Wireless Location Tracking via Complex-Domain Super MDS with Time Series Self-Localization Information

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1. Abstract

Background:

- Internet of things (IoT) applications often rely on networks consisting of large numbers of nodes typically limited in computational capabilities. The importance of location information in such a modern system is increasing.
  - ➡ Low-complexity localization algorithms based on multidimensional information aggregated from the nodes

Conventional methods:

- Multidimensional scaling (MDS) framework
  - ✓ Classical MDS [1] : Localization method based on edge kernel consisting from the distance estimates between nodes
  - ✓ Super MDS (SMDS) [2] : Extending the MDS to handle hybrid information (both distance and angle) simultaneously
  - ✓ Complex-domain SMDS (CD-SMDS) [3] : Casting the SMDS onto the complex domain for further improvements

Focus:

- The improvement in the CD-SMDS is due to a noise reduction effect of the low-rank truncation via singular value decomposition (SVD) of the edge kernel matrix, which in the case of the CD-SMDS algorithm has rank one.
  - ➡ Further improvement can be expected in systems of larger dimensions.
- MDS algorithms are typically limited to snapshot positioning.
  - ➡ Leaving potential for improvement in tracking systems, whereby the temporal dimension is also incorporated.

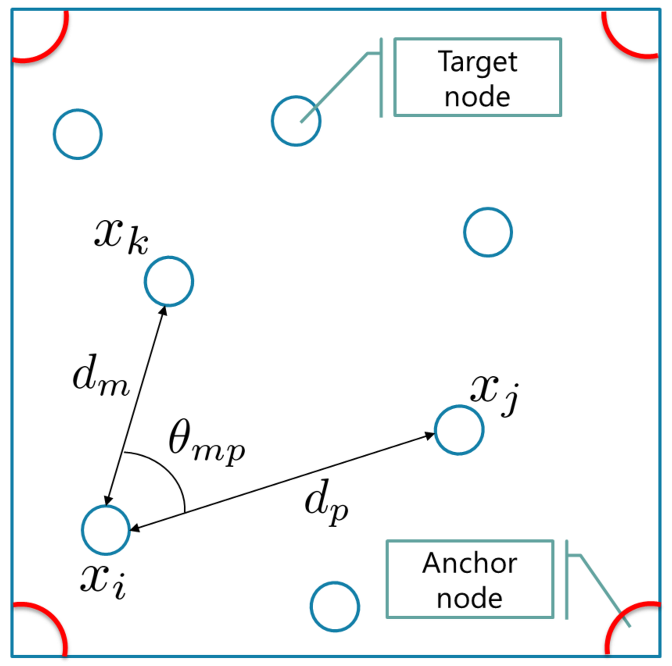
Proposal:

- Recent target devices can easily estimate the distance and direction of their own movement.
- Such inertial information between certain time slots obtained from the self-localization (SL) enables us to bridge the information at time slot *t* with the information at the previous discrete time slot *t* − 1.
  - ➡ Boosting localization accuracy by constructing a larger rank-one edge kernel matrix in the MDS framework, encompassing the time series information of both distance and angle.

Goal:

- The efficacy of the proposed method is confirmed via software simulations, and compared with an SL-aware CRLB.

2. Problem description and CD-SMDS algorithm review



- ◆**Anchor nodes** (ANs) with their locations known a priori
- ◆**Target nodes** (TNs) to be estimated based on localization

Input data: measurements between nodes:

- mutual distances: d_m
- mutual phases: θ_{mp}

◆Complex coordinate vector:

$$\boldsymbol{x} \triangleq [x_1, \ldots, x_N]^{\mathrm{T}} \in \mathbb{C}^{N\times 1}$$

N: Num. of nodes

◆Complex edge vector:

$$\boldsymbol{v} \triangleq [(x_1 - x_2), (x_1 - x_3), \ldots, (x_{N-1} - x_N)]^{\mathrm{T}} = [v_1, \ldots, v_M]^{\mathrm{T}} = \boldsymbol{C}\cdot\boldsymbol{x} \in \mathbb{C}^{M\times 1}$$

