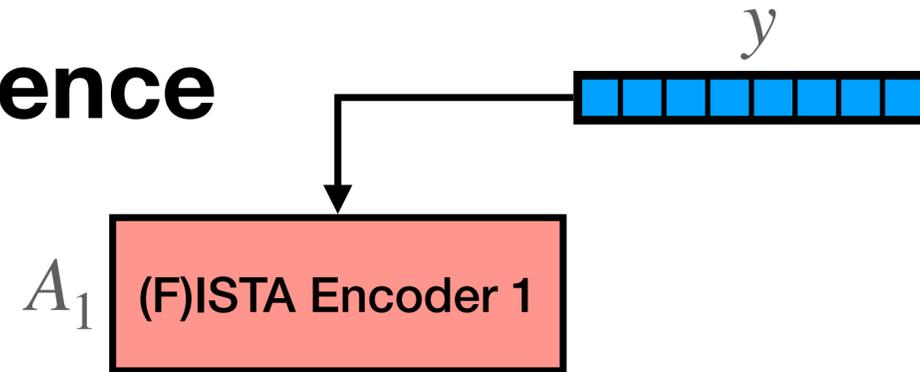
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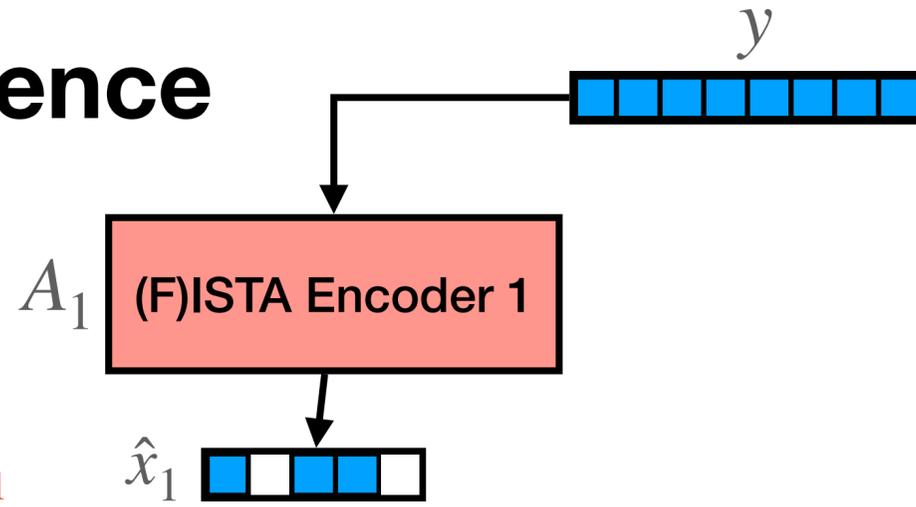


Sparse Coding Problem with  $A_1$

$$\hat{x}_1 = \arg \min_x ||y - A_1 x||_2^2 + \lambda ||x||_1$$

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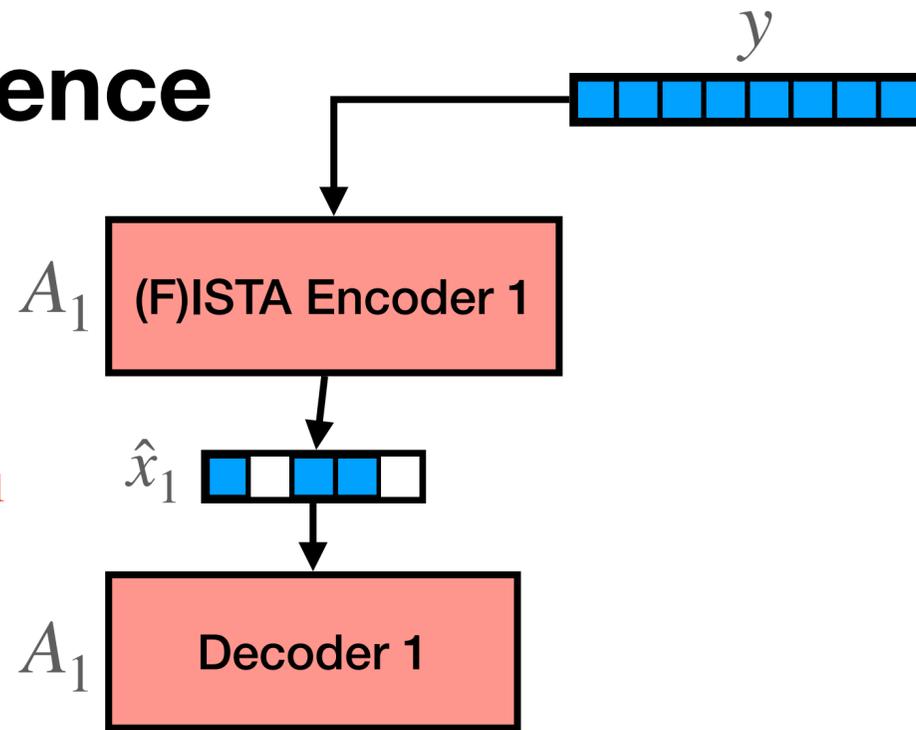


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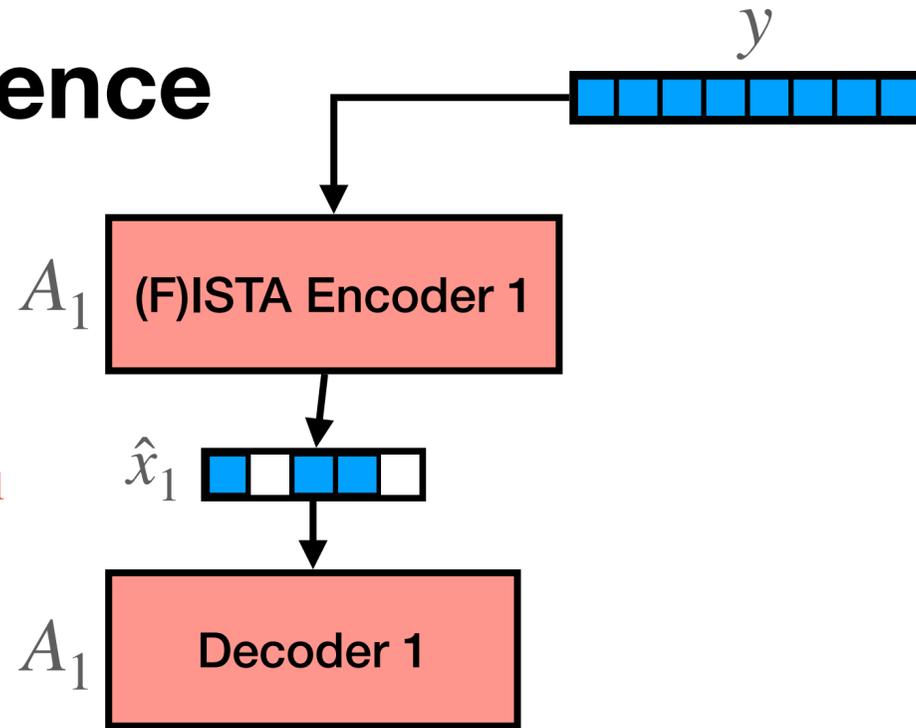


