LiQuiD-MIMO Radar: Distributed MIMO Radar with Low-Bit Quantization

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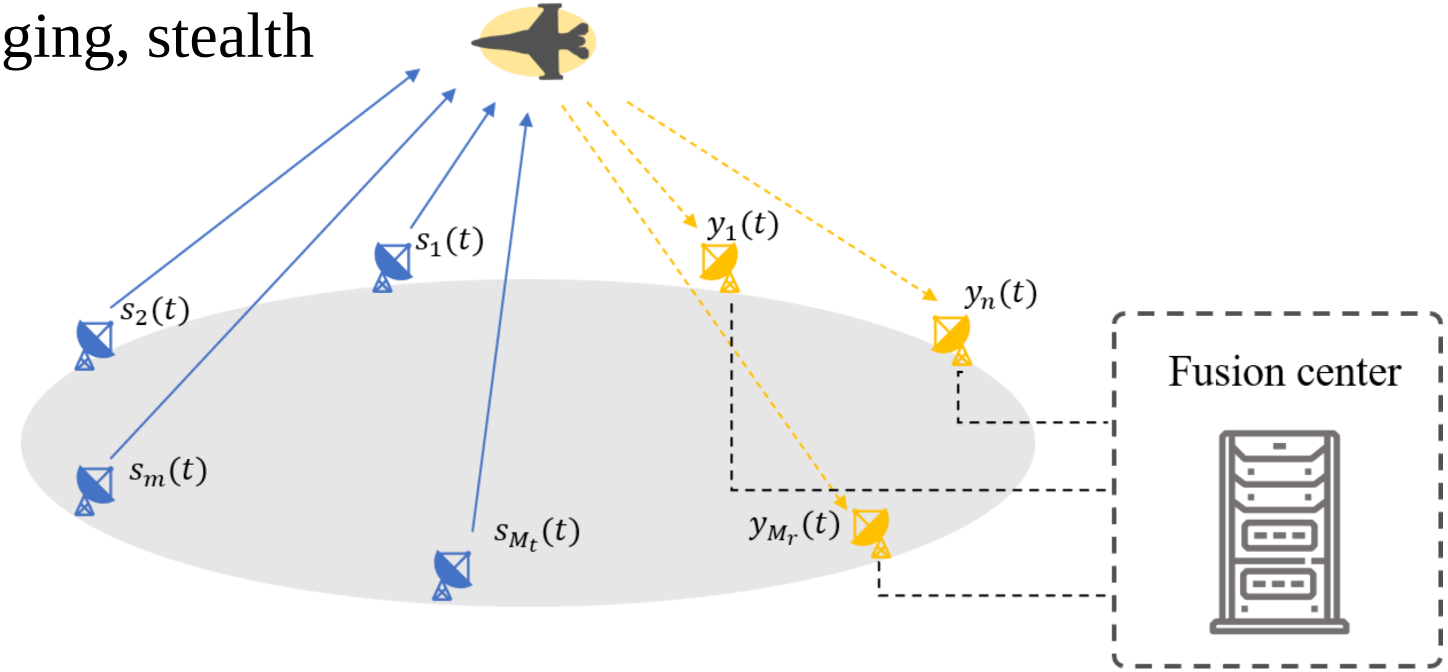
Distributed MIMO Radar

- ☐ Antennas spread out over a larger area
- ☐ Spatial diversity is exploited to improve the target detection performance
- ☐ Joint process at the fusion center by collecting all the data
- ☐ Emerging applications: radar imaging, stealth targets detection, and so on
- ☐ Practical challenges
  - Cost
  - Power consumption
  - System complexity



Cost-Efficient Solutions

- ❑ Analog-to-digital converters (ADCs) are costly and power-consuming at high bit rates
- ❑ A large amount of data has to be transmitted to the fusion center
- ❑ Current solutions mainly for collocated MIMO radar
  - Reduce sampling rate, e.g., sub-Nyquist MIMO radar (Cohen et al TSP18)
  - Reduce quantized bits, e.g., one-bit MIMO radar (Xi et al TSP20)
  - Optimize quantizers, e.g., Bit-Limited MIMO Radar (Xi et al TSP21)
- ❑ Our solution: develop a low-bit quantized distributed MIMO radar (LiQuiD-MIMO radar)

Outline

- ☐ LiQuiD-MIMO Radar Model
- ☐ Quantized Robust PCA (QRPCA) Problem Formulation
- ☐ Our Method: APG-QRPCA Algorithm + LS-based Target Parameter Estimation
- ☐ Numerical results

LiQuiD-MIMO Radar Model

A. Signal Model

We assume that all the targets are distributed in the same 2-D plane where the transmit and receive antennas are located.

- Time delay

$$\tau_{mn}^{(k)} = \frac{\left\|\mathbf{p}^{(k)} - \mathbf{p}_t^{(k)}\right\| + \left\|\mathbf{p}^{(k)} - \mathbf{p}_r^{(k)}\right\|}{c}$$