$M \triangleq \binom{N}{2}$: Num. of edges

$$\boldsymbol{C} \triangleq \left[\begin{array}{c|c|c|c} \mathbf{1}_{N-1\times 1} & \multicolumn{3}{c}{-\boldsymbol{I}_{N-1}} \\ \hline \mathbf{0}_{N-2\times 1} & \mathbf{1}_{N-2\times 1} & \multicolumn{2}{c}{-\boldsymbol{I}_{N-2}} \\ \hline \multicolumn{2}{c|}{\ddots} & \ddots & \ddots \\ \hline \multicolumn{2}{c|}{\mathbf{0}_{1\times N-2}} & 1 & -1 \end{array}\right] \in \mathbb{R}^{M\times N}$$

◆Product of a pair of complex-valued edge:

$$v_m \cdot v_p^* = d_m d_p (\cos\theta_{mp} + \mathrm{j}\sin\theta_{mp}) = d_m d_p e^{\mathrm{j}\theta_{mp}}$$

◆Complex-domain edge kernel matrix:

$$\boldsymbol{K} \triangleq \boldsymbol{v}\cdot\boldsymbol{v}^{\mathrm{H}} = \mathrm{diag}(\boldsymbol{d}) \cdot \begin{bmatrix} e^{\mathrm{j}\theta_{11}} & \cdots & e^{\mathrm{j}\theta_{1M}} \\ \vdots & \ddots & \vdots \\ e^{\mathrm{j}\theta_{M1}} & \cdots & e^{\mathrm{j}\theta_{MM}} \end{bmatrix} \cdot \mathrm{diag}(\boldsymbol{d})$$

where $\boldsymbol{d} \triangleq [d_1, \ldots, d_M]^{\mathrm{T}}$

◆CD-SMDS algorithm:

Step 1: Edge kernel construction based on measured values

$$\tilde{\boldsymbol{K}} = \mathrm{diag}\left(\tilde{\boldsymbol{d}}\right) \cdot \begin{bmatrix} e^{\mathrm{j}\tilde{\theta}_{11}} & \cdots & e^{\mathrm{j}\tilde{\theta}_{1M}} \\ \vdots & \ddots & \vdots \\ e^{\mathrm{j}\tilde{\theta}_{M1}} & \cdots & e^{\mathrm{j}\tilde{\theta}_{MM}} \end{bmatrix} \cdot \mathrm{diag}\left(\tilde{\boldsymbol{d}}\right)$$

Step 2: Edge vector estimation via SVD of edge kernel:

$$\hat{\boldsymbol{v}} = \sqrt{\lambda}\boldsymbol{u}$$

where $(\lambda, \boldsymbol{u})$ is the dominant eigenpair of $\tilde{\boldsymbol{K}}$

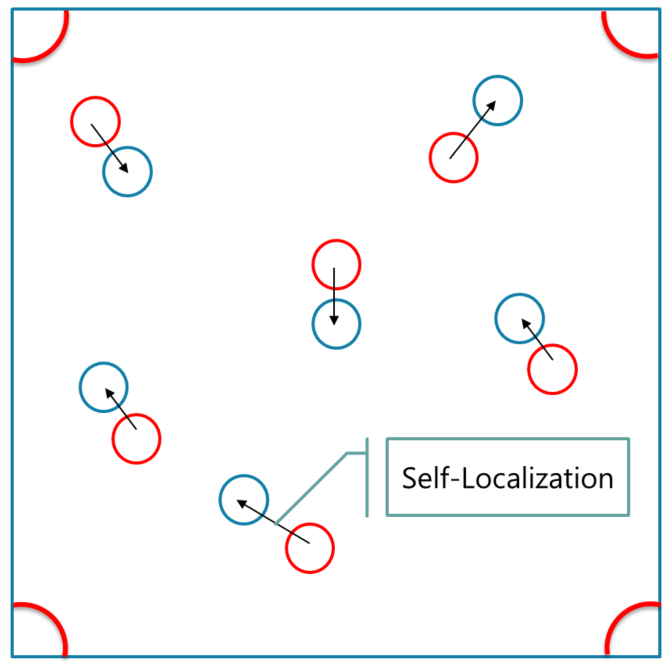
➡ Noise reduction effect of the low-rank truncation via SVD

Step 3: Coordinate vector estimation:

$$\begin{bmatrix} \boldsymbol{x}_{\mathrm{A}} \\ \hat{\boldsymbol{x}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{N_{\mathrm{A}}} \,|\, \mathbf{0}_{N_{\mathrm{A}}\times N_{\mathrm{T}}} \\ \boldsymbol{C} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \boldsymbol{x}_{\mathrm{A}} \\ \hat{\boldsymbol{v}} \end{bmatrix}$$

Finally, a Procrustes transformation [4] may be required to bring the resulting estimate to the same scale, orientation, and coordinate origin of the true coordinates ***x***.