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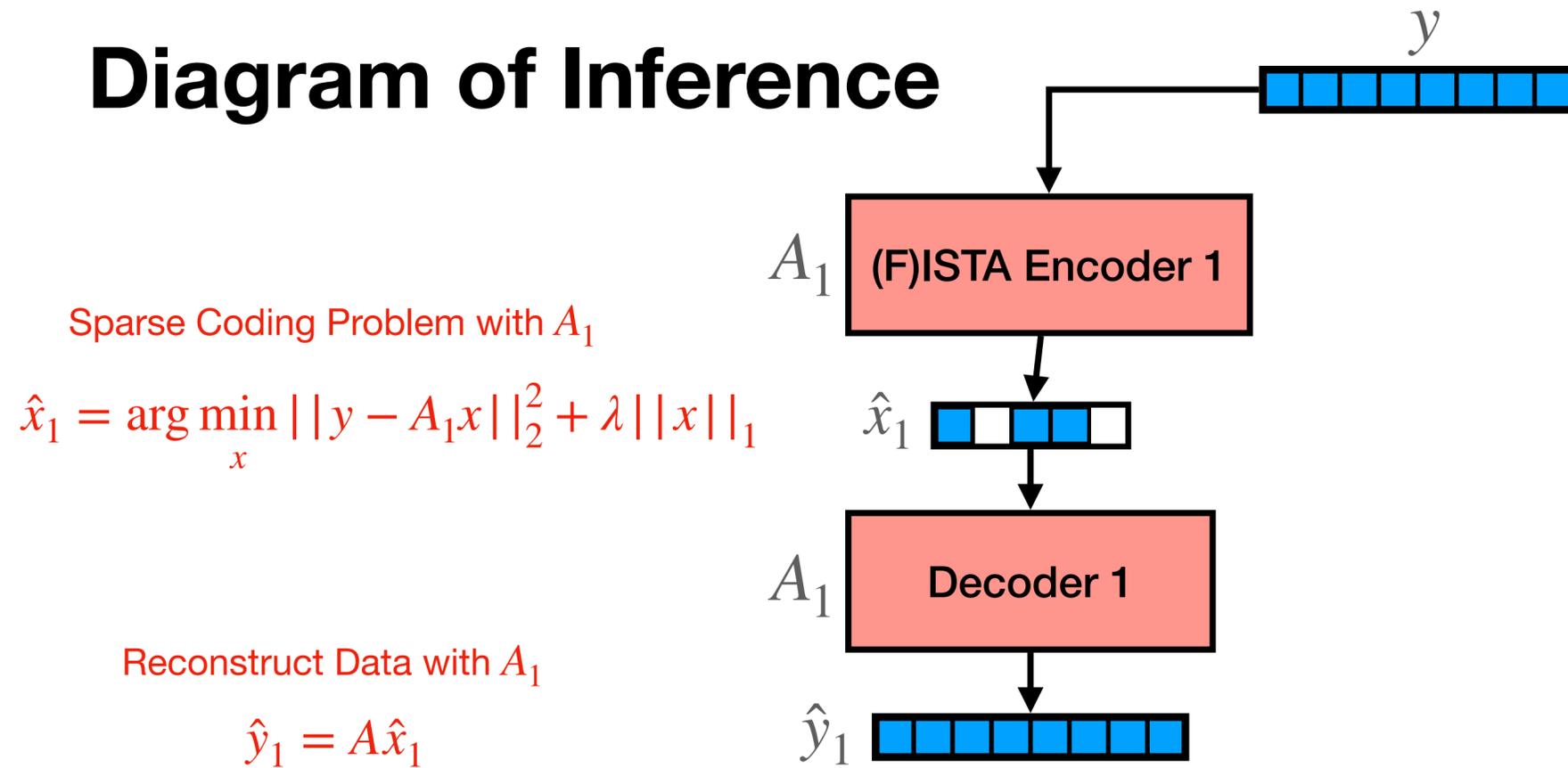
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Reconstruct Data with  $A_1$

$$\hat{y}_1 = A_1 \hat{x}_1$$

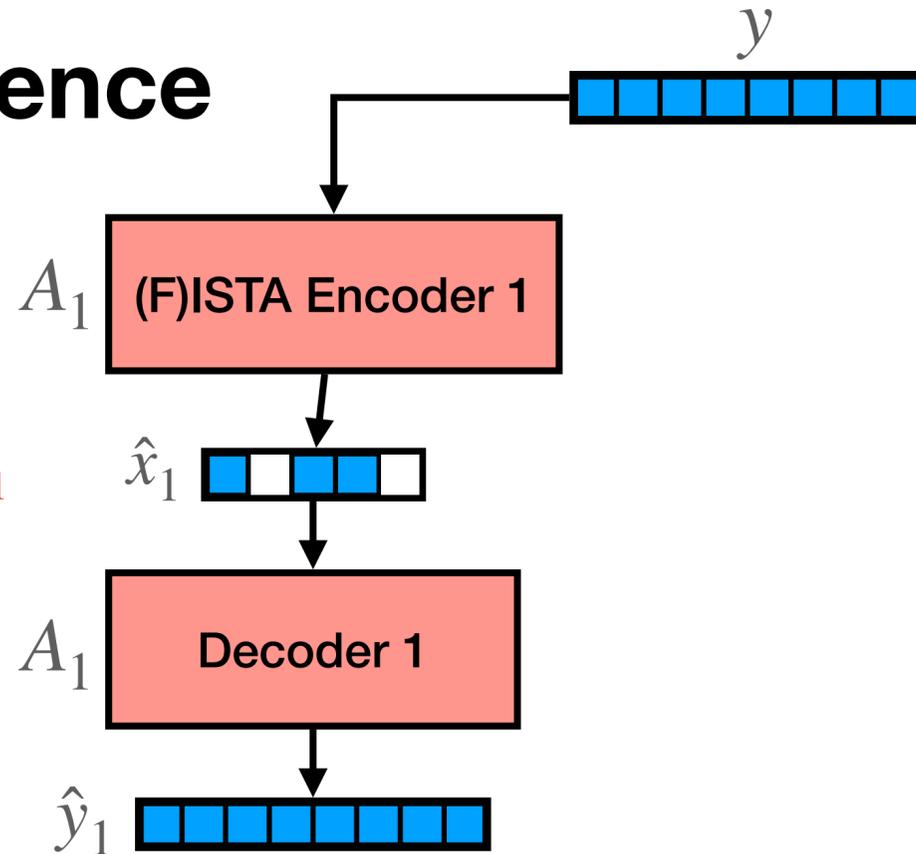
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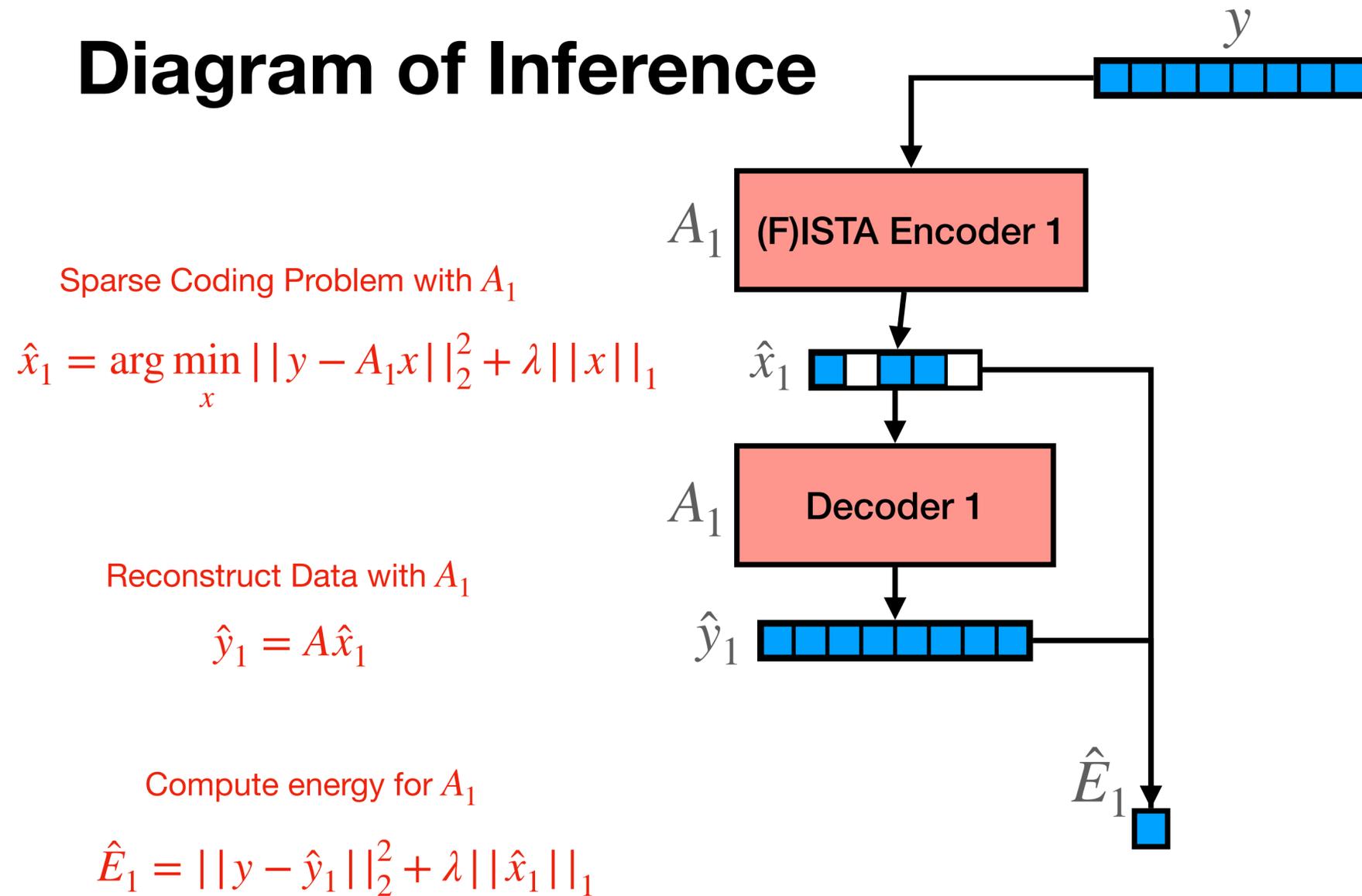
$$\hat{y}_1 = A \hat{x}_1$$