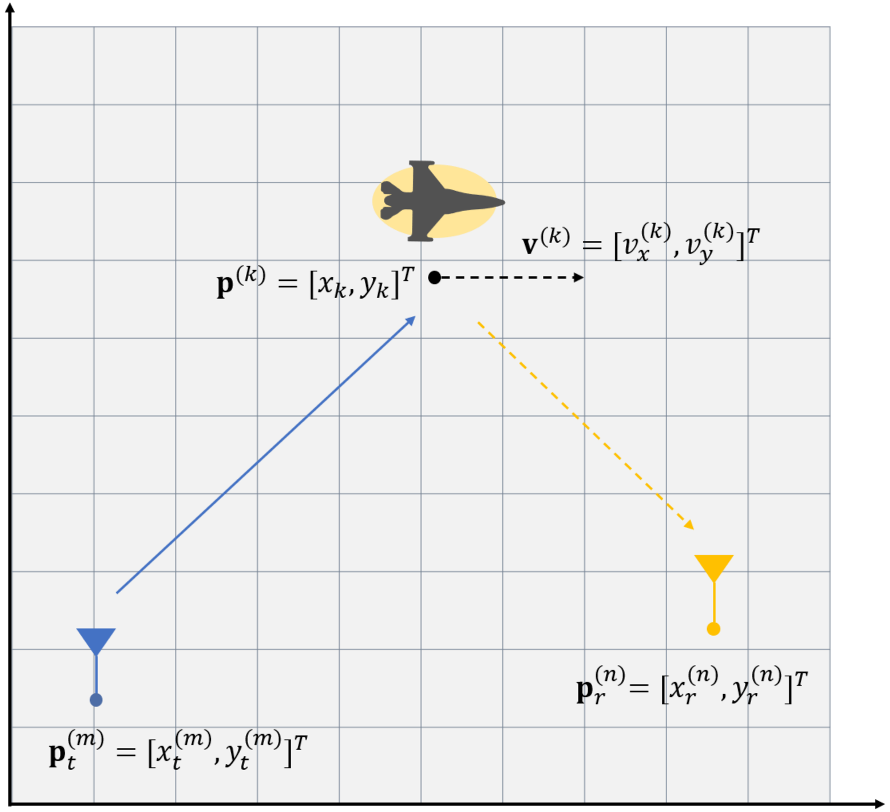
- Doppler frequency

$$f_{mn}^{(k)} = \frac{f_m}{c}\left(\frac{\left\langle \mathbf{v}^{(k)}, \mathbf{p}^{(k)} - \mathbf{p}_t^{(m)} \right\rangle}{\left\|\mathbf{p}^{(k)} - \mathbf{p}_t^{(m)}\right\|} + \frac{\left\langle \mathbf{v}^{(k)}, \mathbf{p}^{(k)} - \mathbf{p}_r^{(m)} \right\rangle}{\left\|\mathbf{p}^{(k)} - \mathbf{p}_r^{(m)}\right\|}\right)$$

- Received signal:

$$y_{mn}(t) = \sum_{q=0}^{Q-1} \sum_{k=1}^{K} \beta_{mn}^{(k)} s_m\left(t - \tau_{mn}^{(k)} - qT_{\mathrm{PRI}}\right) e^{j2\pi f_{mn}^{(k)} qT_{\mathrm{PRI}}} + w_{mn}(t)$$

- $s_m(t)$ could be FDMA waveforms
- GOAL: Resolve the K position and velocity pairs $\{\mathbf{p}^{(k)}, \mathbf{v}^{(k)}\}_{k=1}^{K}$ from received signals.

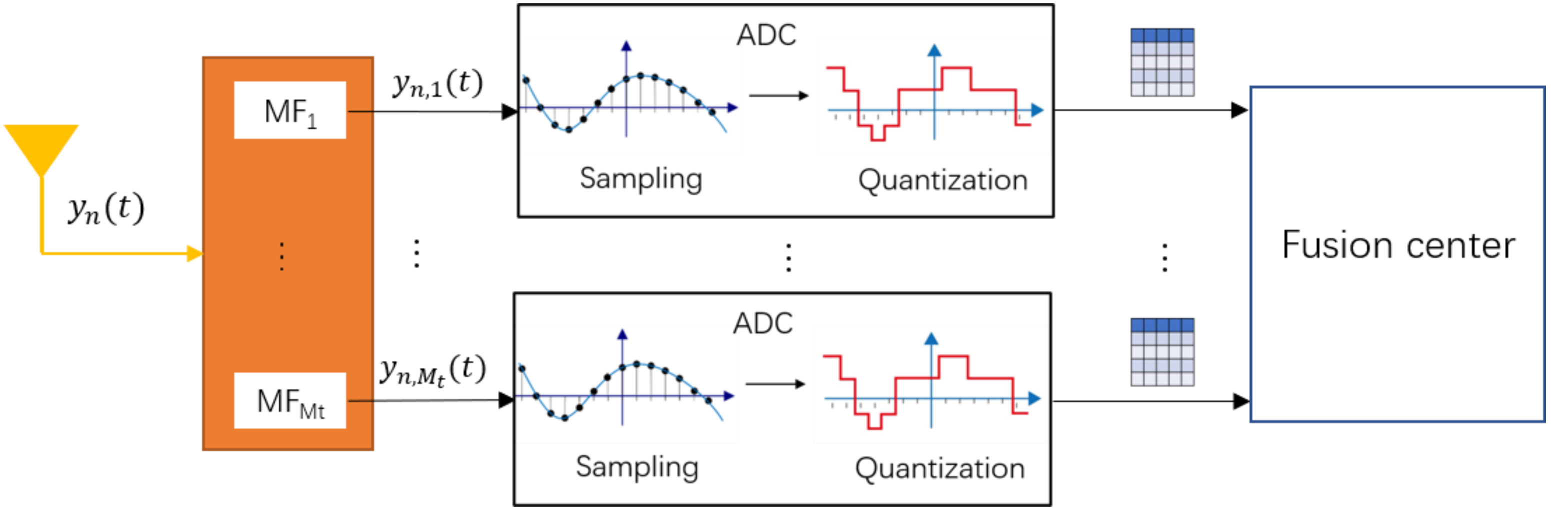


LiQuiD-MIMO Radar Model

B. Sampling and Quantization with Low-Resolution ADCs

- ☐ Using the low-resolution ADCs: each data is quantized into $\tilde{b}$ bits, e.g., $\tilde{b} = 2, 3{,}4$;
- ☐ Send quantized data to fusion center.
- ☐ $\mathbf{X}_{mn}$:Target information matrix (TIM); $\mathbf{W}_{mn}$:White Gaussian Noise (WGN); $\tilde{\mathbf{T}}_{mn}$:Data transmission error (DTE).