3. Proposal: SL-aided CD-SMDS (SL-CD-SMDS)

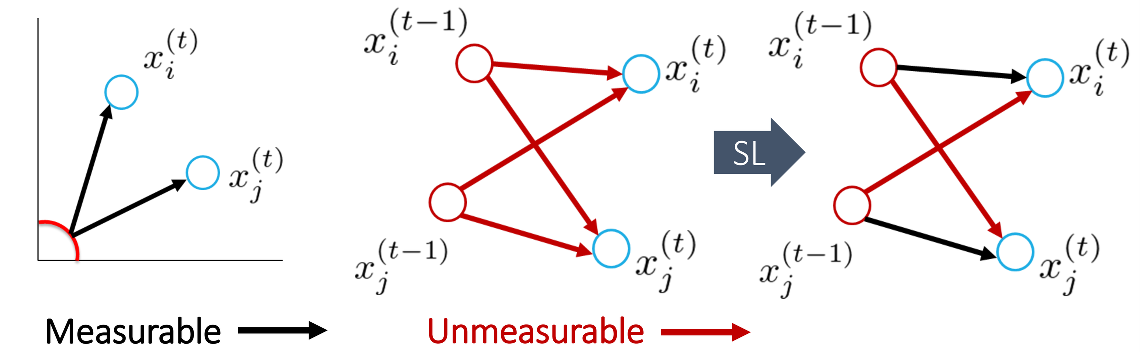


- (red circle): Previously estimated data
- (blue circle): Target of estimation (current location of TNs)

The TNs estimated one time ago is regarded as the ANs

Taking into account measurements from two consecutive time slots by seamlessly integrating available coordinate information

□Between $\boldsymbol{x}_{\mathrm{A}}$ and $\boldsymbol{x}_{\mathrm{T}}^{(t)}$ □Between $\boldsymbol{x}_{\mathrm{T}}^{(t-1)}$ and $\boldsymbol{x}_{\mathrm{T}}^{(t)}$



Measurements between $\boldsymbol{x}_{\mathrm{T}}^{(t-1)}$ and $\boldsymbol{x}_{\mathrm{T}}^{(t)}$ cannot be obtained as the nodes are moving…

➡ Only the part corresponding to their own movements that can be informed by SL can be constructed, enabling to bridge the information of different discrete time slots!

◆Extended complex coordinate vector:

$$\boldsymbol{x}' \triangleq \left[\boldsymbol{x}_{\mathrm{A}}^{\mathrm{T}} \quad \left(\hat{\boldsymbol{x}}_{\mathrm{T}}^{(t-1)}\right)^{\mathrm{T}} \quad \left(\boldsymbol{x}_{\mathrm{T}}^{(t)}\right)^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{(N+N_{\mathrm{T}})\times 1}$$

Used as ANs at time slot *t* | TNs at time slot *t*

◆Corresponding extended complex edge vector:

$$\boldsymbol{v}' \triangleq \left[\boldsymbol{v}_{\mathrm{AA}}^{\mathrm{T}} \quad \boldsymbol{v}_{\mathrm{AT}}^{\mathrm{T}} \quad \boldsymbol{v}_{\mathrm{TT}}^{\mathrm{T}}\right]^{\mathrm{T}} = \boldsymbol{C}'\cdot\boldsymbol{x}' \in \mathbb{C}^{M'\times 1}$$