Compute energy for  $A_1$

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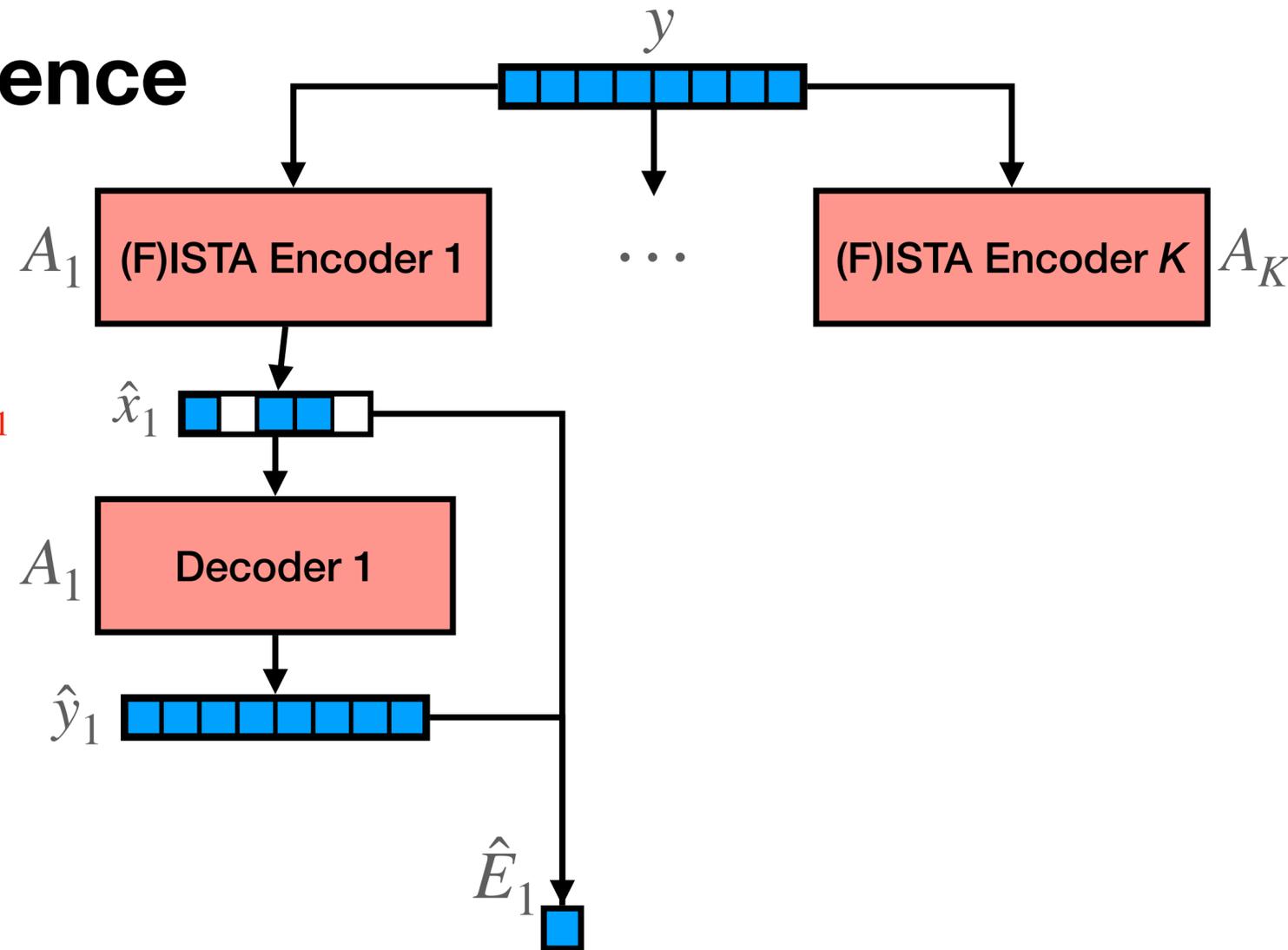
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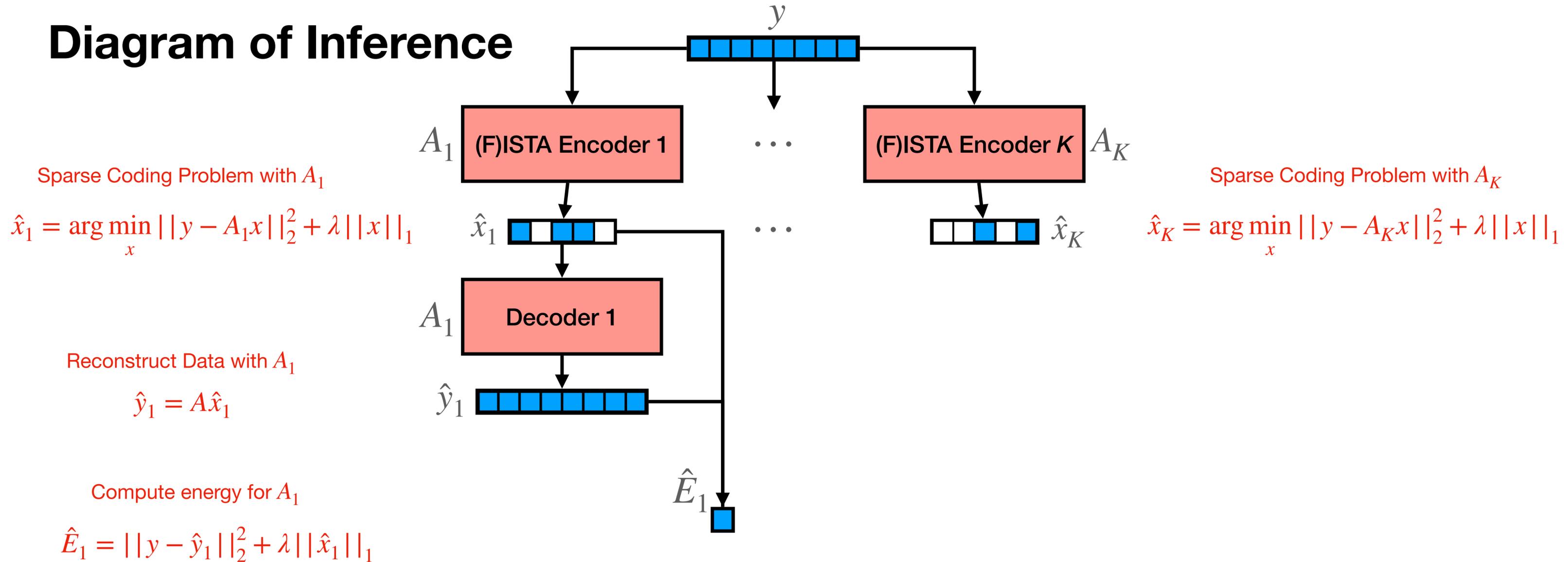
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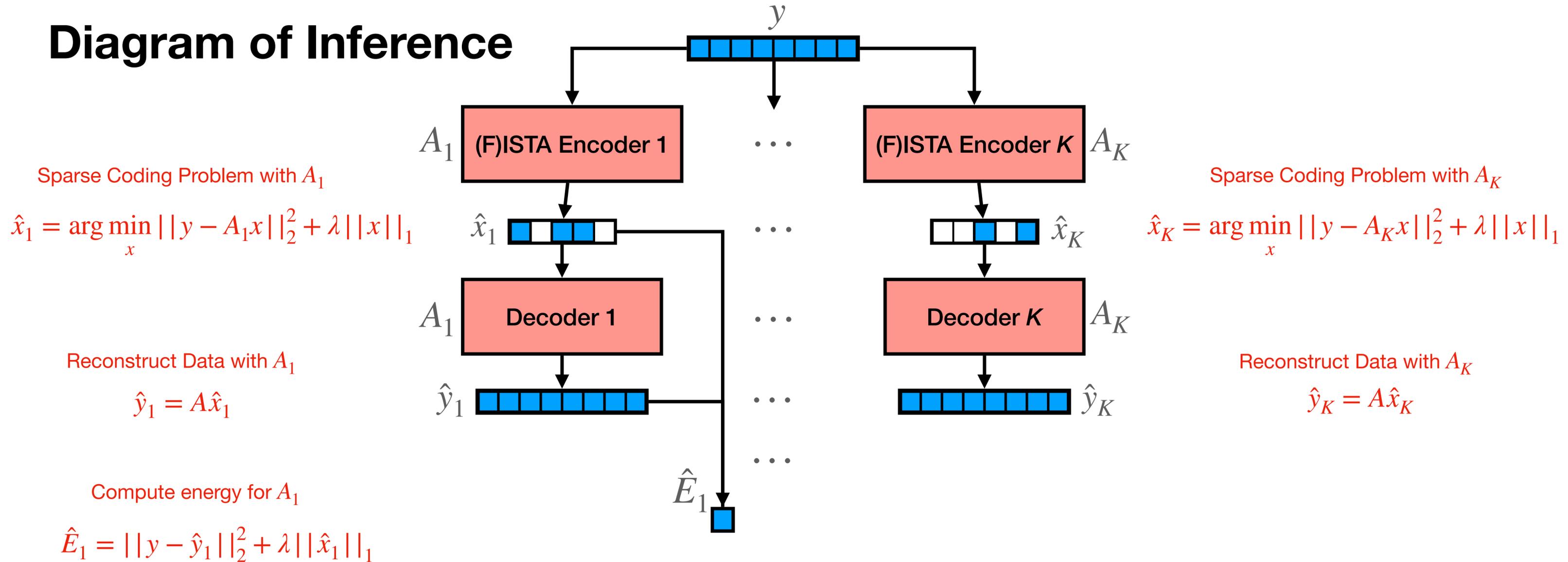
