Restoration of Time-varying Graph Signals Using Deep Algorithm Unrolling

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Introduction

Graph Signal Processing (GSP)

- Signal often have their underlying structures
- GSP can treat the underlying structure of signals
  filtering, sampling, restoration

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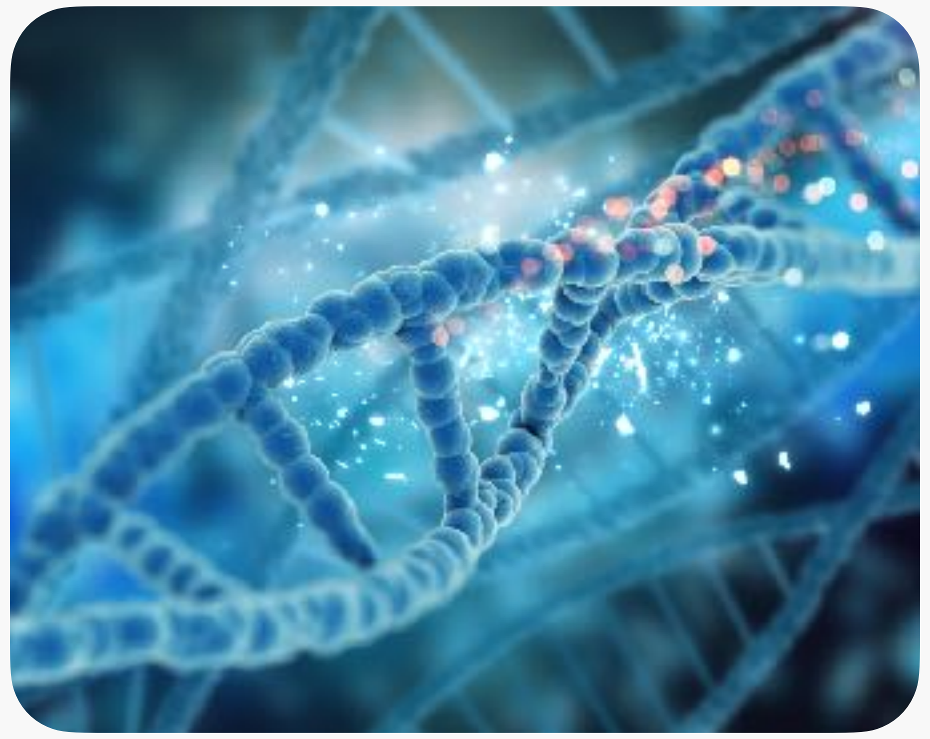
Transportation network



