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LiQuiD-MIMO Radar: Distributed MIMO Radar with Low-Bit Quantization

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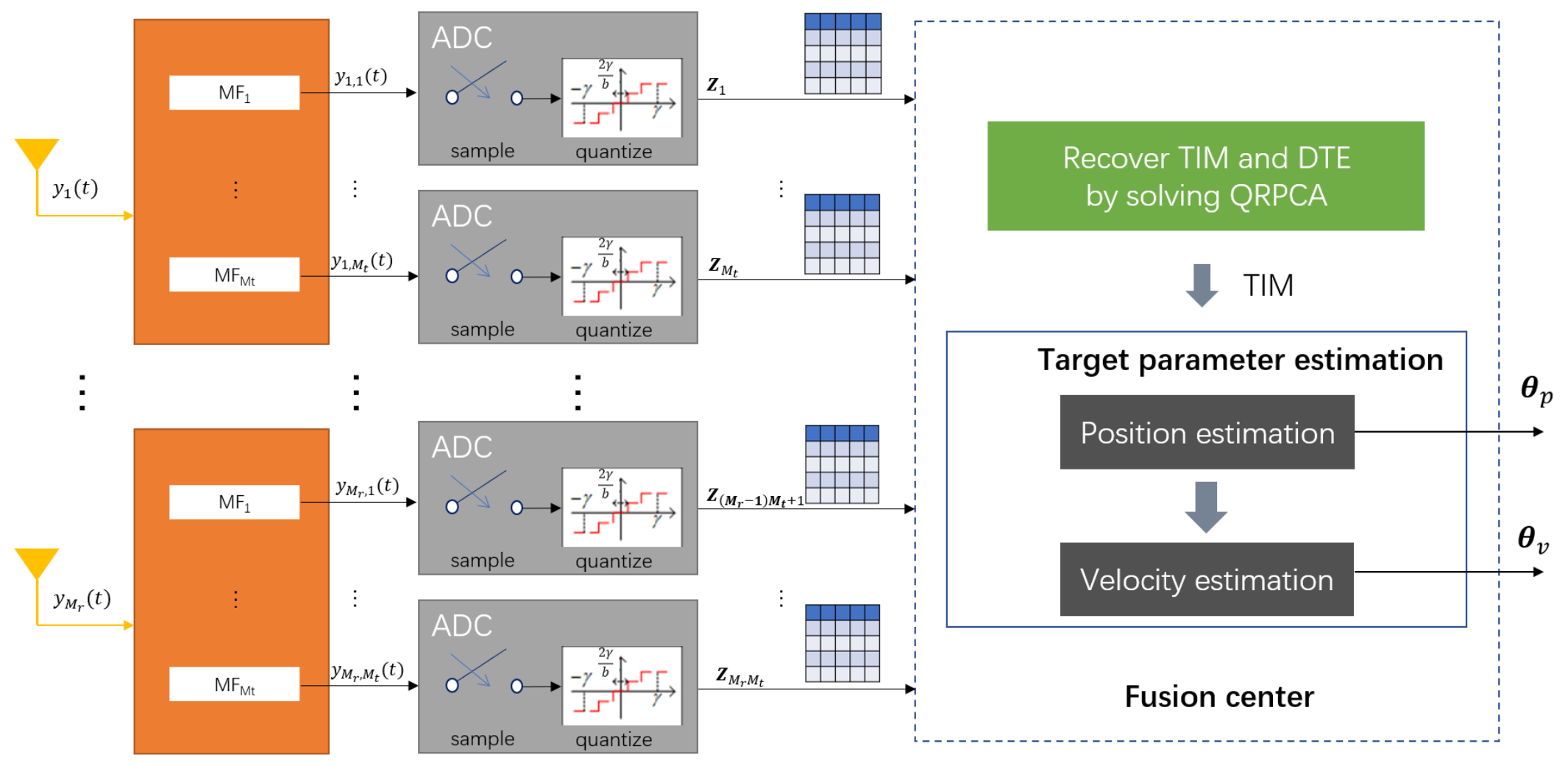
CONTRIBUTIONS