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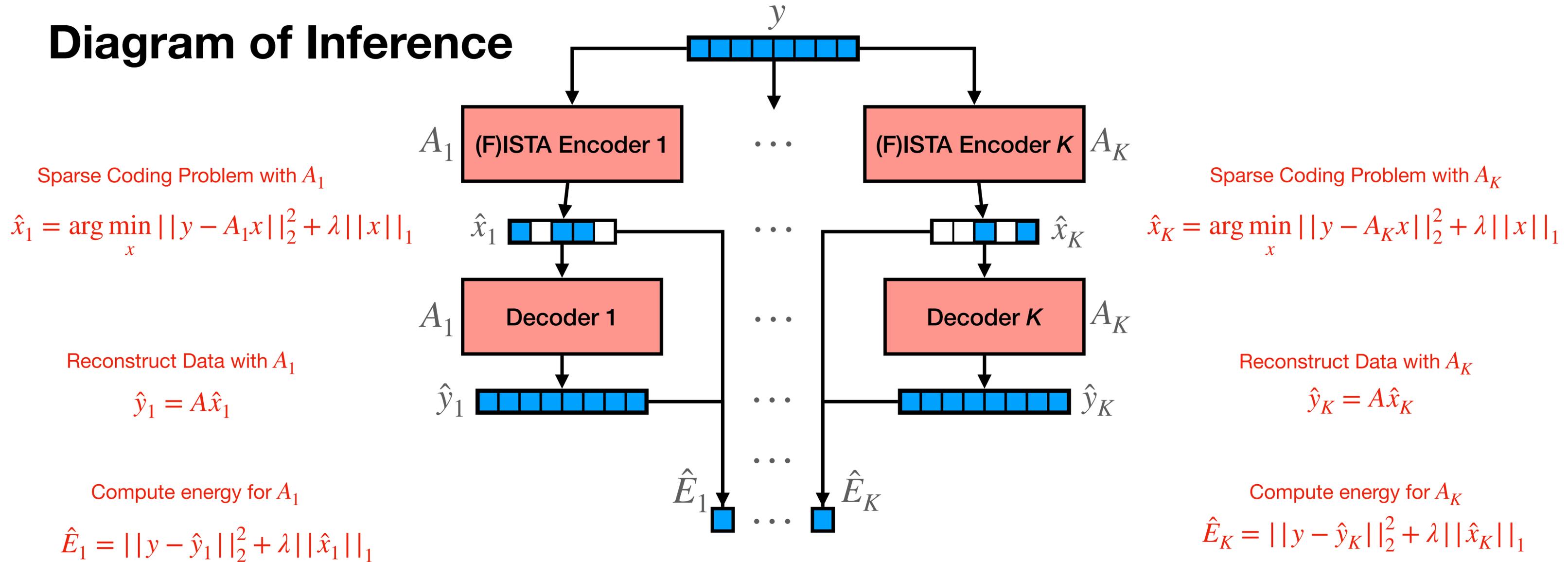
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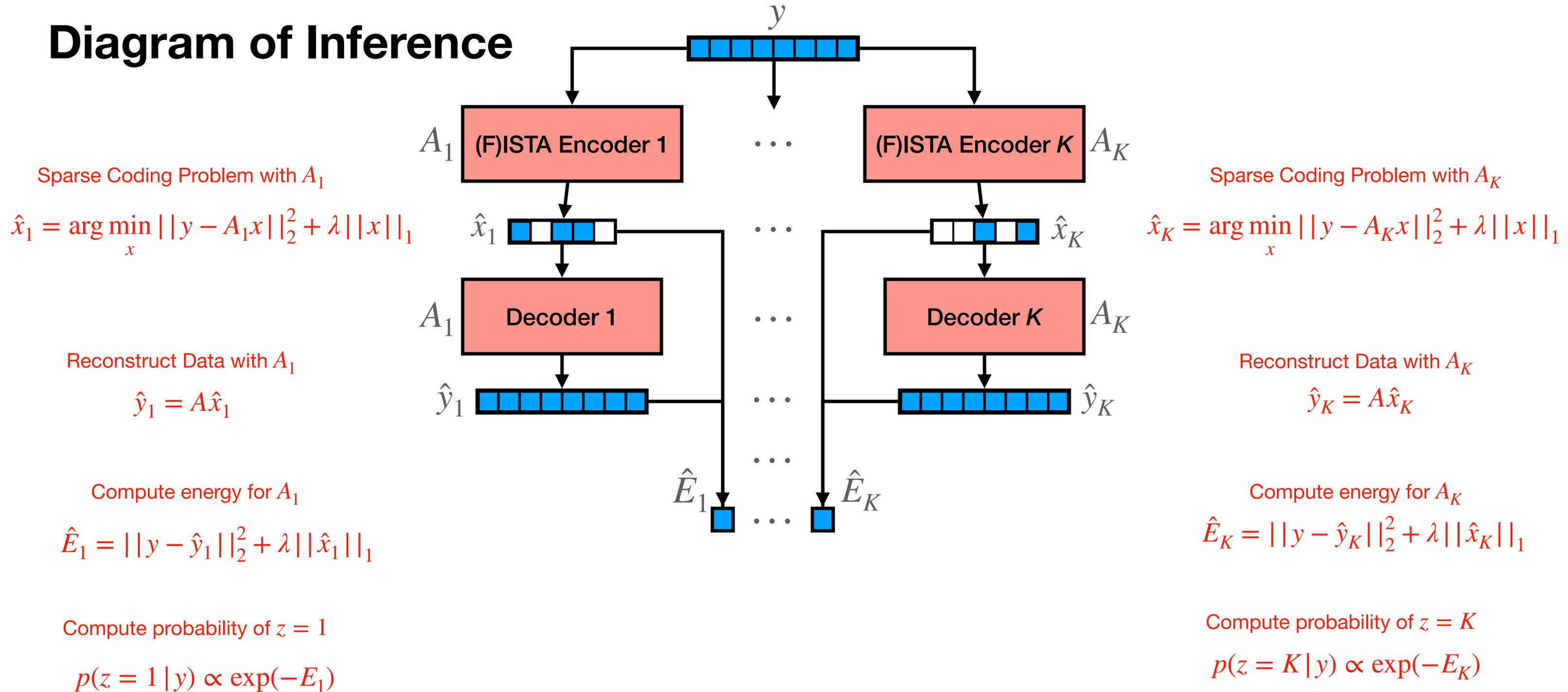
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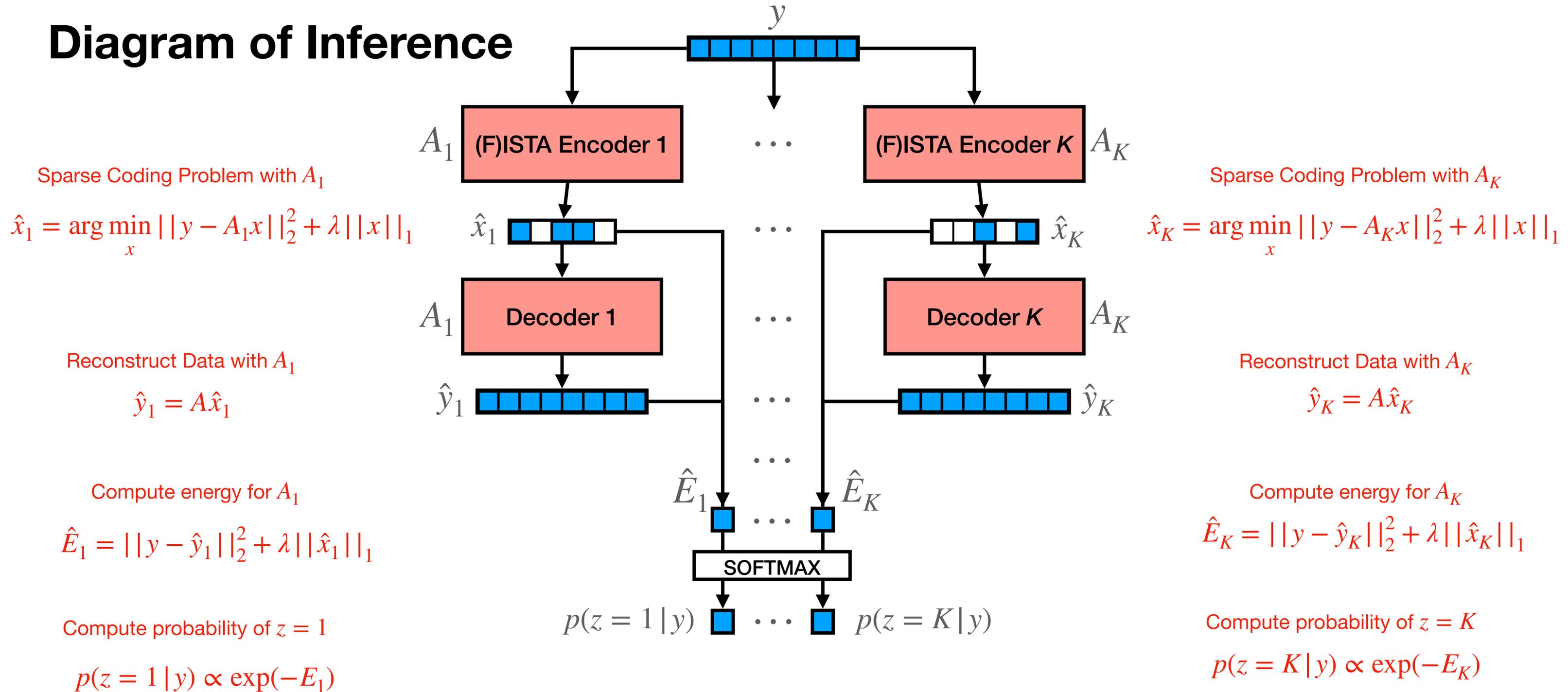