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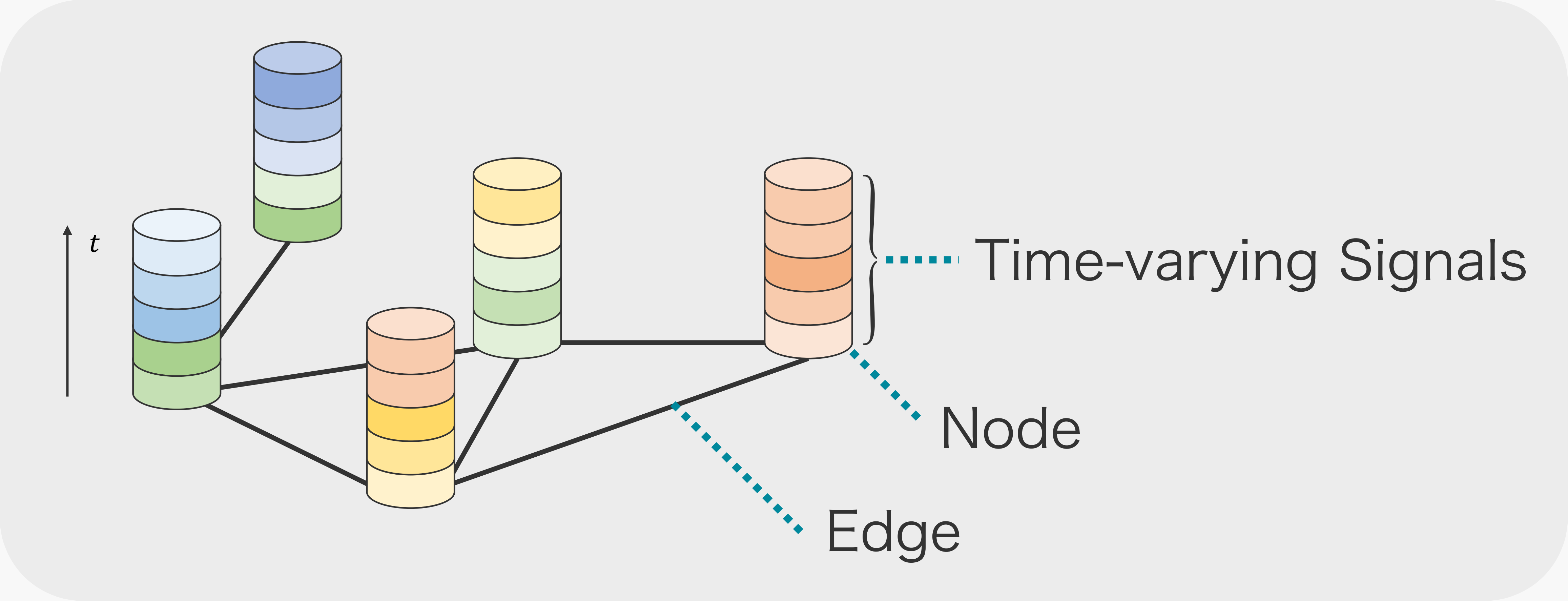
Bioinformatics

Time-varying graph signals

- A time-varying signal defined on a graph
  graph (nodes and edges) + time-varying signals