ANs to ANs | ANs to TNs | TNs to TNs

$$M' \triangleq \binom{N+N_{\mathrm{T}}}{2} - N_{\mathrm{T}}(N_{\mathrm{T}}-1)$$

$$\boldsymbol{C}' \triangleq \left[\boldsymbol{C}_{\mathrm{AA}}^{\mathrm{T}} \quad \boldsymbol{C}_{\mathrm{AT}}^{\mathrm{T}} \quad \boldsymbol{C}_{\mathrm{TT}}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{M'\times(N+N_{\mathrm{T}})}$$

$$\boldsymbol{C}_{\mathrm{AA}} \triangleq \left[\begin{array}{c|c|c|c|c} \mathbf{1}_{N-1\times 1} & \multicolumn{3}{c|}{-\boldsymbol{I}_{N-1}} & \mathbf{0}_{N-1\times N_{\mathrm{T}}} \\ \hline \mathbf{0}_{N-2\times 1} & \mathbf{1}_{N-2\times 1} & \multicolumn{2}{c|}{-\boldsymbol{I}_{N-2}} & \mathbf{0}_{N-2\times N_{\mathrm{T}}} \\ \hline \multicolumn{2}{c|}{\ddots} & \ddots & \ddots & \vdots \\ \hline \multicolumn{2}{c|}{\mathbf{0}_{1\times N-2}} & 1 & -1 & \mathbf{0}_{1\times N_{\mathrm{T}}} \end{array}\right] \in \mathbb{R}^{M\times(N+N_{\mathrm{T}})}$$

$$\boldsymbol{C}_{\mathrm{AT}} \triangleq \left[\begin{array}{c|c|c|c} \mathbf{1}_{N_{\mathrm{T}}\times 1} & \multicolumn{2}{c|}{\mathbf{0}_{N_{\mathrm{T}}\times N-1}} & -\boldsymbol{I}_{N_{\mathrm{T}}} \\ \hline \mathbf{0}_{N_{\mathrm{T}}\times 1} & \mathbf{1}_{N_{\mathrm{T}}\times 1} & \mathbf{0}_{N_{\mathrm{T}}\times N-2} & -\boldsymbol{I}_{N_{\mathrm{T}}} \\ \hline \ddots & \ddots & \ddots & \vdots \\ \hline \mathbf{0}_{N_{\mathrm{T}}\times N_{\mathrm{A}}-1} & \mathbf{1}_{N_{\mathrm{T}}\times 1} & \mathbf{0}_{N_{\mathrm{T}}\times N_{\mathrm{T}}} & -\boldsymbol{I}_{N_{\mathrm{T}}} \\ \hline \multicolumn{2}{c|}{\mathbf{0}_{N_{\mathrm{T}}\times N_{\mathrm{A}}}} & \boldsymbol{I}_{N_{\mathrm{T}}} & -\boldsymbol{I}_{N_{\mathrm{T}}} \end{array}\right] \in \mathbb{R}^{N_{\mathrm{T}}(N_{\mathrm{A}}+1)\times(N+N_{\mathrm{T}})}$$

$$\boldsymbol{C}_{\mathrm{TT}} \triangleq \left[\begin{array}{c|c|c|c|c} \mathbf{0}_{N_{\mathrm{T}}-1\times N} & \mathbf{1}_{N_{\mathrm{T}}-1\times 1} & \multicolumn{3}{c}{-\boldsymbol{I}_{N_{\mathrm{T}}-1}} \\ \hline \mathbf{0}_{N_{\mathrm{T}}-2\times N} & \mathbf{0}_{N_{\mathrm{T}}-2\times 1} & \mathbf{1}_{N_{\mathrm{T}}-2\times 1} & \multicolumn{2}{c}{-\boldsymbol{I}_{N_{\mathrm{T}}-2}} \\ \hline \vdots & \ddots & \ddots & \multicolumn{2}{c}{\ddots} \\ \hline \mathbf{0}_{1\times N} & \multicolumn{2}{c|}{\mathbf{0}_{1\times N_{\mathrm{T}}-2}} & 1 & -1 \end{array}\right] \in \mathbb{R}^{N_{\mathrm{T}}(N_{\mathrm{T}}-1)/2\times(N+N_{\mathrm{T}})}$$