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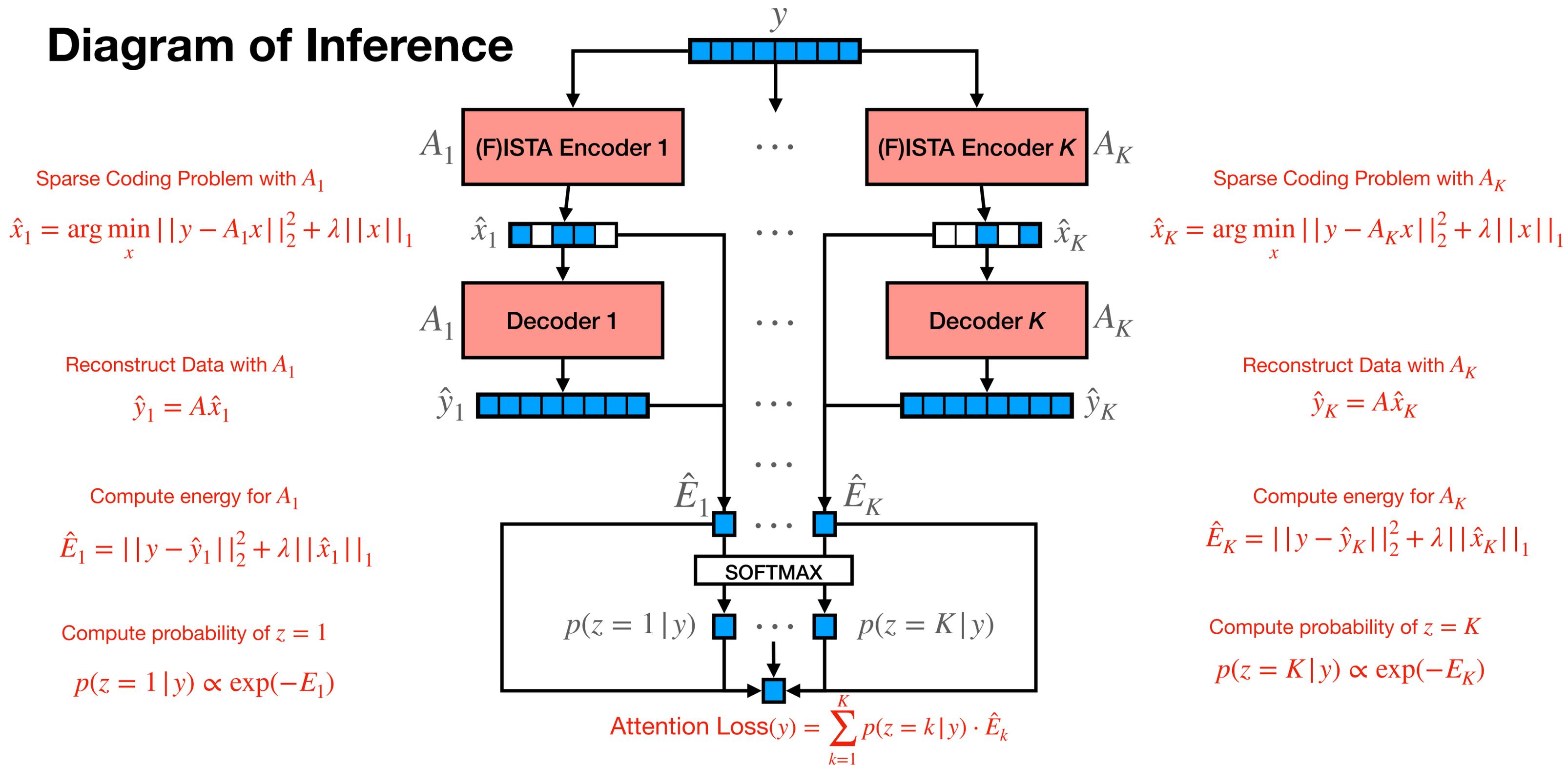
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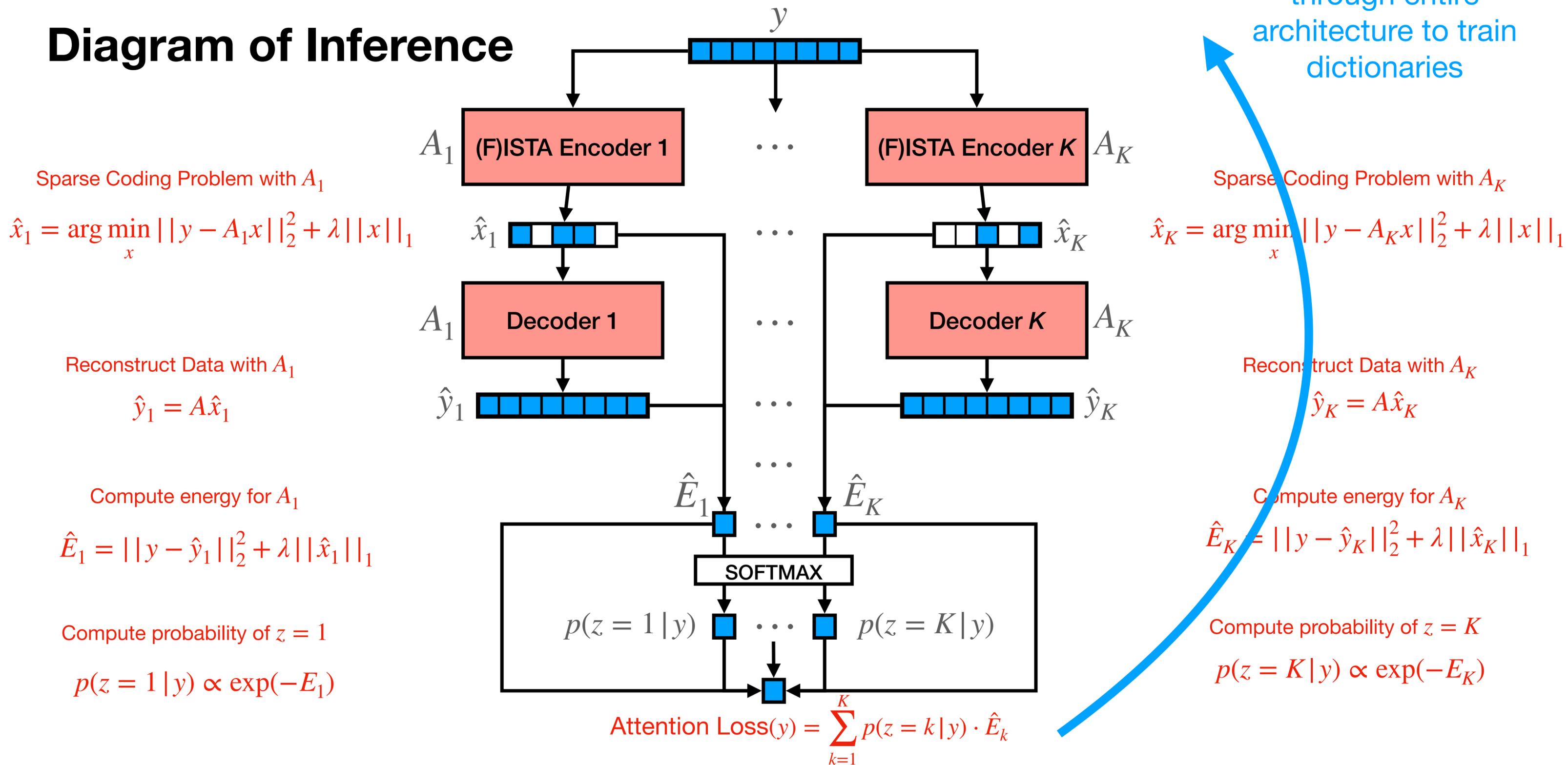
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# Results: Image Clustering Datasets