- ❑ **Motivation:** Reduce the cost, energy consumption, and system complexity of distributed MIMO radar systems
- ❑ **Key idea：**
  - ➢ Using low-resolution ADCs to reduce the data transmission volume
  - ➢ Formulate a QRPCA problem to recover the infinite-precision data
  - ➢ Develop an APG-based algorithm to solve the QRPCA problem
- ❑ **Result:** Develop a low-bit quantized distributed MIMO radar (LiQuiD-MIMO radar)

Low-resolution ADCs + Data recovery + Parameter Estimation



LiQuiD-MIMO Radars Model

Signal Model

- ❑ Time delay

$$\tau_{mn}^{(k)} = \frac{\left\|\mathbf{p}^{(k)} - \mathbf{p}_t^{(k)}\right\| + \left\|\mathbf{p}^{(k)} - \mathbf{p}_r^{(k)}\right\|}{c}$$

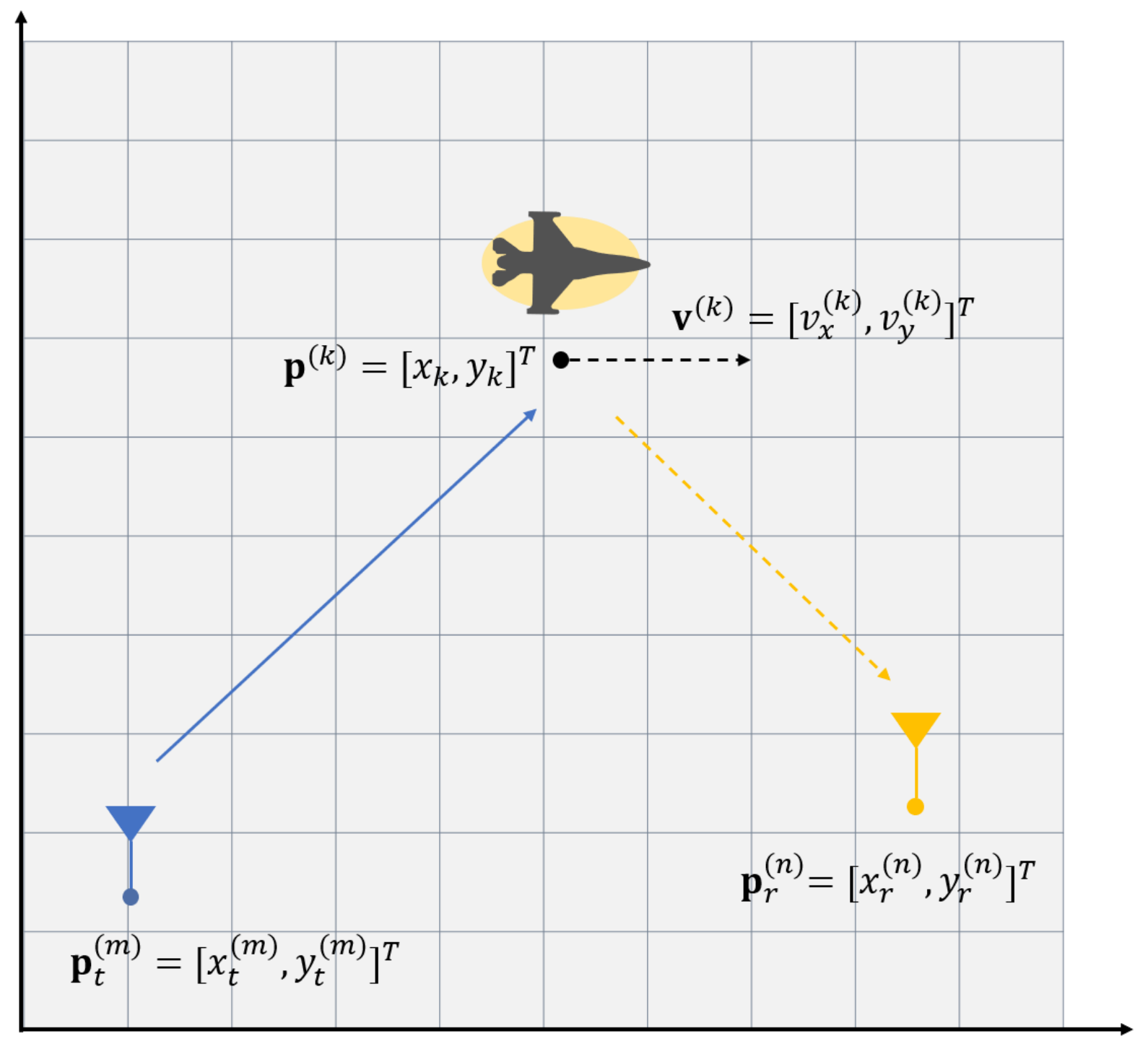
- ❑ Doppler frequency