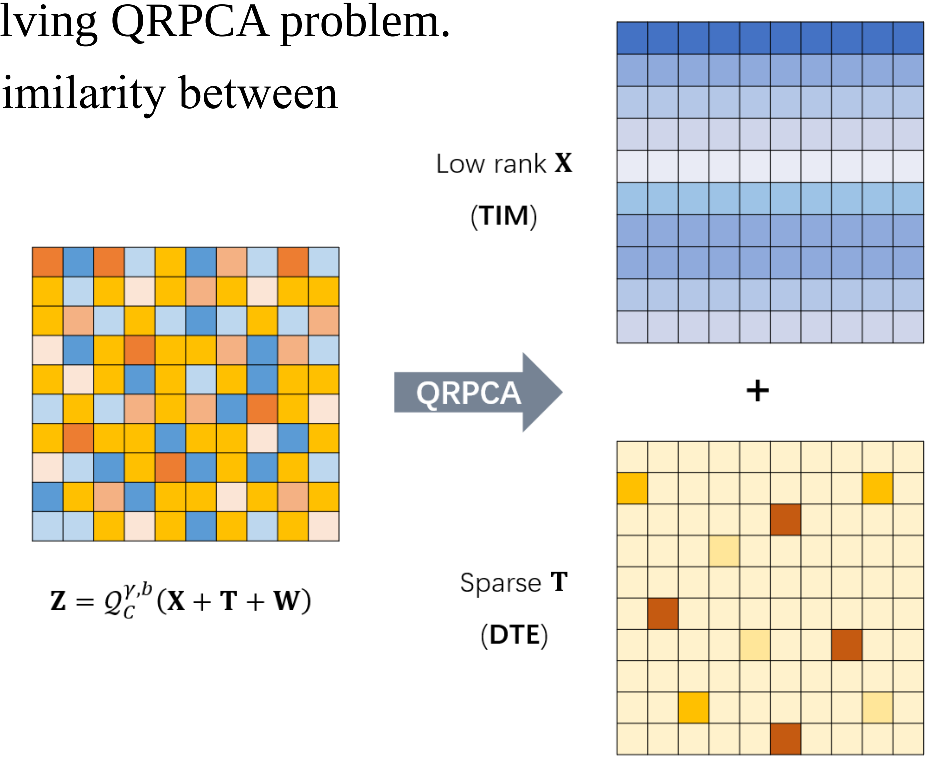
$$\mathbf{Y}_{mn} = \mathbf{X}_{mn} + \mathbf{W}_{mn} \Longrightarrow \tilde{\boldsymbol{Z}}_{mn} = \mathcal{Q}_C^{\gamma,b}(\mathbf{X}_{mn} + \mathbf{W}_{mn}) \Longrightarrow \boldsymbol{Z}_{mn} = \mathcal{Q}_C^{\gamma,b}(\mathbf{X}_{mn} + \mathbf{W}_{mn}) + \tilde{\mathbf{T}}_{mn}$$



QRPCA Problem Formulation

- $\mathbf{Z}=\mathcal{Q}_C^{\gamma,b}(\mathbf{X}+\mathbf{W})+\widetilde{\mathbf{T}}$ can be equivalent to $\mathbf{Z}=\mathcal{Q}_C^{\gamma,b}(\mathbf{X}+\mathbf{T}+\mathbf{W})$ (omitting the subscript mn).
  - $\mathbf{X}$: Low rank. Its rank depends on the number of targets with different distances or different velocities.
  - $\widetilde{\mathbf{T}}$: Sparse. It is generally sparse since the bit error rate (BER) is generally quite low.
  - $\mathbf{T}$: Sparse. It is an equivalent sparse DTE before quantization.
- Recover the low-rank matrix $\mathbf{X}$ and the sparse matrix $\mathbf{T}$ by solving QRPCA problem.
  - Function $D(\cdot,\cdot)$ is similarity metric which measures the similarity between the quantized data and the unquantized data.

$$\min_{\mathbf{X},\mathbf{T}} \quad \frac{1}{2}D(\mathbf{Z},\mathbf{X}+\mathbf{T})+\mu\|\underbrace{\boxed{\mathbf{X}}}_{\text{Low rank}}\|_*+\lambda\|\underbrace{\boxed{\mathbf{T}}}_{\text{Sparse}}\|_1$$



QRPCA Problem Formulation

- The relationship between quantization data **Z** and the unquantization data **Y**=**X**+**T** can be written as

$$-\frac{\Delta}{2} \leq \Re\{\mathbf{Y}-\mathbf{Z}\} \leq \frac{\Delta}{2}$$
$$-\frac{\Delta}{2} \leq \Im\{\mathbf{Y}-\mathbf{Z}\} \leq \frac{\Delta}{2}$$