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<b>MNIST</b>						
NMI	0.80	0.81	<b>0.86</b>	<b>0.86</b>	0.75	<b>0.86</b> $\pm$ 0.03
ARI	0.75	0.75	0.82	0.82	0.72	<b>0.85</b> $\pm$ 0.04
ACC	0.84	0.83	0.88	0.88	0.84	<b>0.92</b> $\pm$ 0.04
Params	2.1 M	2.1 M	22.1 M	21.4 M		0.4 M
<b>Fashion</b>						
NMI	0.54	0.55	0.65	0.65	0.60	<b>0.68</b> $\pm$ 0.02
ARI	0.40	0.42	0.48	0.48	0.44	<b>0.52</b> $\pm$ 0.01
ACC	0.51	0.50	0.60	0.60	0.57	<b>0.63</b> $\pm$ 0.01
Params	N/A	N/A	N/A	N/A		0.4 M
<b>USPS</b>						
NMI	0.77	0.68	0.78	0.80	0.79	<b>0.82</b> $\pm$ 0.01
ARI	N/A	N/A	0.70	0.71	0.73	<b>0.76</b> $\pm$ 0.02
ACC	0.76	0.69	0.75	0.77	0.79	<b>0.81</b> $\pm$ 0.03
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Params	N/A	N/A	N/A	N/A		0.08 M

Clustering metrics  
(higher=better,  
1.0 is best)

MixMate obtains  
best performance  
on all metrics for  
all datasets...

