Rate-Distortion-Classification Model In Lossy Image Compression

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Rate-distortion (RD) theory is a fundamental theory for lossy image compression that treats compressing the original images to a specified bitrate with minimal signal distortion, which is an essential metric in practical application. Moreover, with the development of visual analysis applications (such as classification, detection, segmentation, etc.), the semantic distortion in compressed images are also an important dimension in the theoretical analysis of lossy image compression. In this paper, we model the rate-distortion-classification (RDC) trade-off in lossy image compression based on the previous RD model. Specifically, the classification task is used as a representative image vision analysis task to calculate the semantic distortion. For the joint optimization modeling of RDC, the optimization objective function is the code rate expressed by the mutual information $I(\cdot,\cdot)$ with the constraints of MSE loss $\mathbb{E}[\Delta(\cdot,\cdot)]$ and the classification task error rate ε, where ε is defined by Equation (2). Define the binary classifier as:

$$c(t) = c(t \mid \mathcal{R}) = \begin{cases} \omega_1, & \text{if } t \in \mathcal{R} \\ \omega_2, & \text{otherwise} \end{cases} \tag{1}$$

where $\mathcal{R}$ is the set of values of t corresponding to the category ω_1. Denote X as the original image and $\hat{X}$ as the degraded reconstructed image. Then the classification error rate obtained from the input binary classifier can be expressed as:

$$\begin{aligned} \text{Classification Error Rate } &:= \varepsilon(\hat{X} \mid c) = \varepsilon(\hat{X} \mid \mathcal{R}) \\ &= P_2 \sum_{\hat{x} \in \mathcal{R}} p_{\hat{X}2}(\hat{x}) + P_1 \sum_{\hat{x} \notin \mathcal{R}} p_{\hat{X}1}(\hat{x}) \end{aligned} \tag{2}$$

Definition In lossy image compression, RDC is defined as:

$$\begin{aligned} I(X,\hat{X}) = \min_{P_{\hat{X}|X}} & I(X,\hat{X}), \\ & \text{s.t. } \mathbb{E}[\Delta(X,\hat{X})] \leq D, \varepsilon(\hat{X} \mid C_0) \leq E \end{aligned} \tag{3}$$

where $C_0 = c(\cdot \mid \mathcal{R}_0)$ is the predefined binary classifier, D and E are constraint conditions. Under certain conditions, the RDC model satisfies the monotonic non-increasing and convex function properties, which are first analyzed statistically on a multi-distribution source, and then further discussed through experiments on the MNIST dataset. Our conclusions could be inspiring for human-machine friendly compression methods and emerging Video Coding for Machine (VCM) approaches.

Reference: Yochai Blau and Tomer Michaeli, "Rethinking lossy compression: The rate-distortion-perception tradeoff," in International Conference on Machine Learning (ICML), 2019.
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