IQGAN: Robust Quantum Generative Adversarial Network for Image Synthesis On NISQ Devices

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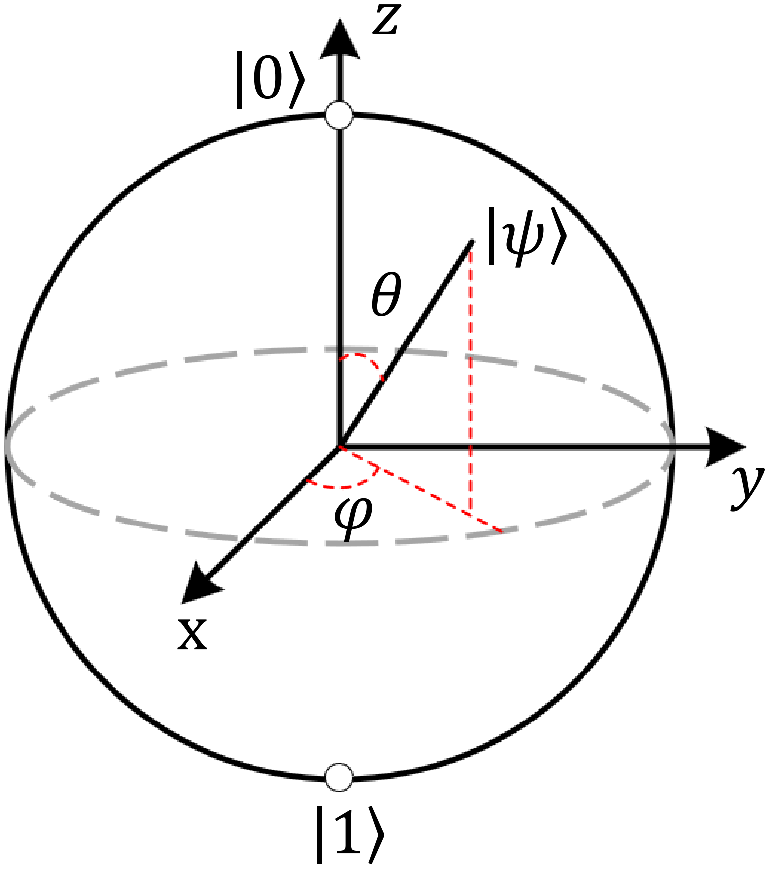
Qubit vs Bit

Classical Bit: 0 1

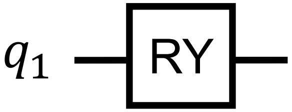
Quantum Bit: 0 1 ◐

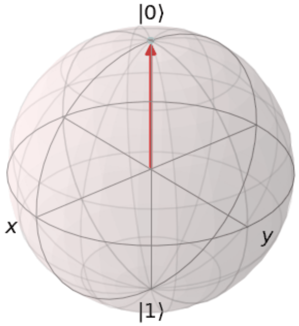
- Quantum Bit (Qubit):
  - $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$
  - $|\psi\rangle = \begin{bmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{bmatrix}$



Quantum Gates

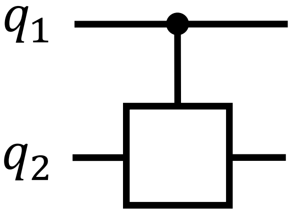
- Quantum gate ⇒ Matrix
  - Single qubit gate ⇒ 2*2
  - Two-qubit gate ⇒ 4*4
  - Multi-qubit gate ⇒ n*n