...with the fewest  
number of  
parameters (up to  
50x fewer)!

[1] Opoichinsky, Y., Chazan, S. E., Gannot, S., & Goldberger, J. (2020, May). K-autoencoders deep clustering. In *ICASSP 2020-2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)* (pp. 4037-4041). IEEE.

# Results: Image Clustering Datasets

Other state-of-the-art deep clustering networks [1]

	DEC*	DCN*	DAMIC*	K-DAE*	MixMate	
					INIT	TRAIN
<b>MNIST</b>						
NMI	0.80	0.81	<b>0.86</b>	<b>0.86</b>	0.75	<b>0.86</b> ± 0.03
ARI	0.75	0.75	0.82	0.82	0.72	<b>0.85</b> ± 0.04
ACC	0.84	0.83	0.88	0.88	0.84	<b>0.92</b> ± 0.04
Params	2.1 M	2.1 M	22.1 M	21.4 M		0.4 M
<b>Fashion</b>						
NMI	0.54	0.55	0.65	0.65	0.60	<b>0.68</b> ± 0.02
ARI	0.40	0.42	0.48	0.48	0.44	<b>0.52</b> ± 0.01
ACC	0.51	0.50	0.60	0.60	0.57	<b>0.63</b> ± 0.01
Params	N/A	N/A	N/A	N/A		0.4 M
<b>USPS</b>						
NMI	0.77	0.68	0.78	0.80	0.79	<b>0.82</b> ± 0.01
ARI	N/A	N/A	0.70	0.71	0.73	<b>0.76</b> ± 0.02
ACC	0.76	0.69	0.75	0.77	0.79	<b>0.81</b> ± 0.03
Params	N/A	N/A	N/A	N/A		0.08 M

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These numbers don't  
change even if 90% of  
images have 25% of  
pixels missing →  
MixMate is robust to  
incomplete data!

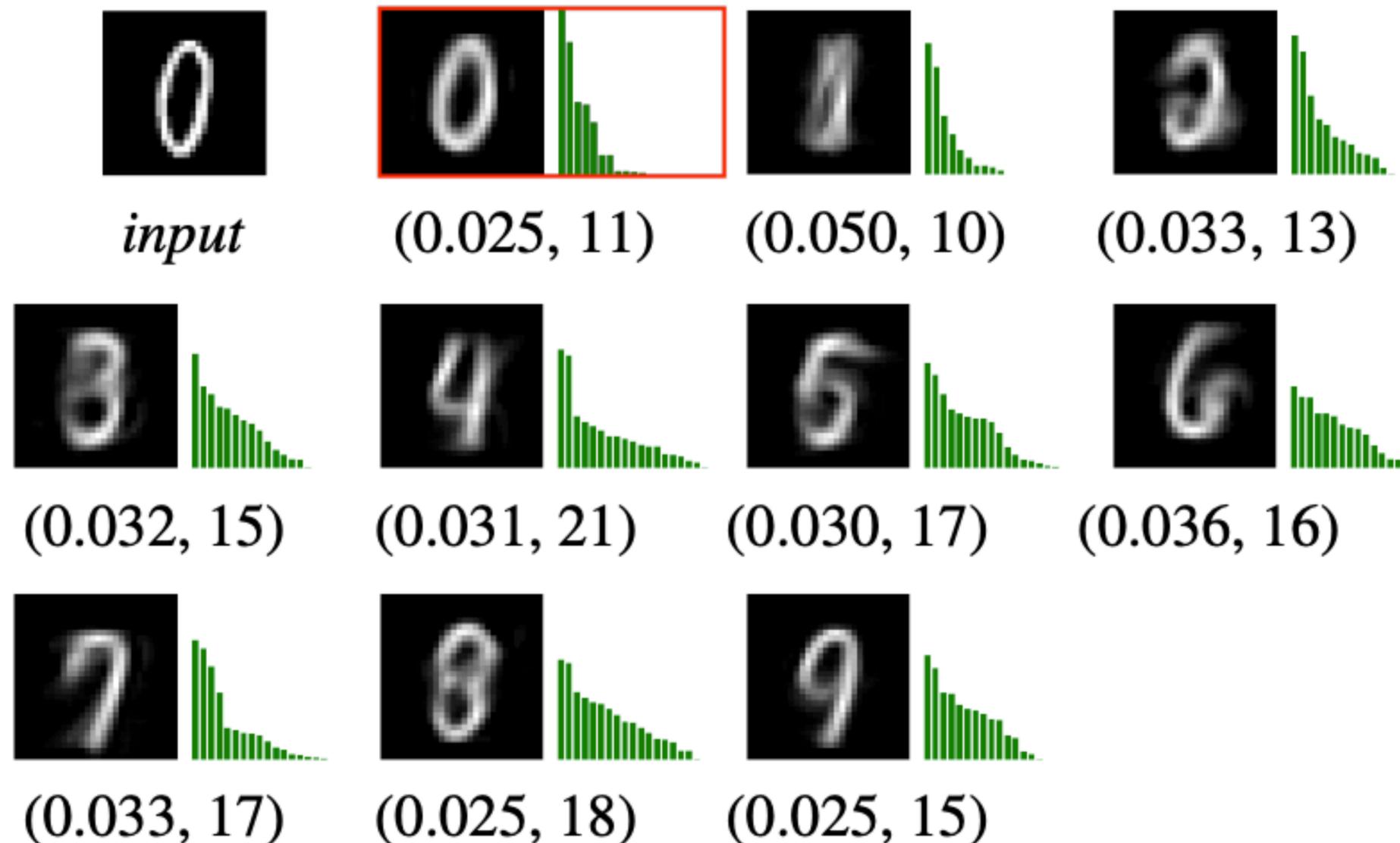
[1] Opoichinsky, Y., Chazan, S. E., Gannot, S., & Goldberger, J. (2020, May). K-autoencoders deep clustering. In *ICASSP 2020-2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)* (pp. 4037-4041). IEEE.

# Why does MixMate work so well?

Latent sparsity really helps!

Each of the 10 auto-encoder's output is labeled with (recon error, L0 norm of code)

Clustering depends on *both* reconstruction error and latent sparsity.