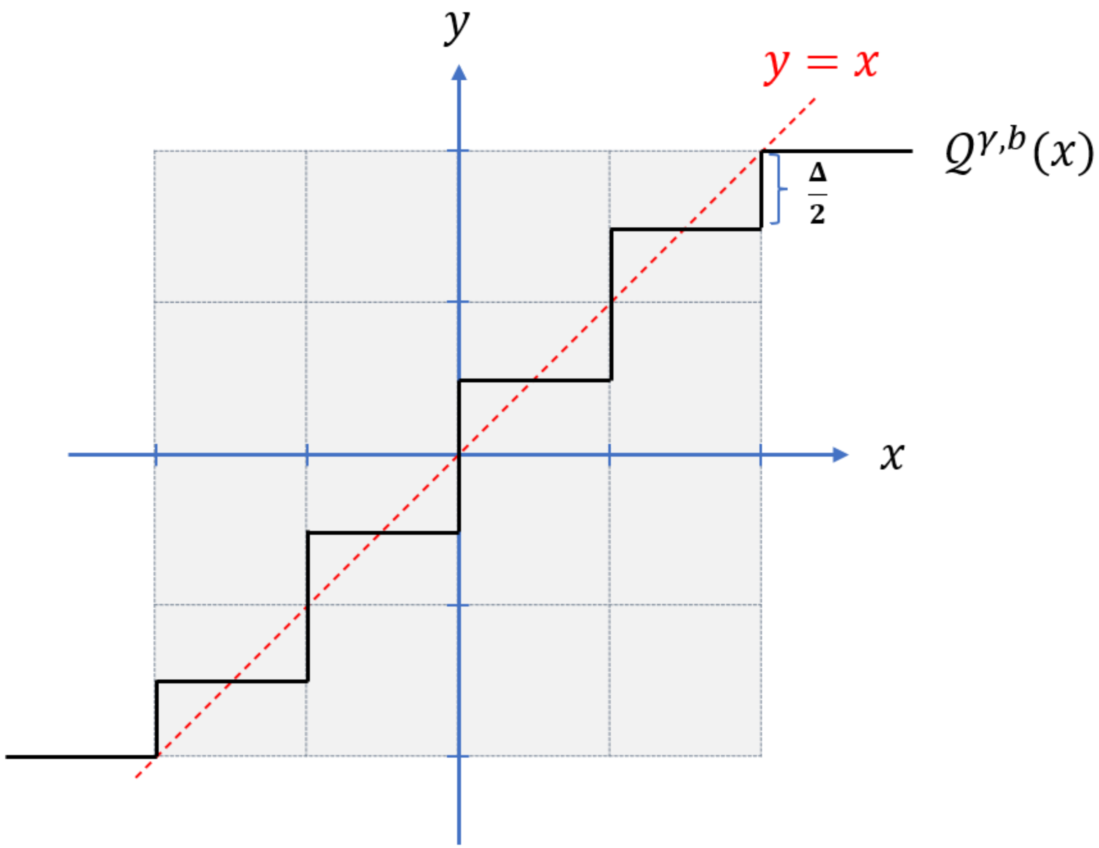
- By considering the unknown noise distortion on the quantized data, we define the similarity metric function $D(\cdot,\cdot)$

$$D(\mathbf{Z},\mathbf{X}+\mathbf{T})$$

$$= \left\|\rho\left(\left[\Re\{\mathbf{X}+\mathbf{T}-\mathbf{Z}\}+\frac{\Delta}{2};\Im\{\mathbf{X}+\mathbf{T}-\mathbf{Z}\}+\frac{\Delta}{2}\right]\right)\right\|_F^2$$

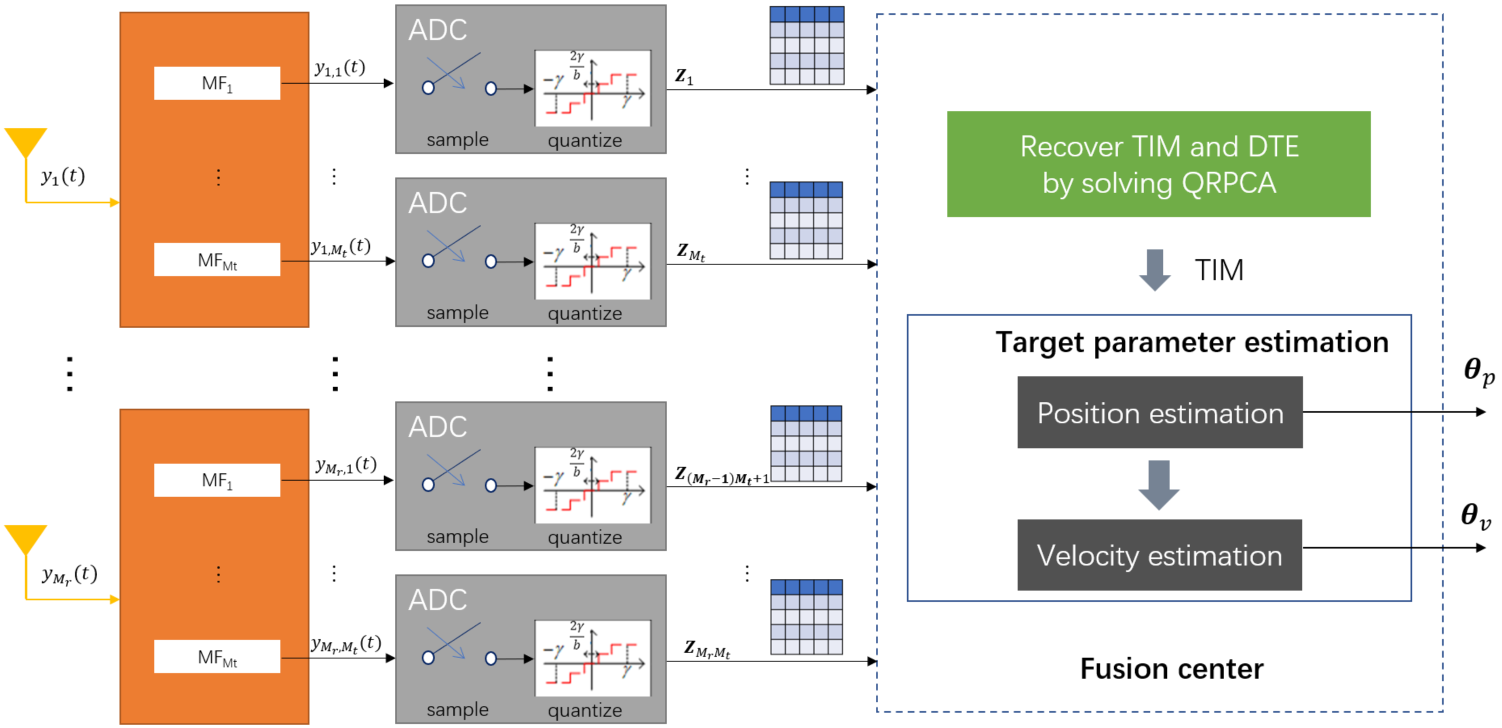
$$+ \left\|\rho\left(\left[\Re\{\mathbf{Z}-\mathbf{X}-\mathbf{T}\}+\frac{\Delta}{2};\Im\{\mathbf{Z}-\mathbf{X}-\mathbf{T}\}+\frac{\Delta}{2}\right]\right)\right\|_F^2$$