4. Simulation results

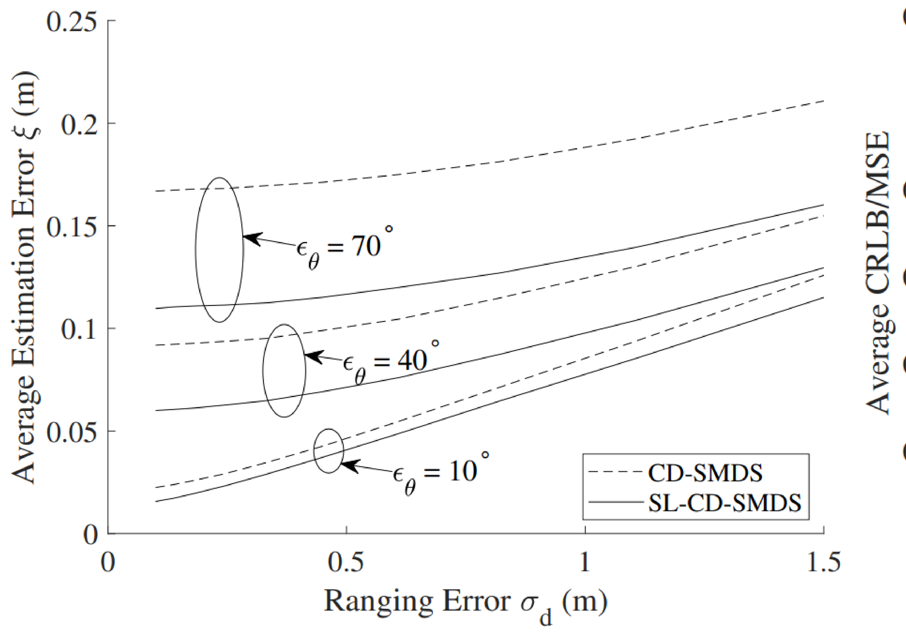


Fig. 1: Localization accuracies v.s. ranging errors

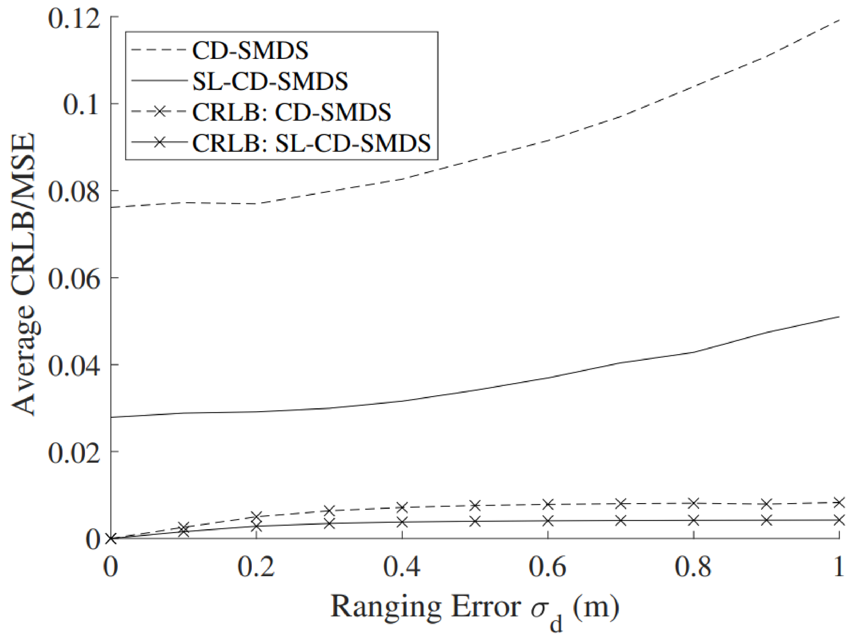


Fig. 2: Average MSE v.s. ranging errors

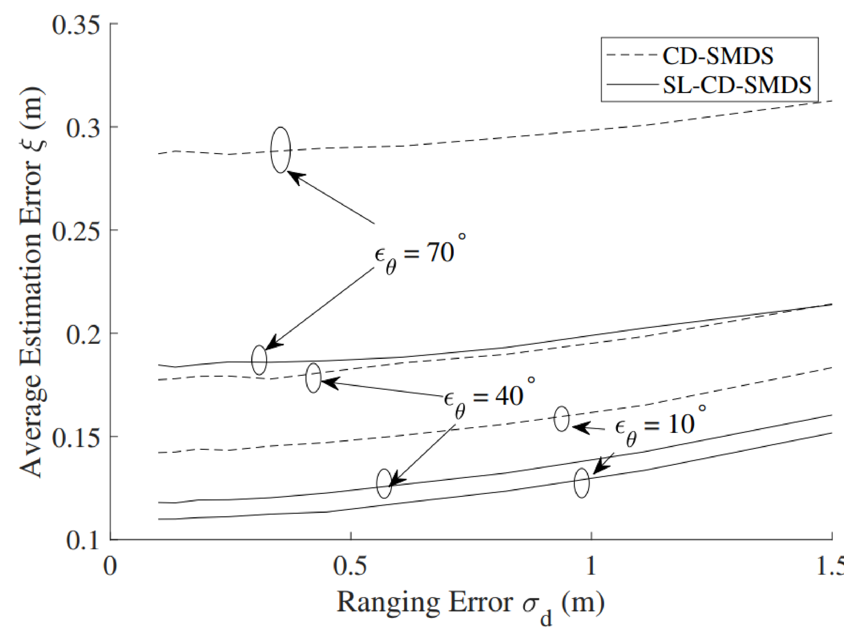


Fig. 3: More practical setting

Parameters	Values
Num. of ANs	4
Num. of TNs	6
Test area setting	10 m × 10 m
Angle errors: Thikhonov-distributed	$p_\Theta\left(\tilde{\theta};\theta;\rho\right) = \frac{1}{2\pi I_0(\rho)}\cdot e^{\rho\cos(\theta-\tilde{\theta})}$ $\epsilon_\theta = \theta_{\mathrm{B}} \Big| \int_{-\theta_{\mathrm{B}}}^{\theta_{\mathrm{B}}} p_\Theta(t;0,\rho)dt = 0.9$
Distance errors: Gamma-distributed	$p_D(d;\alpha,\beta) = \frac{1}{\beta^\alpha(\alpha)}\cdot\tilde{d}^{(\alpha-1)}\cdot e^{\frac{\tilde{d}}{\beta}}$ $\alpha = d^2/\sigma_d^2, \quad \beta = \sigma_d^2/d$
Error metric	$\xi = \frac{1}{N_t} \parallel \hat{X} - X \parallel_F$

- The simulation is performed with the test area of 10 m-by-10 m equipped with 4 ANs, one at each corner and TNs randomly placed in the plane.
- Performance is assessed by the average error between the true position of the target and the estimated value. The TNs follow independent random trajectories generated accordingly to a first-order autoregressive (AR) model.
- □**Fig. 1** compares accuracy of the localization as a function of the standard deviations of distance estimates, where three different levels of Tikhonov-distributed angle estimation errors are considered.