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Human pose

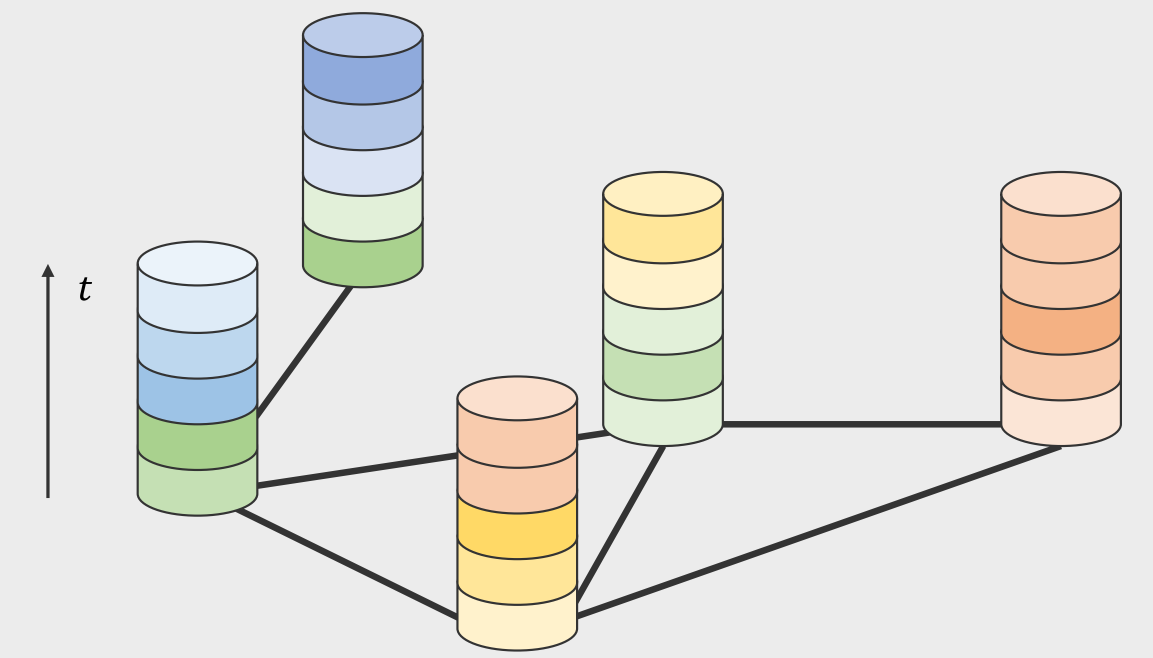
Image courtesy of pixabay.com

Social network

Purpose

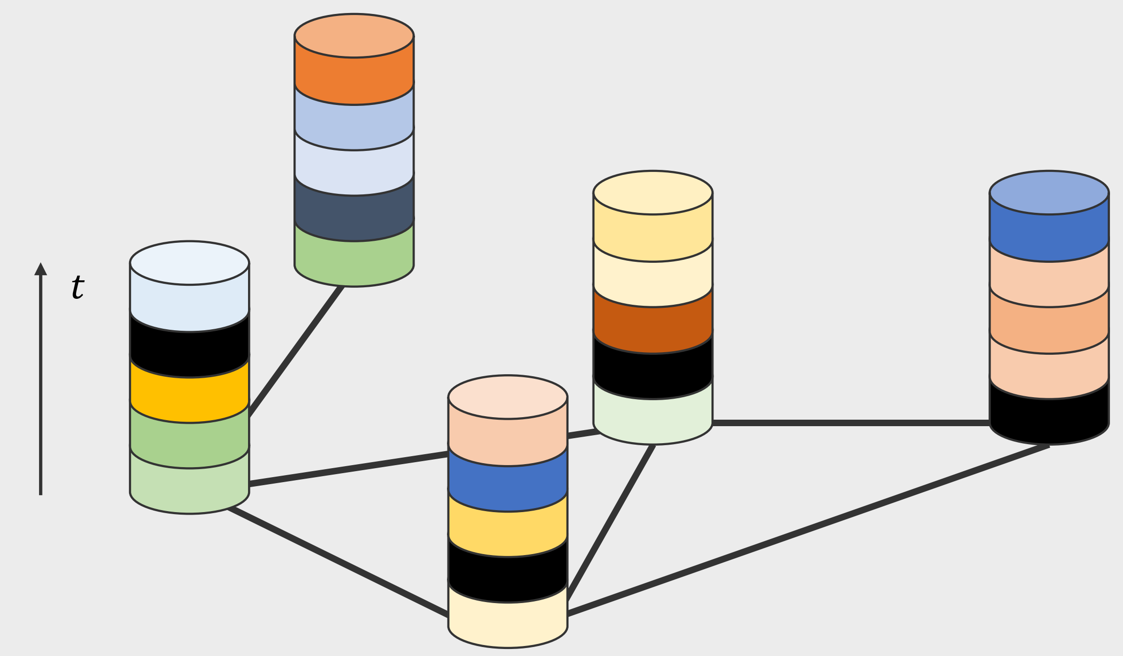
Recover original time-varying graph signals from observed signals

