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Practical construction of sensing matrices for a greedy sparse recovery algorithm over finite fields

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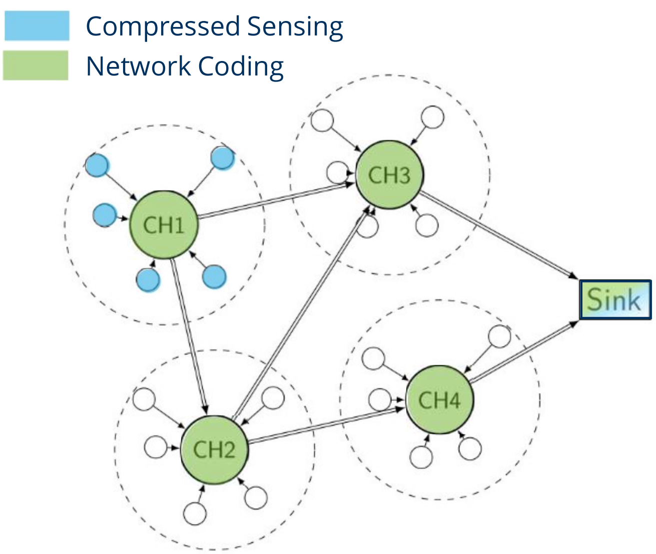
Content

Motivations



Wireless Sensors Clustered Topology [1]

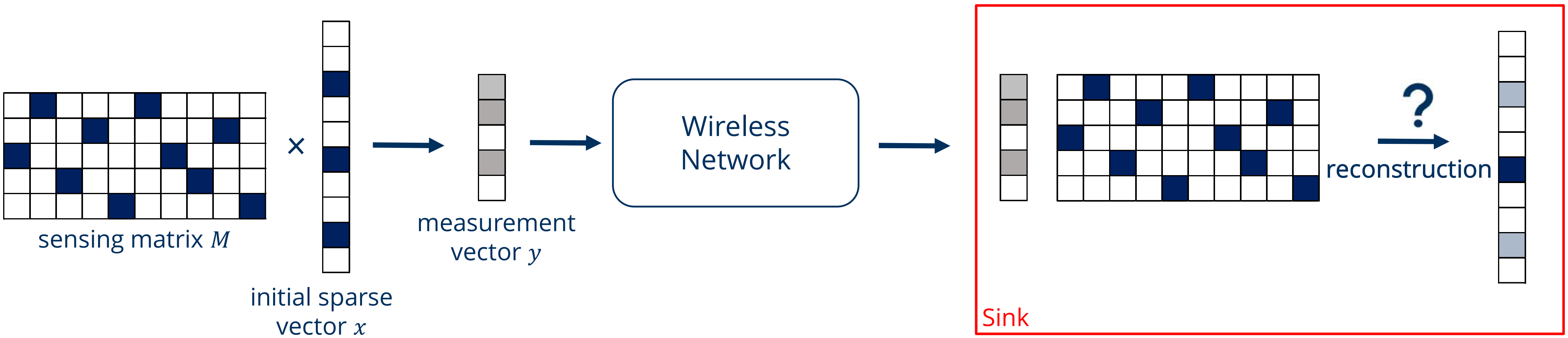
- Source Nodes Compression: to benefit from the spatial/temporal correlation
- Intra-clusters Network Coding: to increase robustness, and reduce retransmission cost
- Joint reconstruction: to overcome the all-or-nothing problem [2]

  - ➢ Real field or finite fields?
  - ➢ Challenges of Compressed Sensing over finite fields
  - ➢ F2OMP for practical use: first steps of the work

[1] Taghouti M, Kumar Chorppath A, Waurick T, and Fitzek F H.P. “Practical Compressed Sensing and Network Codingfor Intelligent Distributed Communication Networks.” 4th International Wireless Communications & Mobile Computing Conference (IWCMC), 2018, 962–68

[2] Soheil F, and Medard M. “A Power Efficient Sensing/Communication Scheme: Joint Source-Channel-Network Coding by Using Compressive Sensing.” In 49th Annual Allerton Conference on Communication, Control, and Computing (Allerton), 1048–54, 2011.