where $\rho(\cdot)$ is an element-wise function with $\rho(x) = \max\{-x, 0\}$.



Method

- ❑ Step1: Accelerated Proximal Gradient (APG) Algorithm for QRPCA Problem
  - Recover the low-rank matrix **X** and the sparse matrix **T**.
- ❑ Step2: Least Square (LS)-based Target Parameter Estimation
  - Estimate the unknown target parameters $\{\mathbf{p}^{(k)}, \mathbf{v}^{(k)}\}_{k=1}^{K}$ from recovered matrix **X**.



Method

A. APG-QRPCA Algorithm

- Define $h(\mathbf{X},\mathbf{T}) = \mu\|\mathbf{X}\|_* + \lambda\|\mathbf{T}\|_1$ and $g(\mathbf{X},\mathbf{T}) = \frac{1}{2}D(\mathbf{Z},\mathbf{X}+\mathbf{T})$, where $h(\mathbf{X},\mathbf{T})$ is convex and $g(\mathbf{X},\mathbf{T})$ is differentiable.
- The QPRCA problem can be rewritten as

$$\min_{\mathbf{X},\mathbf{T}} \quad h(\mathbf{X},\mathbf{T}) + g(\mathbf{X},\mathbf{T})$$

- Iteratively calculate
  - Calculate momentum

$$\bar{\mathbf{X}}_l = \mathbf{X}_l + \frac{\zeta_{l-1}}{\zeta_l}(\mathbf{X}_l - \mathbf{X}_{l-1}), \bar{\mathbf{T}}_l = \mathbf{T}_l + \frac{\zeta_{l-1}}{\zeta_l}(\mathbf{T}_l - \mathbf{T}_{l-1})$$

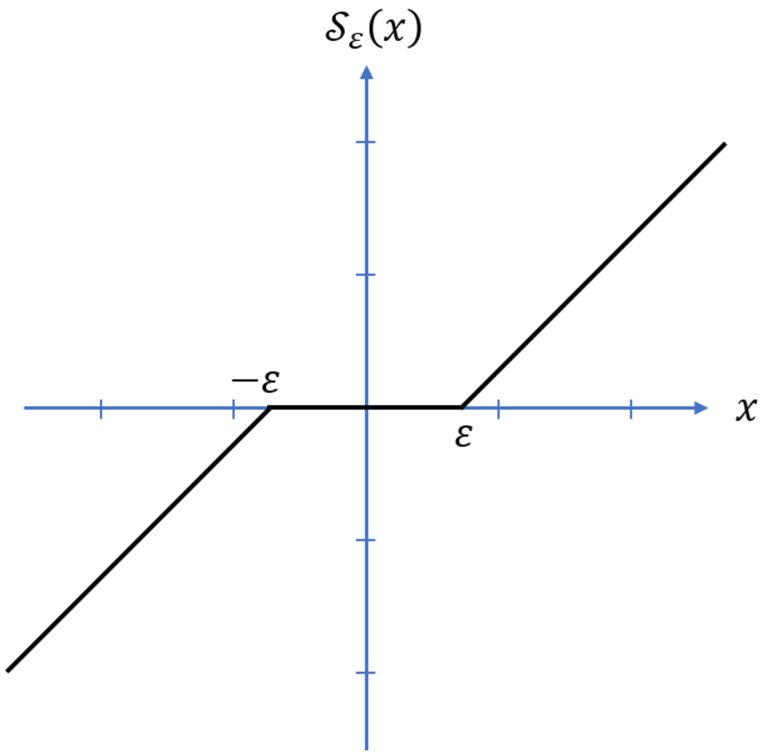
  - Gradient descent

$$\mathbf{X}_p = \bar{\mathbf{X}}_l - \delta\nabla_{\mathbf{X}} g(\bar{\mathbf{X}}_l, \bar{\mathbf{T}}_l), \mathbf{T}_p = \bar{\mathbf{T}}_l - \delta\nabla_{\mathbf{T}} g(\bar{\mathbf{X}}_l, \bar{\mathbf{T}}_l)$$

  - Proximal map

$$\mathbf{X}_{l+1} = \arg\min_{\mathbf{X}} \left\{\mu\|\mathbf{X}\|_* + \frac{1}{2\delta}\left\|\mathbf{X}-\mathbf{X}_p\right\|_F^2\right\} = \mathbf{U}_p\mathcal{S}_{\mu\delta}(\mathbf{\Sigma}_p)\mathbf{V}_p^T,$$

$$\mathbf{T}_{l+1} = \arg\min_{\mathbf{T}} \left\{\lambda\|\mathbf{T}\|_1 + \frac{1}{2\delta}\left\|\mathbf{T}-\mathbf{T}_p\right\|_F^2\right\} = \mathcal{S}_{\lambda\delta}(\mathbf{T}_p)$$