- Quantum gate operation
  - Matrix Multiplication

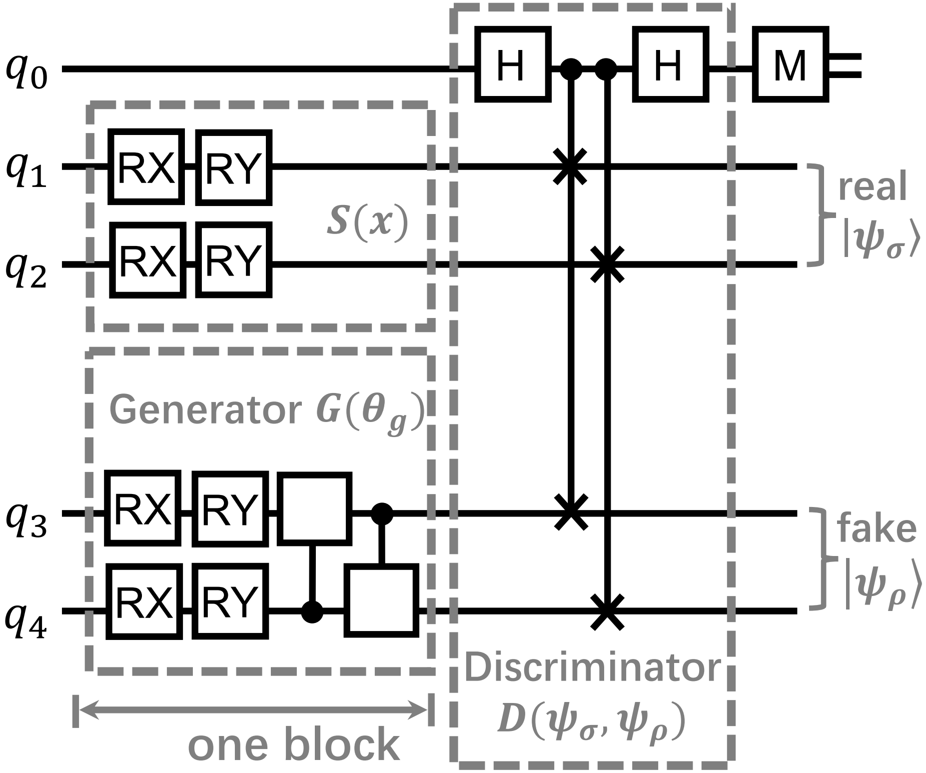
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} b_0' \\ b_1' \end{bmatrix}$$



$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

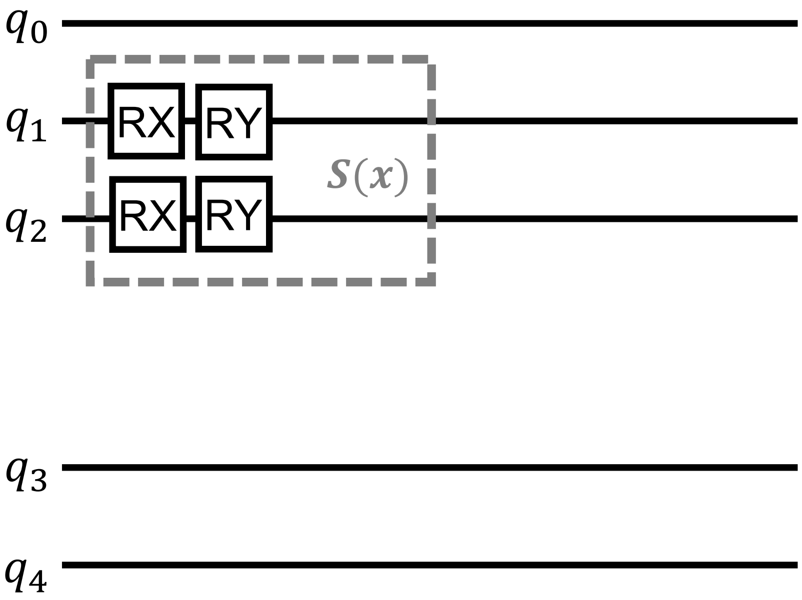
Quantum GANs framework

- Data encoder $\boldsymbol{S(x)}$
    - Embed a classical input $\boldsymbol{x}$ to a quantum state $|\boldsymbol{\psi_\sigma}\rangle$.
- Generator $\boldsymbol{G(\theta_g)}$
    - Generate synthetic data $|\boldsymbol{\psi_\sigma(\theta_g)}\rangle$.
- Discriminator $\boldsymbol{D(\psi_\sigma, \psi_\rho)}$
    - Measure the fidelity between the real data $|\boldsymbol{\psi_\sigma}\rangle$ and the fake data $|\boldsymbol{\psi_\rho}\rangle$.
- Training Objective
    - $\min_{\theta_g} L(\theta_g) = \min_{\theta_g}[1 - \langle\psi_\sigma|\psi_\rho(\theta_g)\rangle^2]$