Compressed Sensing: quick overview

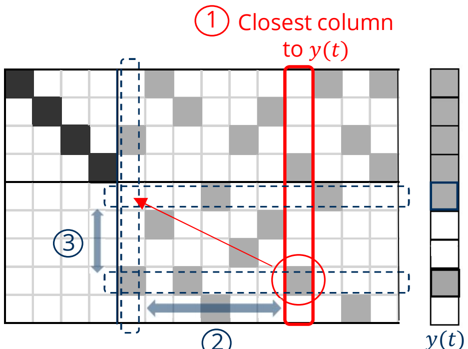


l_0-minimization problem: $\min_x \|x\|_0$ subject to $y = Mx$ with $\|x\|_0 = card(\{i : x_i \neq 0\})$

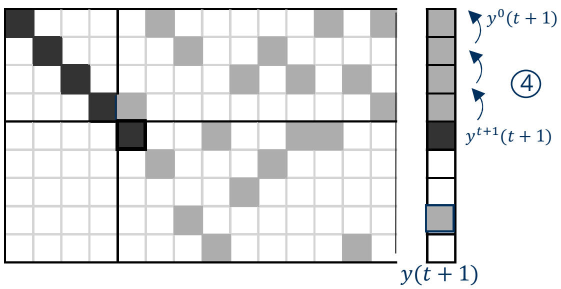
- Reconstruction algorithms:
  - l_1-minimization
  - Belief Propagation (BP)
  - (Orthogonal) Matching Pursuit (OMP)
  - …

State of the art : F2OMP a recovery algorithm over finite fields

Orthogonal Matching Pursuit for Compressed Sensing over finite fields [3]



Sensing Matrix at step t



Sensing Matrix at step $t+1$

At step t :

- Find the column with the minimum hamming distance to $y(t)$
- Swap rows/columns to have a non-zero pivot
- Gaussian elimination/substitution to calculate $y(t+1)$

Algorithm stops when $t = m$ or $(m - t)$ final components of y are equal to 0

[3] Valerio Bioglio, Giulio Coluccia, and Enrico Magli. "Sparse image recovery using compressed sensing over finite alphabets," in Proceedings of the IEEE International Conference on Image Processing (ICIP), Oct. 2014.

F2OMP – Loop

Problem:
- several vectors are at the same minimal distance to y(t)
- once a decision is made - no way back

➢ F2OMP-Loop: repeat when it is obviously wrong

$$\begin{pmatrix} 0 & 6 & 0 & 1 & 0 & 4 \\ 7 & 0 & 1 & 0 & 3 & 0 \\ 5 & 0 & 2 & 3 & 0 & 0 \\ 0 & 4 & 0 & 0 & 7 & 5 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 3 \\ 0 \end{pmatrix} \xrightarrow[\text{arithmetic operations}]{\text{F2OMP}} \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 5 \\ 0 \end{pmatrix}$$

Sensing Matrices $GF(2^4)$

Initial Vector $GF(2^2)$

$(1, 0, 2, 0)^T$: Might be correct → stop

$(1, 0, \mathbf{5}, 0)^T$: Obviously wrong: $5 \notin GF(2^2)$ → start over

Sensing Matrices for F2OMP

$$\begin{pmatrix} 0 & \mathbf{6} & 0 & \mathbf{1} & 0 & \mathbf{4} \\ \mathbf{7} & 0 & \mathbf{1} & 0 & \mathbf{3} & 0 \\ 0 & 0 & \mathbf{2} & 0 & 0 & \mathbf{5} \\ \mathbf{5} & 0 & 0 & \mathbf{3} & 0 & 0 \\ 0 & \mathbf{4} & 0 & 0 & \mathbf{7} & 0 \end{pmatrix}$$

Sensing Matrices M

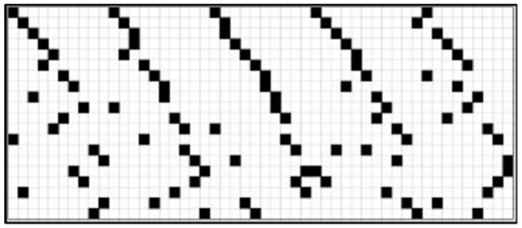
$GF(2^4)$; $d = 2$

F : Finite field of the form $GF(2^p)$ with $p \in \{1, 2, 4, 8, 16\}$
M : Sparse matrix with elements in F
d : Number of non-zero elements per columns

- How to build these matrices in practice?
- How to choose the parameters?
- Do all these matrices have the same recovery performance?