Method

B. LS-based Target Parameter Estimation

- $M_t \times M_r$ TIM matrixes can be recovered at the fusion center.
- A sequential LS method introduced to sequentially estimate the position and velocity parameters.
    - $\boldsymbol{\theta}_p = \left\{\mathbf{p}^{(k)}\right\}_{k=1}^{K}$ and $\boldsymbol{\theta}_v = \left\{\mathbf{v}^{(k)}\right\}_{k=1}^{K}$ are implicitly determined by the matrices $\mathbf{A}_{mn}$ and $\mathbf{B}_{mn}$

Original problem:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \sum_{m=1}^{M_t}\sum_{n=1}^{M_r}\left\|\hat{\mathbf{X}}_{mn} - \mathbf{A}_{mn}(\boldsymbol{\theta}_p)\boldsymbol{\Lambda}_{mn}\mathbf{B}_{mn}(\boldsymbol{\theta}_v)\right\|_2^2$$

Two subproblems:

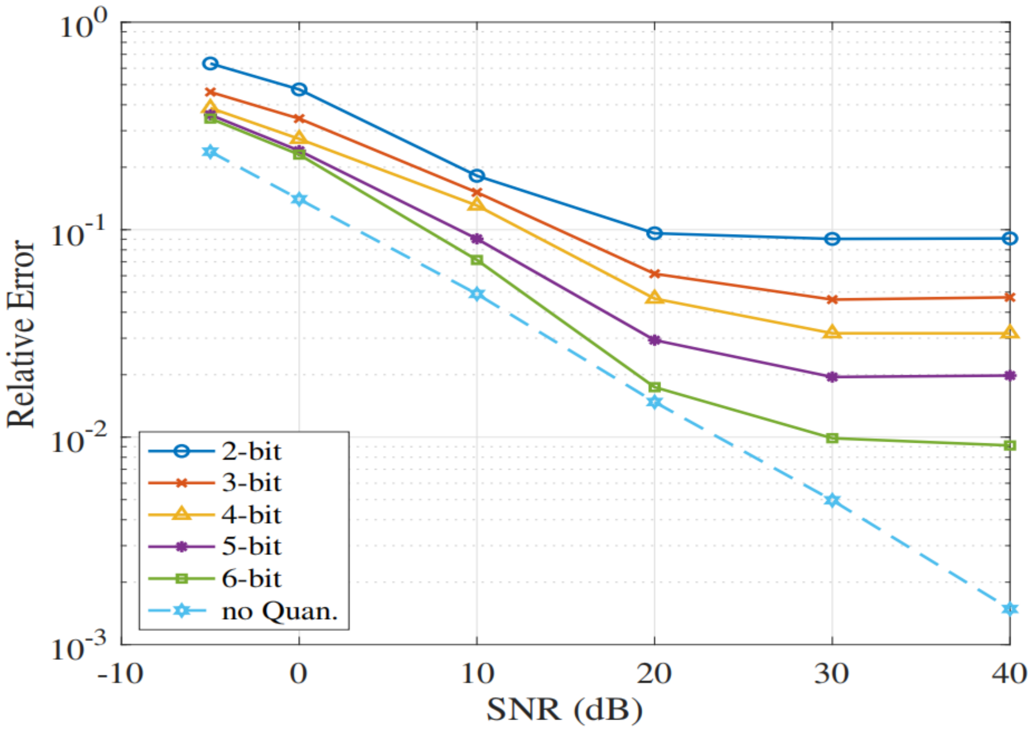
$$\hat{\boldsymbol{\theta}}_p = \arg\min_{\boldsymbol{\theta}_p} \sum_{m=1}^{M_t}\sum_{n=1}^{M_r}\left\|\mathbf{P}^{\perp}_{p,mn}(\boldsymbol{\theta}_p)\hat{\mathbf{X}}_{mn}\right\|_2^2$$

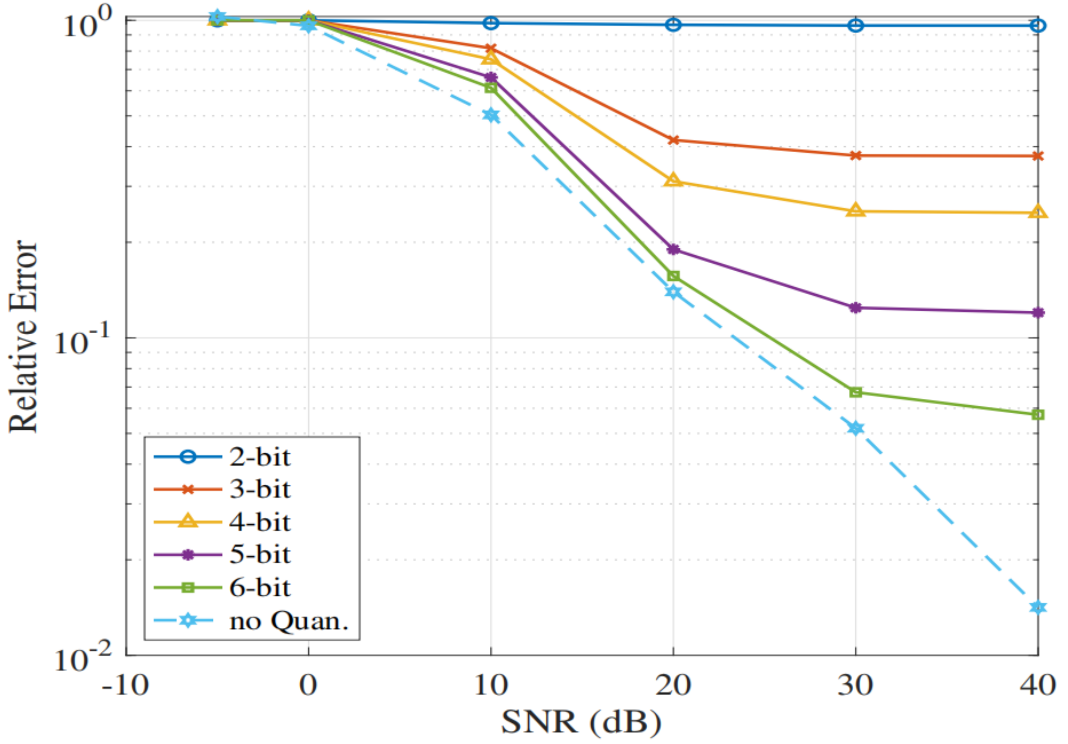
$$\hat{\boldsymbol{\theta}}_v = \arg\min_{\boldsymbol{\theta}_v} \sum_{m=1}^{M_t}\sum_{n=1}^{M_r}\sum_{k=1}^{K}\left(\hat{f}_{mn}^{(k)} - f_{mn}^{(k)}\left(\mathbf{v}^{(k)}\right)\right)^2$$