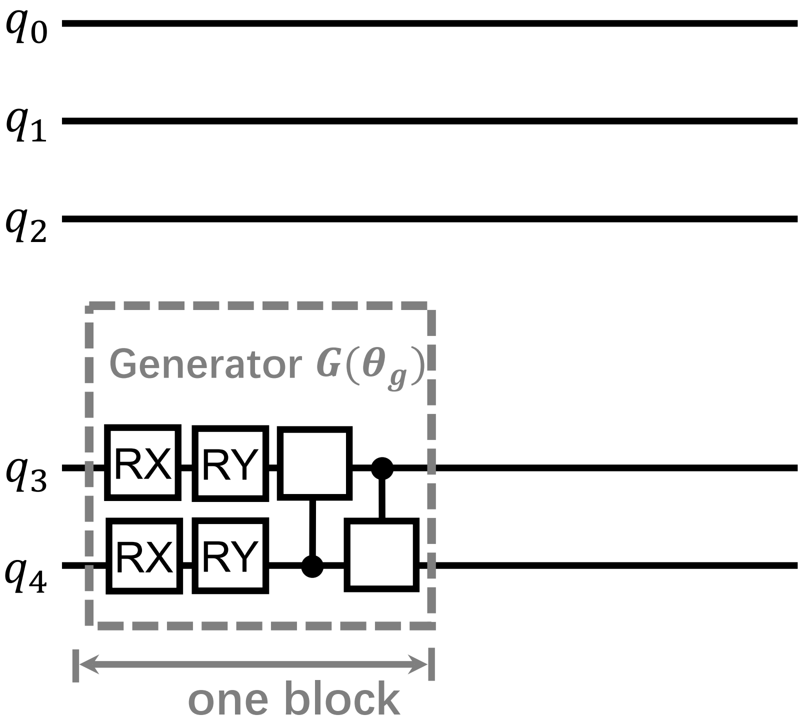
Data encoder $\boldsymbol{S(x)}$

- Encoding Methods
  - **Angle encoding**
  - Amplitude encoding
  - …
- Angle Encoding
  - Simple implementation.
  - Noise immunity.
- Angle Encoding formulation
  - $|\psi_X\rangle = \bigotimes_{i=0}^{N-1} R(x_i)|0\rangle$
  - $R \in \{RX, RY, RX\}$



Generator $G(\theta_g)$

- VQC Circuit Ansatz
  - Parameterized single-qubit gates
  - Two-qubit CNOT gates
- Parameterized single-qubit gates
  - RX, RY, RZ, H, U1, Rot, …
- Two-qubit CNOT gates
  - Provide maximal entanglement between the two target qubits.

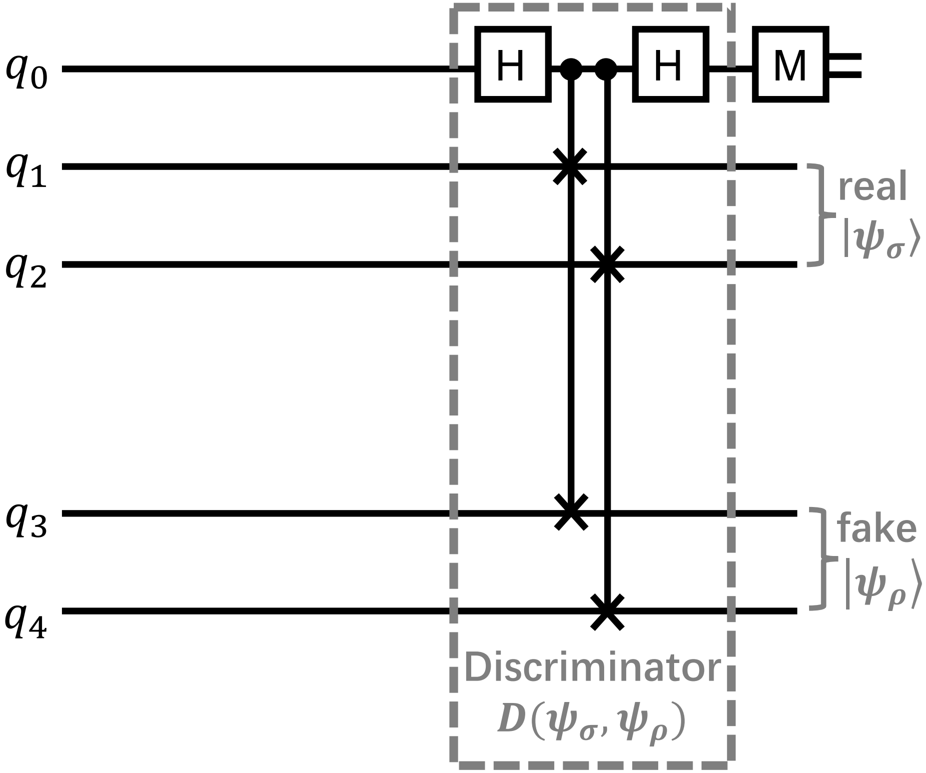


Discriminator $D(\psi_\sigma, \psi_\rho)$

- SWAP Test Circuit
  - One ancillary qubit q_0
  - Two Hadamard gates (H gate)
  - Several controlled SWAP gates