$$f_{mn}^{(k)} = \frac{f_m}{c}\left(\frac{\left\langle \mathbf{v}^{(k)}, \mathbf{p}^{(k)} - \mathbf{p}_t^{(m)}\right\rangle}{\left\|\mathbf{p}^{(k)} - \mathbf{p}_t^{(m)}\right\|} + \frac{\left\langle \mathbf{v}^{(k)}, \mathbf{p}^{(k)} - \mathbf{p}_r^{(m)}\right\rangle}{\left\|\mathbf{p}^{(k)} - \mathbf{p}_r^{(m)}\right\|}\right)$$

- ❑ Received signal:

$$y_{mn}(t) = \sum_{q=0}^{Q-1}\sum_{k=1}^{K} \beta_{mn}^{(k)} s_m(t - \tau_{mn}^{(k)} - qT_{\mathrm{PRI}}) e^{j2\pi f_{mn}^{(k)} qT_{\mathrm{PRI}}} + w_{mn}(t)$$

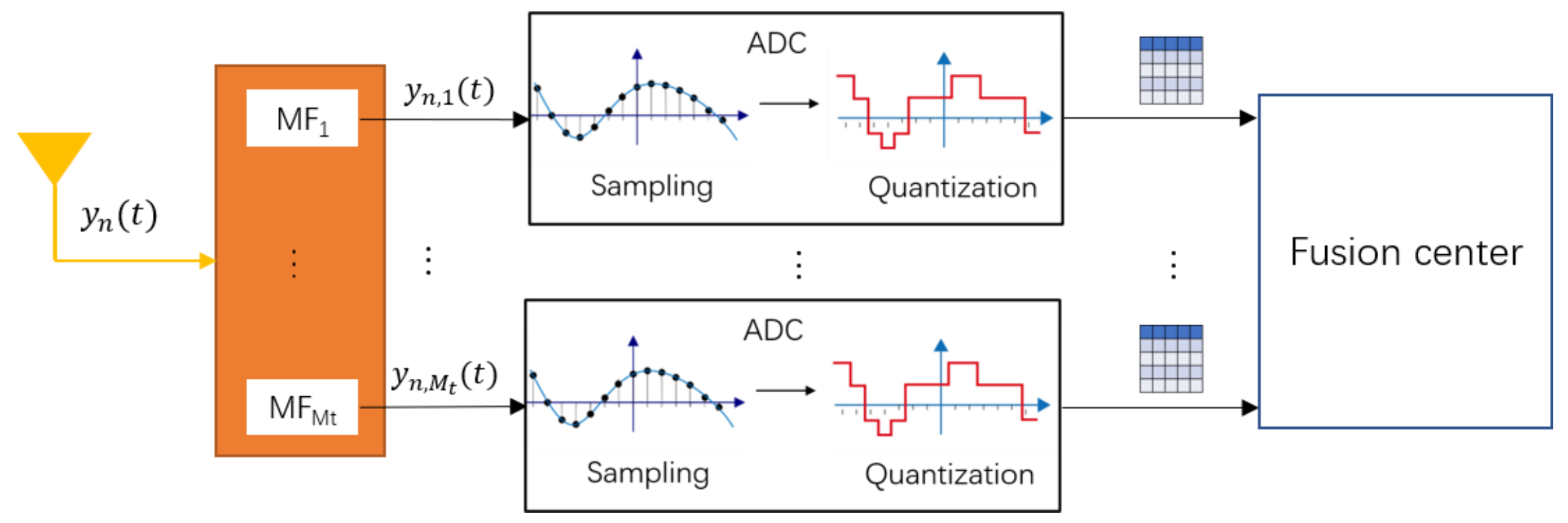
- ❑ $s_m(t)$ could be FDMA waveforms
- ❑ Task: Resolve the K position and velocity pairs $\{\mathbf{p}^{(k)}, \mathbf{v}^{(k)}\}_{k=1}^{K}$ from received signals.