Sparse binary matrices over the real field [4]

- Parity-check matrices of Low-Density Parity-Check (LPDC) code
- Construction method: Progressive Edge Growth (PEG)
- Compressed Sensing over the real field (OMP)



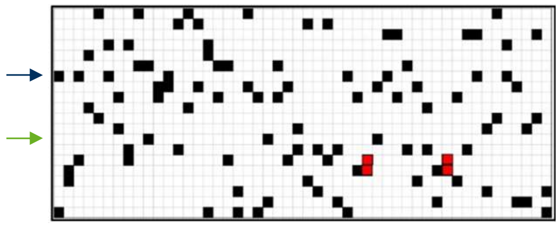
Parity-check matrix of LPDC code

- ➢ Outperform Gaussian matrices and sparse random matrices.
- ➢ Optimal value for d that is parameter-dependent.
- ➢ Perform better when number of *4-cycles* is minimum.

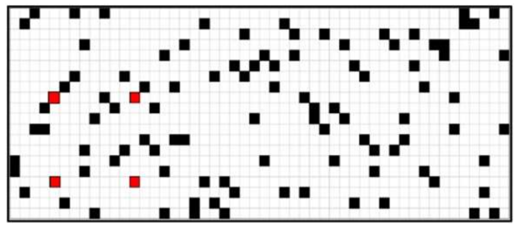
$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

[4] Weizhi Lu, Kidiyo Kpalma, Joseph Ronsin. Sparse Binary Matrices of LDPC codes for Compressed Sensing. Data Compression Conference (DCC), Apr 2012, Snowbird (Utah), United States. 10 p. ffhal-00659236.

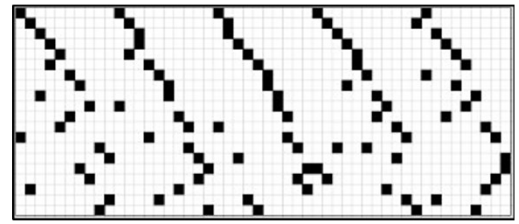
Practical construction of sensing matrices over finite fields



generated with Evencol



generated with Evenboth



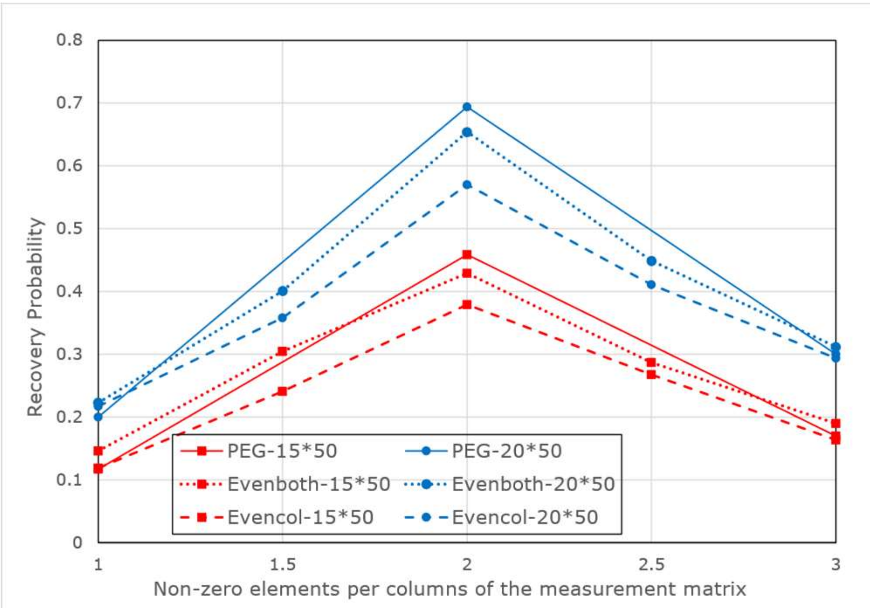
generated with PEG

- Comparison of 3 construction methods of parity-check matrices [5] [6]
- Recovery over finite fields: “Success or Failure”
- Simulation with matrices of different sizes and over different fields
  - ➢ Changing the **position** of the non zero elements has a **higher impact on the recovery** performance than changing the **values** in the matrix