- Quantum fidelity measurement
  - Measure the ancillary qubit q_0.
  - Get fidelity P_0. (The probability it yields a measured output $|0\rangle$)

$$P_0 = \frac{1 + \langle \psi_\sigma | \psi_\rho \rangle^2}{2}$$



Quantum GANs Comparison

Scheme	Loss	Conv	MultiQ	Quality	Cost
QGAN [6]	N/A	N/A	N/A	N/A	N/A
QuGAN18 [7]	Trace	✓	✗	Med.	High
QuGAN21 [8]	Fidelity	✓	✓	Low	High
EQ-GAN [9]	Fidelity	✓	✗	Low	Med.
IQGAN	Fidelity	✓	✓	High	Low

[6] Seth Lloyd_Phys. Rev. Lett' 2018, [7] Pierre-Luc_Phys.Rev. A' 2018 ,

[8] Samuel A._QCE' 2021 , [9] Murphy Yuezhen Niu_arXiv' 2021

Preliminary Analysis

- Low-quality Generated Images
  - The de facto angle encoding (even with normalization) fails to ensure high-quality output in a fidelity-based GAN framework.
  - $\langle\psi_\rho|\psi_\sigma\rangle^2 = \left(\left[\cos\frac{\theta_\rho}{2},\ \sin\frac{\theta_\rho}{2}\right]\begin{bmatrix}\cos\frac{\theta_\sigma}{2}\\ \sin\frac{\theta_\sigma}{2}\end{bmatrix}\right)^2 = \left(\cos\frac{\theta_\rho}{2}\cos\frac{\theta_\sigma}{2}+\sin\frac{\theta_\rho}{2}\sin\frac{\theta_\sigma}{2}\right)^2$

$$= \left(\cos\frac{\theta_\rho+2n\pi}{2}\cos\frac{\theta_\sigma+2n\pi}{2}+\sin\frac{\theta_\rho+2n\pi}{2}\sin\frac{\theta_\sigma+2n\pi}{2}\right)^2$$

$$\Longrightarrow \theta_\rho + 2n\pi = \theta_\rho,\ \ \theta_\sigma + 2n\pi = \theta_\sigma \quad ?$$

- Complex and High-Cost Generator
  - Investigate the effect of two-qubit gates on the performance of a generative ansatz and set out to reduce the circuit complexity of a generator.

Trainable Multiqubit Quantum Encoder

- Variational encoder function
  - $S(\arcsin(x * \theta_s))$
- Pre-trained param θ_s.
  - Pre-trained data set.

    $\mathcal{T} = \{(x_i, y_i) | 0 \leq i \leq N-1\}$
  - Prepare quantum ensemble.

    $\sigma_{y_k} = \frac{1}{N_k} \sum_{j=0}^{j=N_k-1} |\psi_\sigma(x_j)\rangle \langle \psi_\sigma(x_j)|$
  - Train the $\boldsymbol{\theta_s}$ to maximize distance between $\boldsymbol{\sigma_{y_k}}$ and $\boldsymbol{\sigma_{y_m}}$, when $k \neq m$.

Task	Input Size	Qubit #	Accuracy (%)	
			FE	TE
MNIST-2	4*4	16	89.5	**90.9**
MNIST-4	2*2	4	43.2	**45.6**
MNIST-4	4*4	16	45.9	**49.4**
MNIST-8	4*4	16	23.25	**24.3**

Fixed Encoder (FE), Trainable Encoder (TE)

Compact Quantum Generator

2QGate	Block #	Nor. Cost	Fidelity	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
CNOT	2	2.8×	0.813	N/A							
ISWAP	2	3.7×	0.954	N/A							
CRX(θ)	2	6×	0.969	-0.007	-0.023	-0.228	0.034	-0.093	-0.154	-0.015	3.348
				0.003	0.036	1.799	0.018	-0.118	0.013	0.033	3.167
CROT (ϕ, θ, ω)	2	6×	0.969	0.911	-0.057	2.049	0.059	-1.883	0.013	2.827	0.09
				0.079	-0.086	-0.068	-0.971	-0.043	0.098	0.262	-0.062
w/o	N/A	1×	0.969	N/A							

- The generator w/o 2QGate in the table performs the same as CRX and CROT with 6× reduced hardware cost.