Sampling and Quantization with Low-Resolution ADCs

- ❑ Using the low-resolution ADCs: each data is quantized into $\tilde{b}$ bits, e.g., $\tilde{b} = 2, 3,4$;
- ❑ Send quantized data to fusion center.
- ❑ $\mathbf{X}_{mn}$:Target information matrix (TIM); $\mathbf{W}_{mn}$:White Gaussian Noise (WGN); $\widetilde{\mathbf{T}}_{mn}$:Data transmission error (DTE).

$$\mathbf{Y}_{mn} = \mathbf{X}_{mn} + \mathbf{W}_{mn} \Longrightarrow \widetilde{\mathbf{Z}}_{mn} = \mathcal{Q}_C^{\gamma,b}(\mathbf{X}_{mn} + \mathbf{W}_{mn}) \Longrightarrow \mathbf{Z}_{mn} = \mathcal{Q}_C^{\gamma,b}(\mathbf{X}_{mn} + \mathbf{W}_{mn}) + \widetilde{\mathbf{T}}_{mn}$$



QRPCA Problem Formulation

- ❑ $\mathbf{Z} = \mathcal{Q}_C^{\gamma,b}(\mathbf{X} + \mathbf{W}) + \widetilde{\mathbf{T}}$ can be equivalent to $\mathbf{Z} = \mathcal{Q}_C^{\gamma,b}(\mathbf{X} + \mathbf{T} + \mathbf{W})$ (omitting the subscript mn).
  - ■ $\mathbf{X}$: Low rank. Its rank depends on the number of targets with different distances or different velocities.
  - ■ $\widetilde{\mathbf{T}}$: Sparse. It is generally sparse since the bit error rate (BER) is generally quite low.
  - ■ $\mathbf{T}$: Sparse. It is an equivalent sparse DTE before quantization.
- ❑ Recover the low-rank matrix $\mathbf{X}$ and the sparse matrix $\mathbf{T}$ by solving QRPCA problem.
  - ■ Function $D(\cdot,\cdot)$ is similarity metric which measures the similarity between the quantized data $\mathbf{Z}$ and the unquantized data**Y**=**X**+**T** .

$$\begin{aligned}-\frac{\Delta}{2} &\le \Re\{\mathbf{Y} - \mathbf{Z}\} \le \frac{\Delta}{2}\\ -\frac{\Delta}{2} &\le \Im\{\mathbf{Y} - \mathbf{Z}\} \le \frac{\Delta}{2}\end{aligned} \Longrightarrow \begin{aligned} &D(\mathbf{Z}, \mathbf{X} + \mathbf{T})\\ &= \left\|\rho\left(\left[\Re\{\mathbf{X} + \mathbf{T} - \mathbf{Z}\} + \frac{\Delta}{2}; \Im\{\mathbf{X} + \mathbf{T} - \mathbf{Z}\} + \frac{\Delta}{2}\right]\right)\right\|_F^2\\ &+ \left\|\rho\left(\left[\Re\{\mathbf{Z} - \mathbf{X} - \mathbf{T}\} + \frac{\Delta}{2}; \Im\{\mathbf{Z} - \mathbf{X} - \mathbf{T}\} + \frac{\Delta}{2}\right]\right)\right\|_F^2\end{aligned}$$

where $\rho(\cdot)$ is an element-wise function with $\rho(x) = \max\{-x, 0\}$.