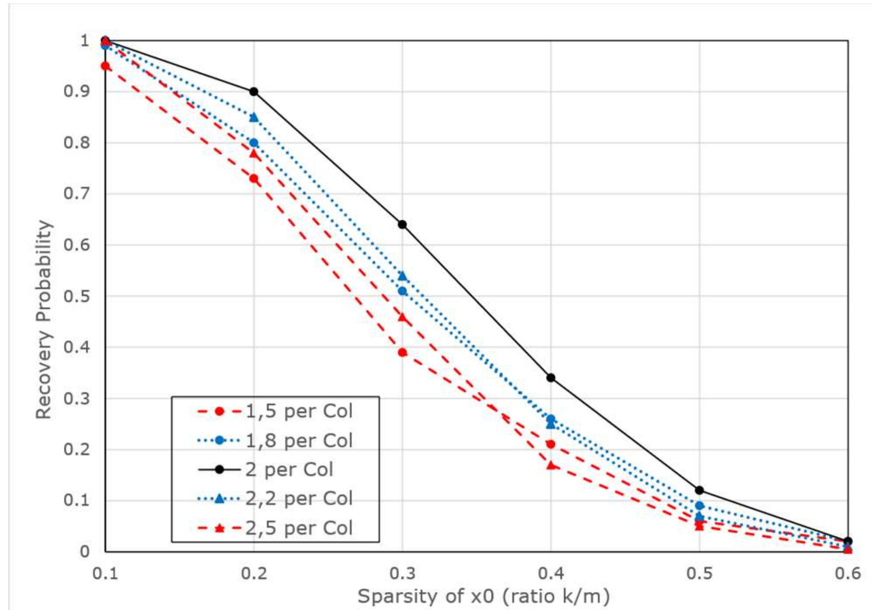
[5] D. J. C. MacKay and R. M. Neal, “Near Shannon limit performance of low density parity check codes,” Electronics Letters, vol. 32, no. 18, pp. 1645–1646, Aug. 1996.

[6] Xiao Yu Hu, Evangelos Eleftheriou, and Dieter M. Arnold, “Regular and irregular progressive edge-growth tanner graphs,” IEEE Transactions on Information Theory, vol. 51, no. 1, pp. 386–398, Jan. 2005.