- □**Fig. 2** shows the MSE performances as a function of the standard deviations of Gamma-distributed distance estimates.
- □**Fig. 3** shows the localization performances for a case where 40% of the entries in the kernels are randomly erased.

Summary: A novel wireless location tracking algorithm, SL-CD-SMDS, is proposed, which is an extension of the CD-SMDS to location tracking problems under the assumption that time series information of distance and angle simultaneously can be utilized. Simulation results show that the proposed method significantly outperforms the conventional method, especially when the measurement error is large and/or the information is partially unattainable.

[1] W. S. Torgerson, "Multidimensional scaling: I. theory and method," IEEE Trans. Signal Processing, vol. 17, no. 4, pp. 401–419, 1952. [2] G. T. F. de Abreu and G. Destino, "Super MDS: Source location from distance and angle information," in in Proc. IEEE Wireless Commun. Netw. Conf. (WCNC), Mar 2007, vol. 2, p. 4430–4434.
[3] A. Ghods and G. T. F. de Abreu, "Complex-domain super MDS: A new framework for wireless localization with hybrid information," IEEE Trans. Wireless Commun., vol. 17, no. 11, pp. 7364–7378, 2018. [4] P.D. Fiore, "Efficient linear solution of exterior orientation," in IEEE Commun. Surveys Tuts, Feb 2001, vol. 23, pp. 140–148.