$$\min_{\mathbf{X},\mathbf{T}} \quad \frac{1}{2} D(\mathbf{Z}, \mathbf{X} + \mathbf{T}) + \mu \|\underbrace{\mathbf{X}}_{\text{Low rank}}\|_* + \lambda \|\underbrace{\mathbf{T}}_{\text{Sparse}}\|_1$$

Method

APG-QRPCA Algorithm

- ❑ Define $h(\mathbf{X}, \mathbf{T}) = \mu\|\mathbf{X}\|_* + \lambda\|\mathbf{T}\|_1$ and $g(\mathbf{X}, \mathbf{T}) = \frac{1}{2} D(\mathbf{Z}, \mathbf{X} + \mathbf{T})$,
- ❑ $h(\mathbf{X}, \mathbf{T})$ is convex , and $g(\mathbf{X}, \mathbf{T})$ is differentiable.
- ❑ The QPRCA problem can be rewritten as $\min_{\mathbf{X},\mathbf{T}} \quad h(\mathbf{X}, \mathbf{T}) + g(\mathbf{X}, \mathbf{T})$
- ❑ Iteratively calculate
  - ■ Calculate momentum $\overline{\mathbf{X}}_l = \mathbf{X}_l + \frac{\zeta_{l-1}}{\zeta_l}(\mathbf{X}_l - \mathbf{X}_{l-1}), \overline{\mathbf{T}}_l = \mathbf{T}_l + \frac{\zeta_{l-1}}{\zeta_l}(\mathbf{T}_l - \mathbf{T}_{l-1})$
  - ■ Gradient descent $\mathbf{X}_p = \overline{\mathbf{X}}_l - \delta\nabla_{\mathbf{X}} g(\overline{\mathbf{X}}_l, \overline{\mathbf{T}}_l), \mathbf{T}_p = \overline{\mathbf{T}}_l - \delta\nabla_{\mathbf{T}} g(\overline{\mathbf{X}}_l, \overline{\mathbf{T}}_l)$
  - ■ Proximal map

$$\mathbf{X}_{l+1} = \arg\min_{\mathbf{X}} \left\{\mu\|\mathbf{X}\|_* + \frac{1}{2\delta}\left\|\mathbf{X} - \mathbf{X}_p\right\|_F^2\right\} = \mathbf{U}_p \mathcal{S}_{\mu\delta}(\boldsymbol{\Sigma}_p)\mathbf{V}_p^T,$$

$$\mathbf{T}_{l+1} = \arg\min_{\mathbf{T}} \left\{\lambda\|\mathbf{T}\|_1 + \frac{1}{2\delta}\left\|\mathbf{T} - \mathbf{T}_p\right\|_F^2\right\} = \mathcal{S}_{\lambda\delta}(\mathbf{T}_p)$$

LS-based Target Parameter Estimation

- ❑ $M_t \times M_r$ TIM matrixes can be recovered at the fusion center.
- ❑ A sequential LS method introduced to sequentially estimate the position and velocity parameters.
  - ■ $\boldsymbol{\theta}_p = \{\mathbf{p}^{(k)}\}_{k=1}^{K}$ and $\boldsymbol{\theta}_v = \{\mathbf{v}^{(k)}\}_{k=1}^{K}$ are implicitly determined by $\mathbf{A}_{mn}$ and $\mathbf{B}_{mn}$

Original problem:

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \sum_{m=1}^{M_t}\sum_{n=1}^{M_r} \left\|\widehat{\mathbf{X}}_{mn} - \mathbf{A}_{mn}(\boldsymbol{\theta}_p)\boldsymbol{\Lambda}_{mn}\mathbf{B}_{mn}(\boldsymbol{\theta}_v)\right\|_2^2 \Longrightarrow$$

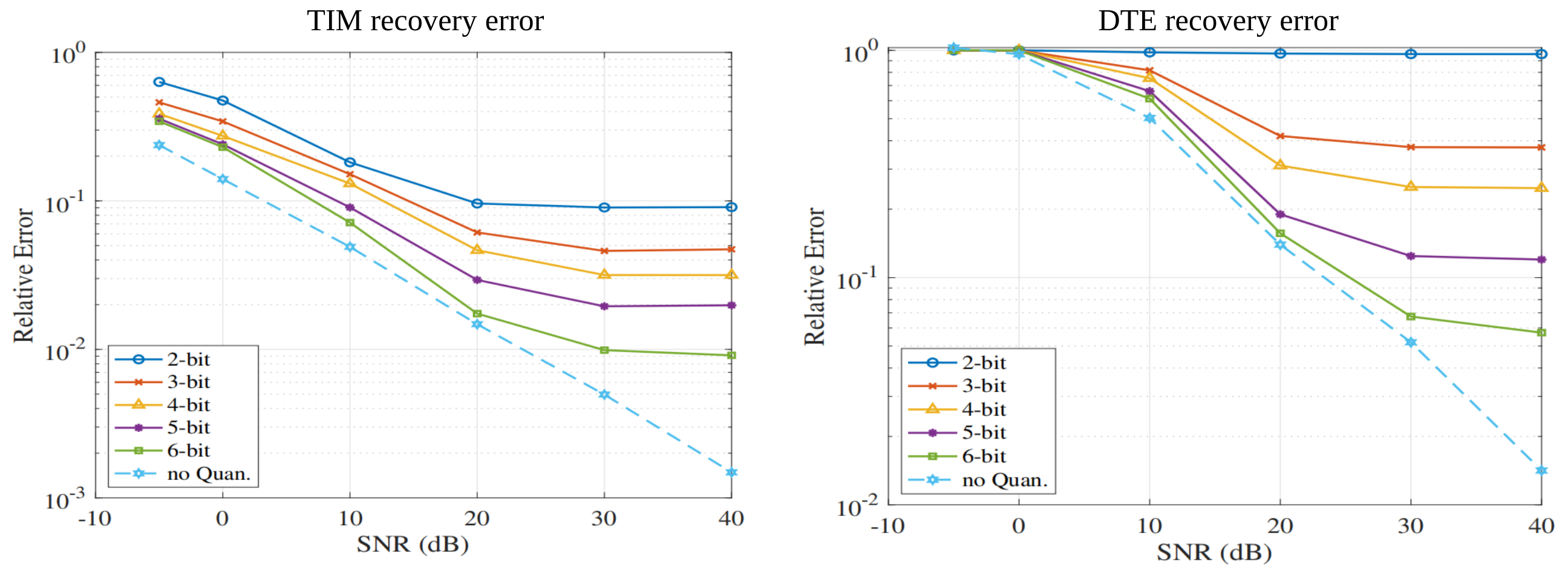
Two subproblems:

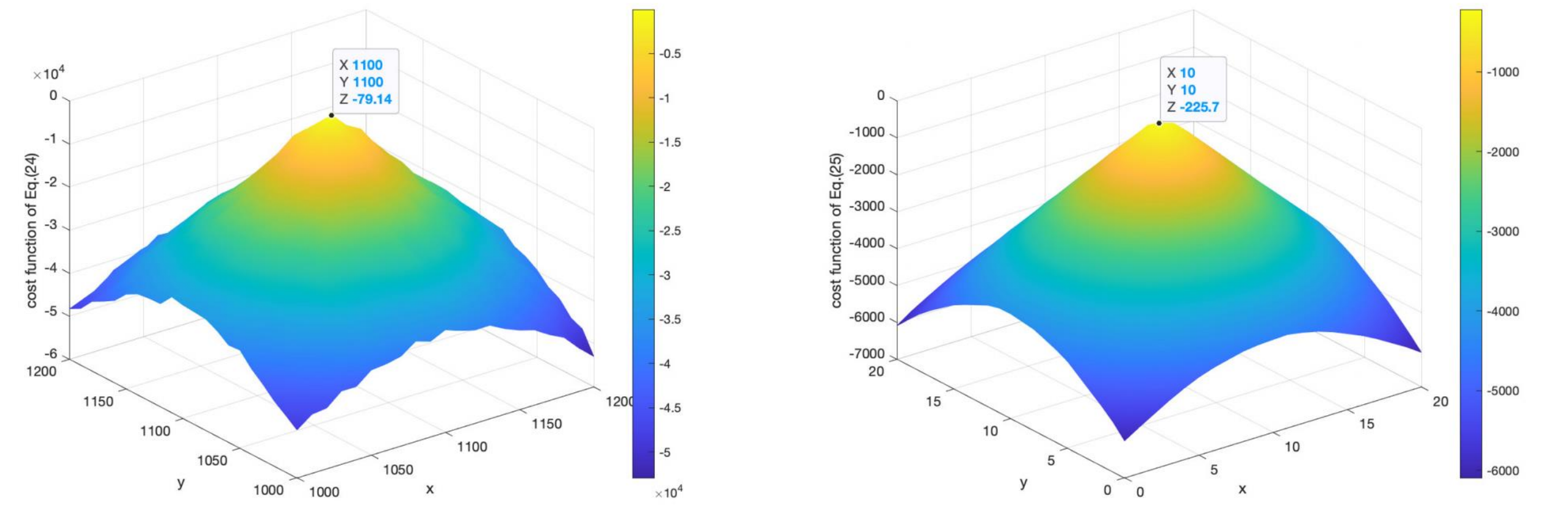
$$\widehat{\boldsymbol{\theta}}_p = \arg\min_{\boldsymbol{\theta}_p} \sum_{m=1}^{M_t}\sum_{n=1}^{M_r} \left\|\mathbf{P}_{p,mn}^{\perp}(\boldsymbol{\theta}_p)\widehat{\mathbf{X}}_{mn}\right\|_2^2$$

$$\widehat{\boldsymbol{\theta}}_v = \arg\min_{\boldsymbol{\theta}_v} \sum_{m=1}^{M_t}\sum_{n=1}^{M_r}\sum_{k=1}^{K} \left(\hat{f}_{mn}^{(k)} - f_{mn}^{(k)}(\mathbf{v}^{(k)})\right)^2$$

Numerical Example

$M_t = 3$ transmit antennas, $M_r = 10$ receive antennas, uniformly distributed on the concentric circles with radius 5km and 3km, respectively. The reference carrier frequency parameters $f_0 = 5\mathrm{GHz}$ and the frequency increment $\Delta f = 50\mathrm{MHz}$. One CPI consists of Q=128 pulses with $T_{\mathrm{PRI}} = 0.5\mathrm{ms}$ and $T_p = 6.4\mu s$. The transmitters emit Hadamard sequences with length of $N = 64$. 1% symbol error rate is assumed to lead sparse data transmission error matrix. One target is located at $\mathrm{p}^{(1)} = [1100, 1100]^T\mathrm{m}$ with $v^{(1)} = [10, 10]^T\mathrm{m/s}$.





(a) Position estimation

(b) Velocity estimation

Conclusions

- ❑ Propose a low-bit quantized distributed MIMO radar system;
- ❑ Formulate a QRPCA problem to recover the infinite-precision target information matrix and the data transmission errors simultaneously;
- ❑ Demonstrate the feasibility of implementing a low-bit quantized distributed MIMO radar system;
- ❑ The analysis of the performance bound of the proposed LiQuiD-MIMO radar will be our future work.