Simulation Results with different construction methods

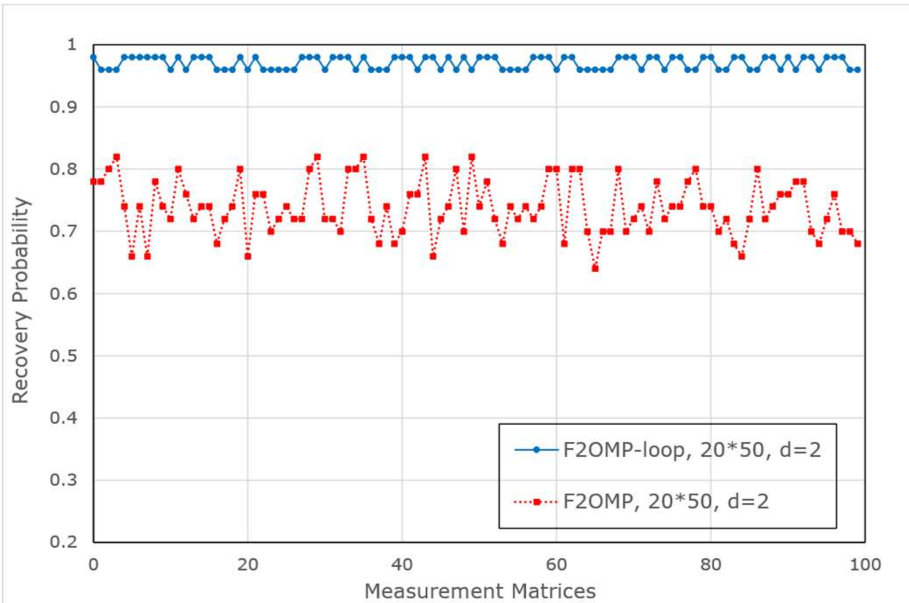


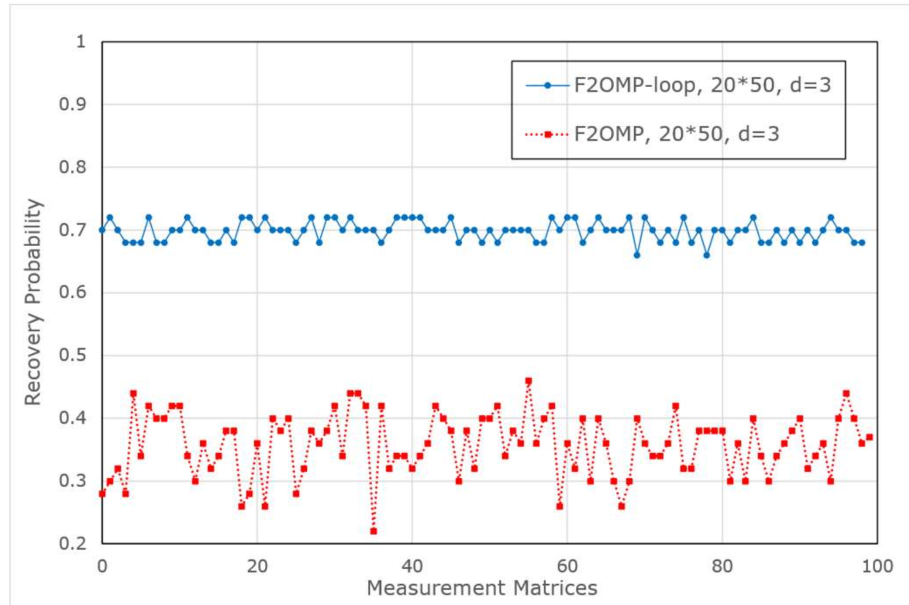
Recovery probability depending on the number of non-zero elements per column for 50 matrices of size 15×50 (red) and 20×50 (blue) generated via PEG, Evenboth and Evencol.



Recovery probability depending on the sparsity of X0 for 50 matrices of size 20×50 (blue) generated via Evenboth with various distribution of non-zero elements per column.

Simulation Results for up to 20 repetitions of F2OMP





Recovery probability of 100 matrices of size 20×50 generated via PEG when applying F2OMP (red) and F2OMP-loop (blue) with 2 non-zero elements per column (left) and 3 non-zero elements per column (right).

Conclusion and outlook

- F2OMP-loop based on some prior knowledge on the initial vector
- Overview of sensing matrices for F2OMP
- Construction of efficient sensing matrices
- Simulations to demonstrate the gain in reliability of F2OMP-loop

➢ Practical requirements to operate Compressed Sensing over finite fields

➢ Conditions on the sensing / coding matrices

➢ Integration of F2OMP into a joint scheme

References

- [1] Taghouti, Kumar Chorppath A, Waurick T, and Fitzek F H.P.”Practical Compressed Sensing and Network Codingfor Intelligent Distributed Communication Networks.” 4th International Wireless Communications & Mobile Computing Conference (IWCMC), 2018, 962–68.
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- [5] D. J. C. MacKay and R. M. Neal. “Near Shannon limit performance of low density parity check codes,” Electronics Letters, vol. 32, no. 18, pp. 1645–1646, Aug. 1996
- [6] Xiao Yu Hu, Evangelos Eleftheriou, and Dieter M. Arnold. “Regular and irregular progressive edge-growth tanner graphs,” IEEE Transactions on Information Theory, vol. 51, no. 1, pp. 386–398, Jan. 2005.
- [7] Mégane Gammoudi, Christian Scheunert, Giang T. Nguyen, and Frank Fitzek.” Practical Construction of Sensing Matrices for a Greedy Sparse recovery algorithm over Finite Fields”. Data Compression Conference (DCC), Mar. 2023, Snowbird (Utah), United States.