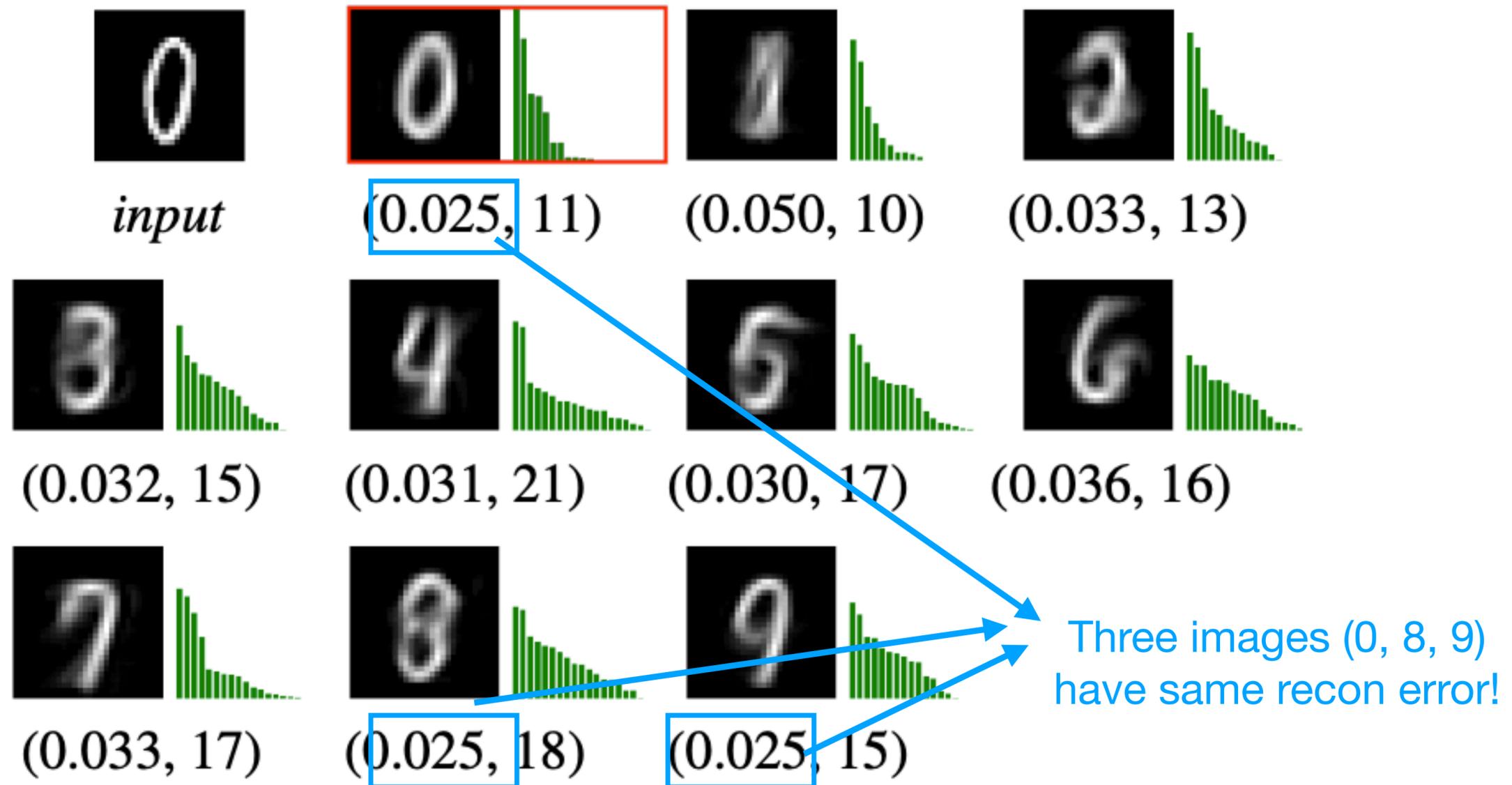


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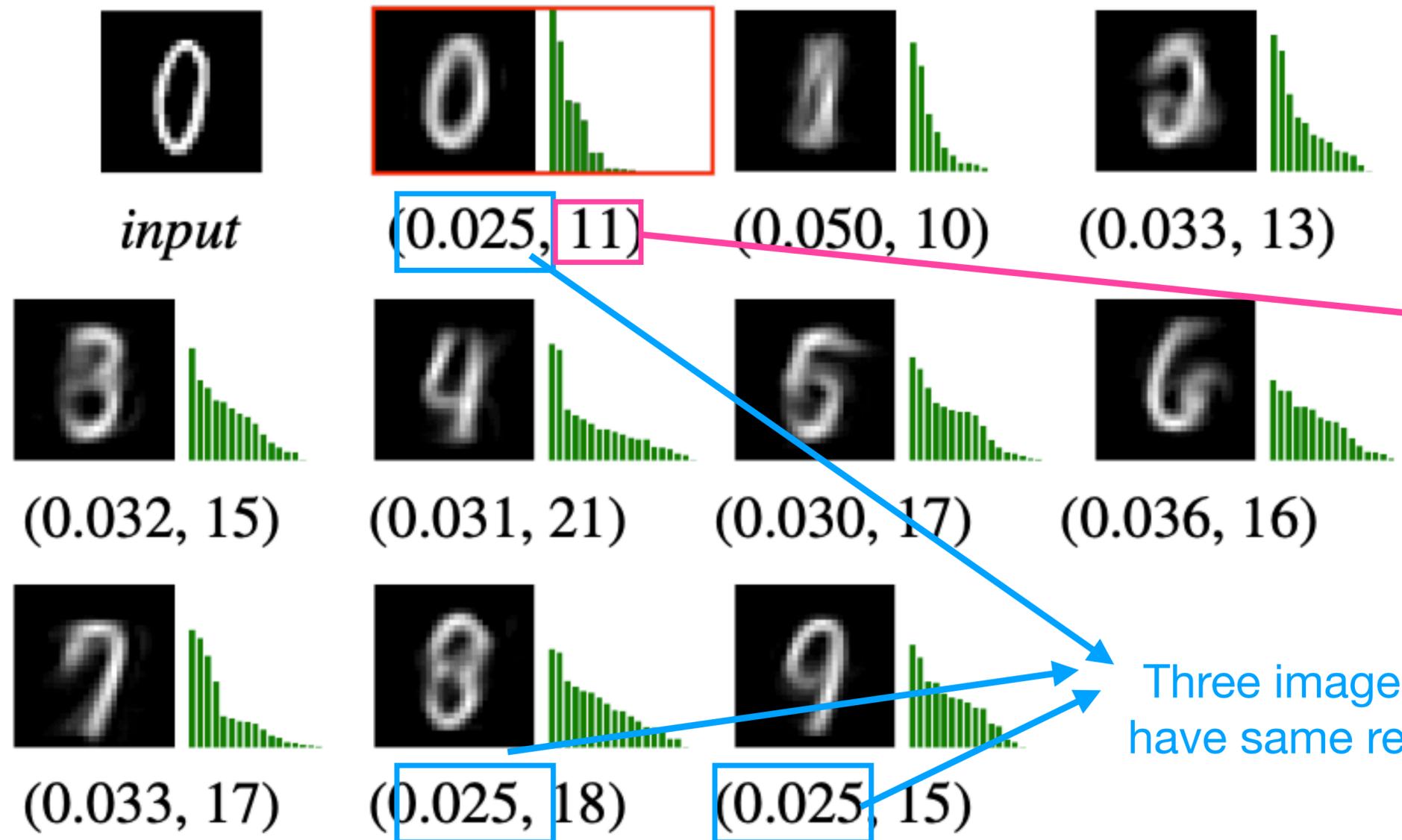


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Cluster 0 has the sparsest code → data is clustered correctly as cluster 0!

Three images (0, 8, 9) have same recon error!

# For more information...

- Paper: <https://ieeexplore.ieee.org/document/9747848>
  - Lin, A., Song, A. H., & Ba, D. (2022, May). **Mixture Model Auto-Encoders: Deep Clustering through Dictionary Learning**. In *ICASSP 2022-2022 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)* (pp. 3368-3372). IEEE.
  - More information on theory behind our MixMate architecture, initialization scheme, tuning the sparsity level, etc.
- Code: <https://github.com/al5250/mixmate>