Experimental setup

- Schemes and Benchmarks

Scheme	Qubit#	1QG#	2QG#	Param.#
QuGAN21 [8]	2n+1	nb+1	4nb	5nb
EQ-GAN [9]	2n+1	2nb+n+2	(b+1)n	2nb
IQGAN	Fidelity	2nb+n+2	n	2nb

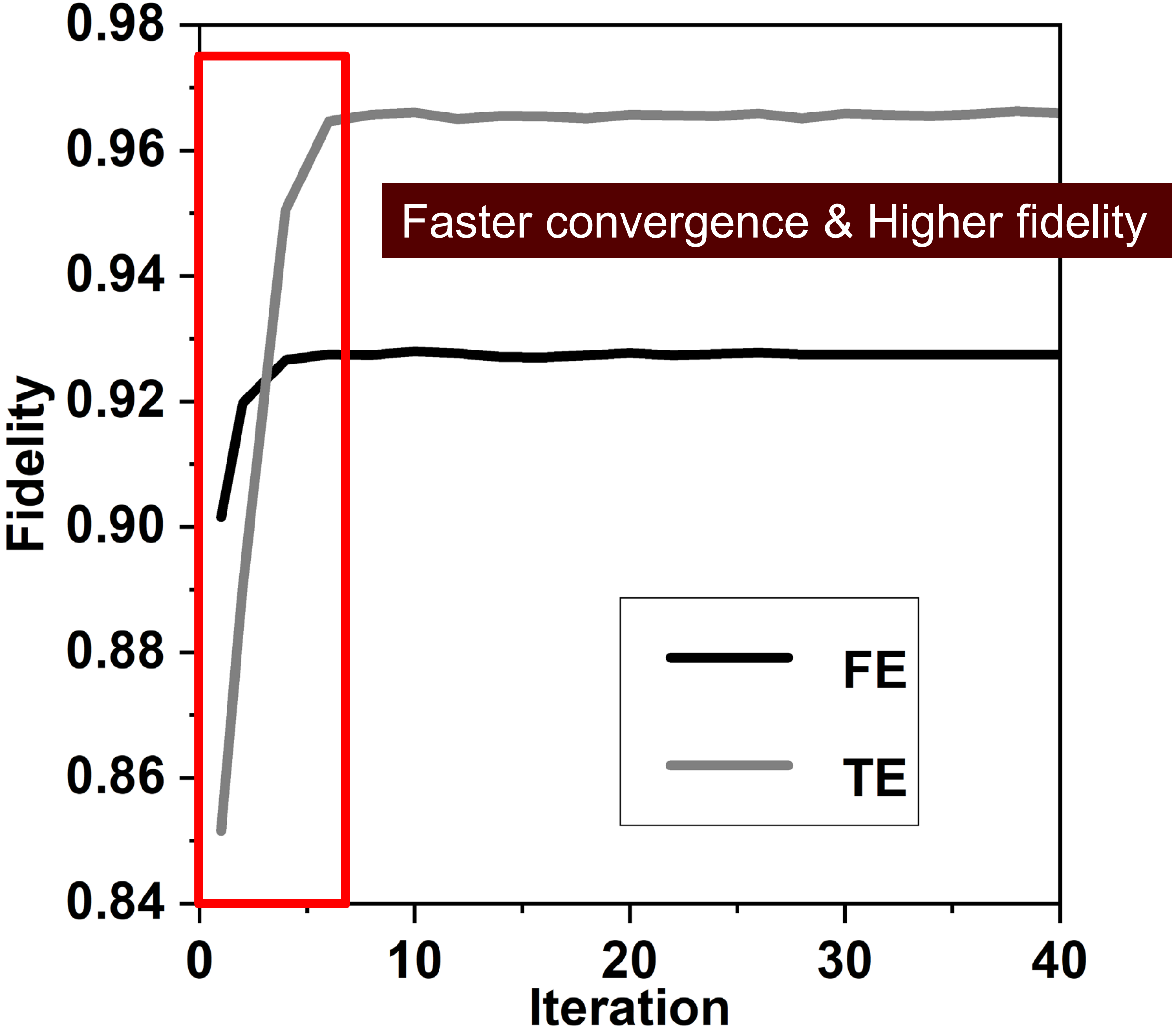
- Software support.
  - PennyLane, Pytorch
- Hyperparameters
  - Learning rate = 0.001
  - Batch size = 32
  - Epoch = 30
  - Learning rate scheduler. *CosineAnnealingLR* with a T_{max} of 30