Numerical Results

- $M_t = 3$ transmit antennas, $M_r = 10$ receive antennas, uniformly distributed on the concentric circles with radius 5km and 3km, respectively.
- The reference carrier frequency parameters $f_0 = 5\text{GHz}$ and the frequency increment $\Delta f = 50\text{MHz}$.
- One CPI consists of $Q = 128$ pulses with $T_{\text{PRI}} = 0.5\text{ms}$ and $T_p = 6.4\mu s$.
- The transmitters emit Hadamard sequences with length of $N = 64$.
- 1% symbol error rate is assumed to lead sparse data transmission error matrix.
- One target is located at $\mathbf{p}^{(1)} = [1100, 1100]^T\text{m}$ with $\mathbf{v}^{(1)} = [10, 10]^T\text{m/s}$.

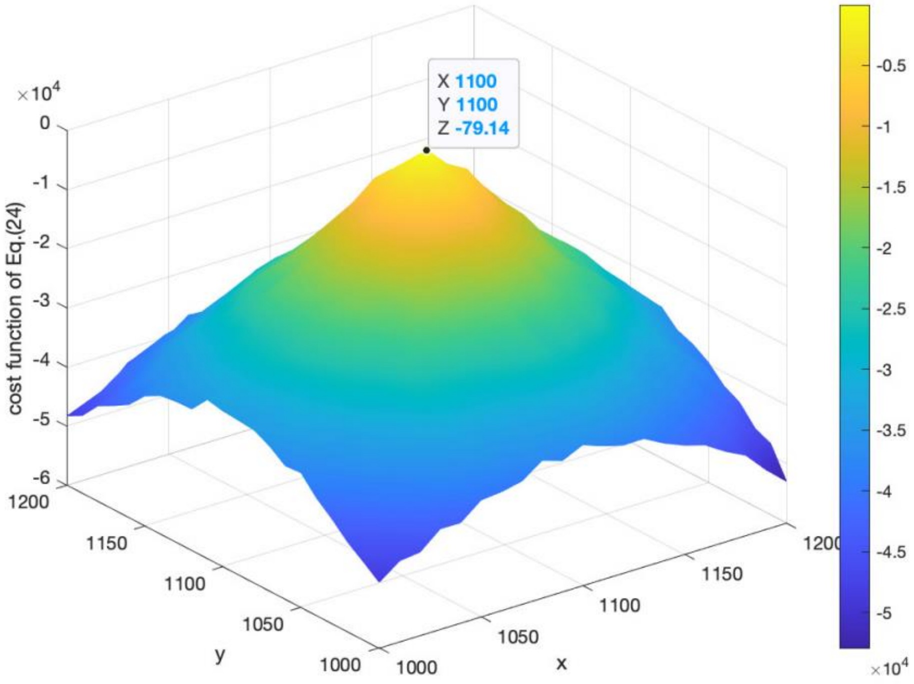
Numerical Results



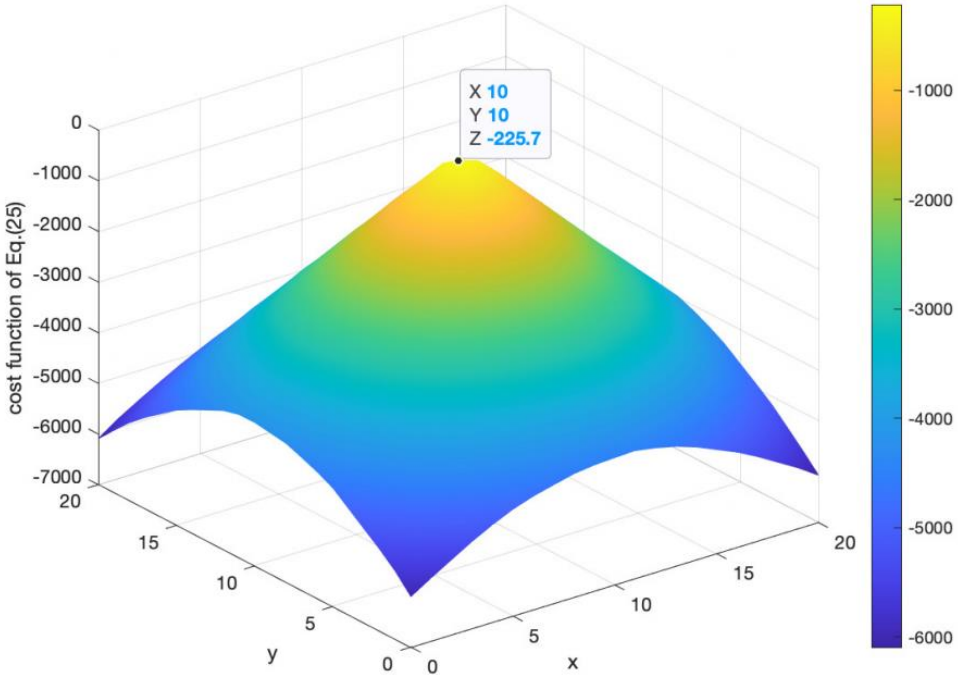


- ☐ It is shown that it is possible to simultaneously recover the matrices $\mathbf{X}_{mn}$ and $\mathbf{T}_{mn}$ from the low-bit quantized data.