[8] Samuel A._QCE' 2021 , [9] Murphy Yuezhen Niu_arXiv' 2021

Comparison of Image Quality

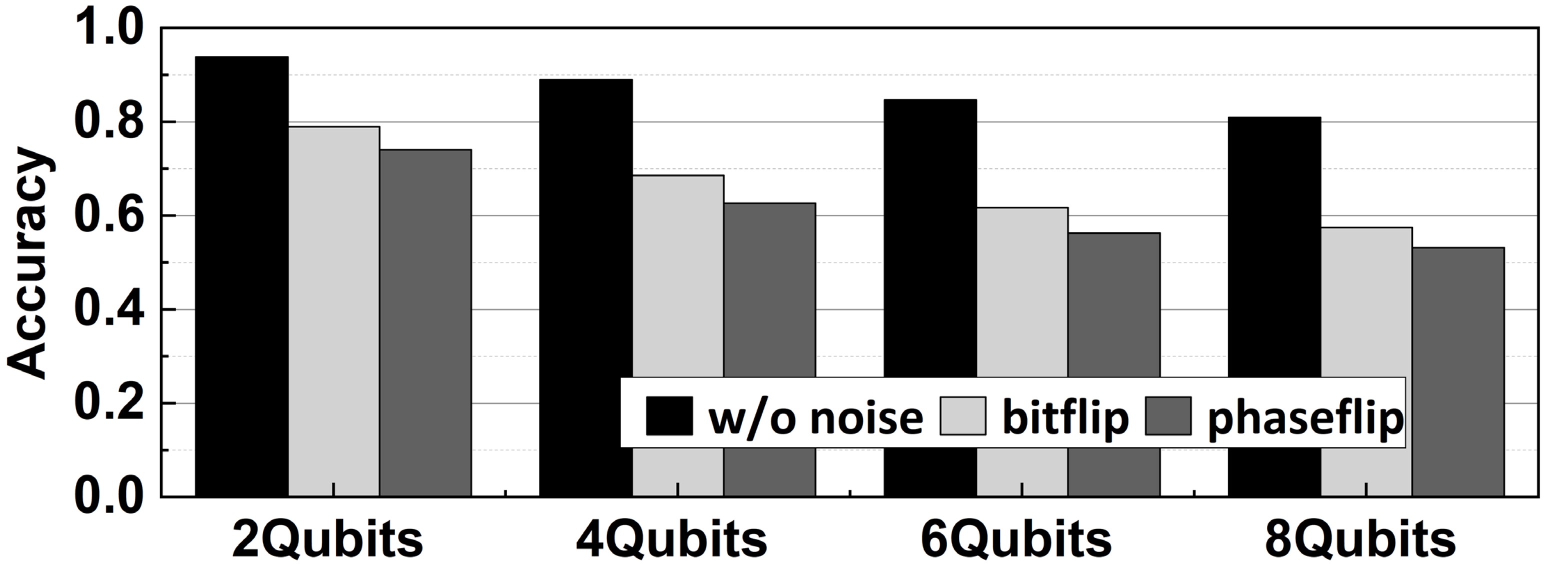
- Compared with previous quantum GANs, IQGAN achieves a stable and consistently high-quality output.

Convergence of IQGAN



Faster convergence & Higher fidelity

Impact of Quantum Errors



- Input size (↑) → Data dimension (↑) → Required generative capability (↑)
- Input size (↑) → Qubit number (↑) → Noise impact (↑)

Conclusions

- We study the reasons for the low generative performance in previous work and conclude that the standard quantum encoders limit the generative ability of a quantum GAN.
- We propose a trainable multiqubit quantum encoder that achieves SOTA quality on the generated data.
- We present a compact generator circuit ansatz that reduces hardware cost and circuit depth compared with previous work.
- We demonstrate that IQGAN can be efficiently implemented on NISQ devices and provide the training procedure.