- ☐ When the SNR is less than 20dB, the performance of 6-bit quantization is very close to that without quantization, proving the effectiveness of low-bit quantization.

Numerical Results



(a) Position estimation



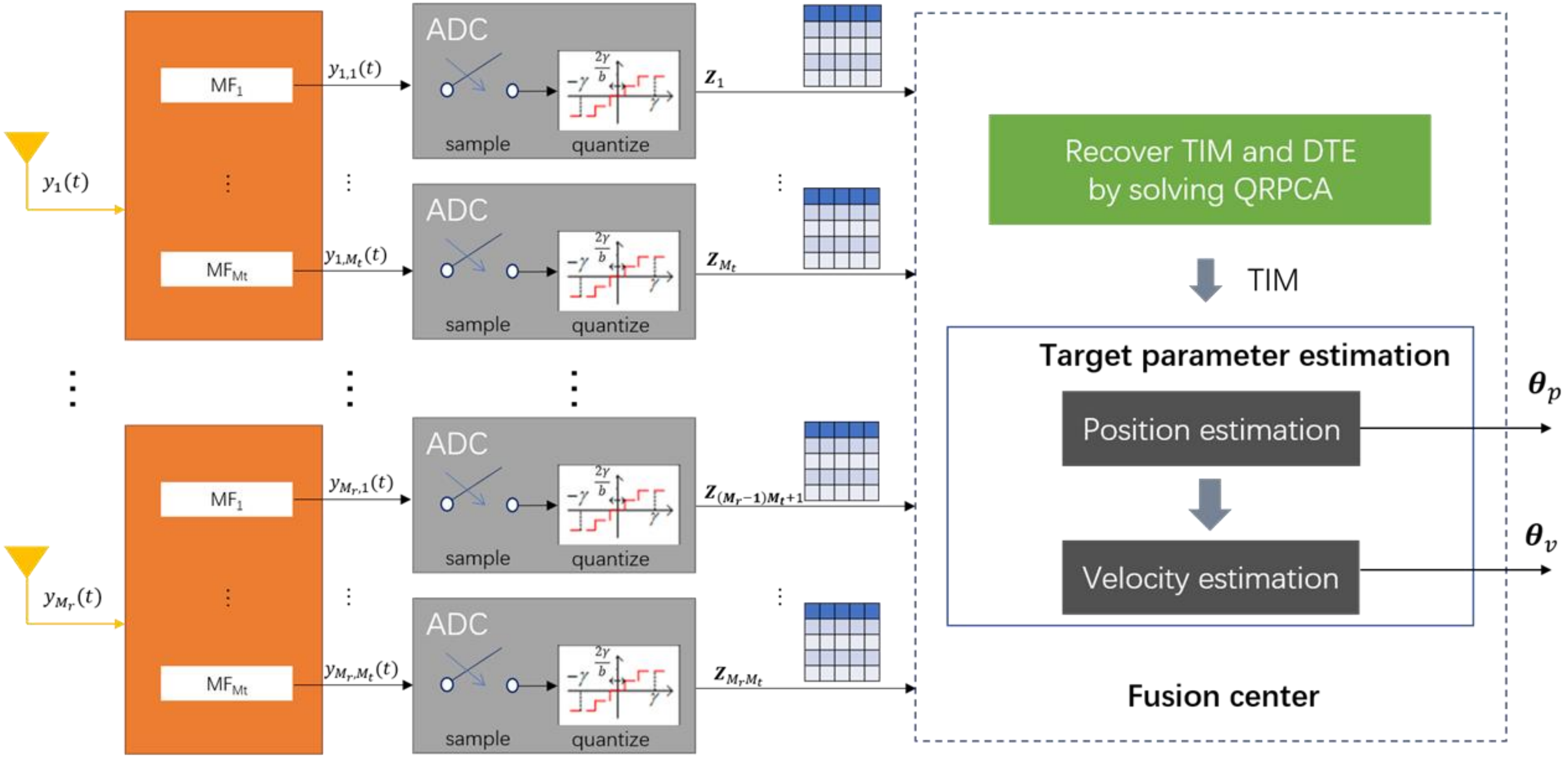
(b) Velocity estimation

- It can be seen that the position and velocity of the target can be accurately estimated by the LS-based method, using 4-bit quantization at SNR=20dB.

Conclusions

- ❑ LiQuiD MIMO radar
  - ▪ Propose a low-bit quantized distributed MIMO radar system;
  - ▪ Formulate a QRPCA problem to recover the infinite-precision target information matrix and the data transmission errors simultaneously;
  - ▪ Demonstrate the feasibility of implementing a low-bit quantized distributed MIMO radar system.
- ❑ Future work
  - ▪ Derive the performance bound of the proposed LiQuiD-MIMO radar.

Thank you!



Recover TIM and DTE by solving QRPCA

Target parameter estimation

Position estimation

Velocity estimation

Fusion center