Original signals



Noise・Missing

Observed Signals



: Missing values

Restoration

Restoration Signals

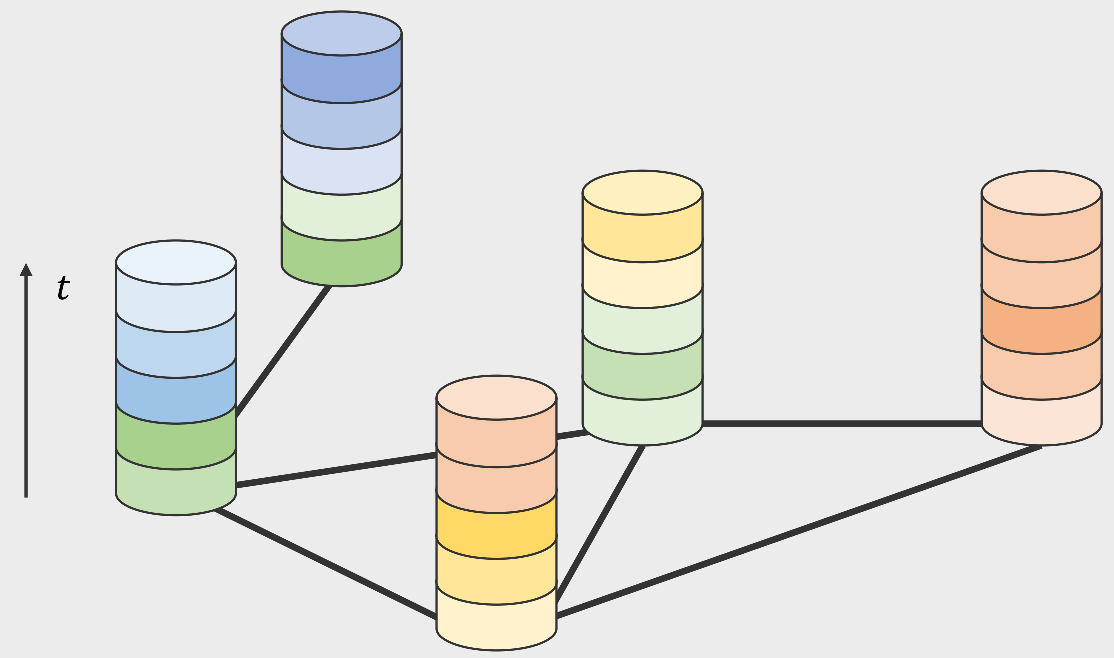


Photo by Viktor Hanacek on picjumbo

measurement error

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connection error

- Low-rank matrix completion
- Optimization method
- Deep neural network
- Graph neural network

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Proposed Method : Design of optimization problems

Framework Design of optimization problems + Unrolled iterations

Optimization Problems

$$\min_{\mathbf{X}} \frac{1}{2}\underbrace{\|\mathbf{J} \circ \mathbf{X} - \mathbf{Y}\|_F^2}_{\text{Data fidelity}} + \frac{\lambda}{2} \underbrace{\mathrm{tr}\left((\mathbf{X}\mathbf{D}_L)^\top \mathbf{L}(\mathbf{X}\mathbf{D}_L)\right)}_{\text{Spatiotemporal smoothness of L-steps graph signals}}$$

: Hyperparameter

$\mathbf{X} \in \mathbb{R}^{N \times T}$: Restoration Signals
$\mathbf{Y} \in \mathbb{R}^{N \times T}$: Observed Signals
$\mathbf{J} \in \{0,1\}^{N \times T}$: Sampling Operator
$\mathbf{L} \in \mathbb{R}^{N \times N}$: Graph Laplacian
N : # Nodes
T : # Times

$$\mathbf{D}_L = \begin{bmatrix} -d_1 & & & \\ -d_2 & -d_1 & & \\ \vdots & \vdots & & \\ -d_L & -d_{L-1} & & \\ 1 & -d_L & \ddots & \vdots \\ & 1 & \ddots & -d_{L-1} \\ & & \ddots & -d_L \\ & & & 1 \end{bmatrix} \in \mathbb{R}^{T \times (T-L)}$$

$$d_1, \dots, d_L \in \mathbb{R}_{\geq 0} \qquad \sum_{l=1}^{L} d_l = 1$$

- ✔ Regularization terms ensure spatiotemporal smoothness
- ✔ L-tap FIR filter for considering past signals
- ✔ Can be solved by iterative algorithm
- ✔ Need to determine hyperparameters

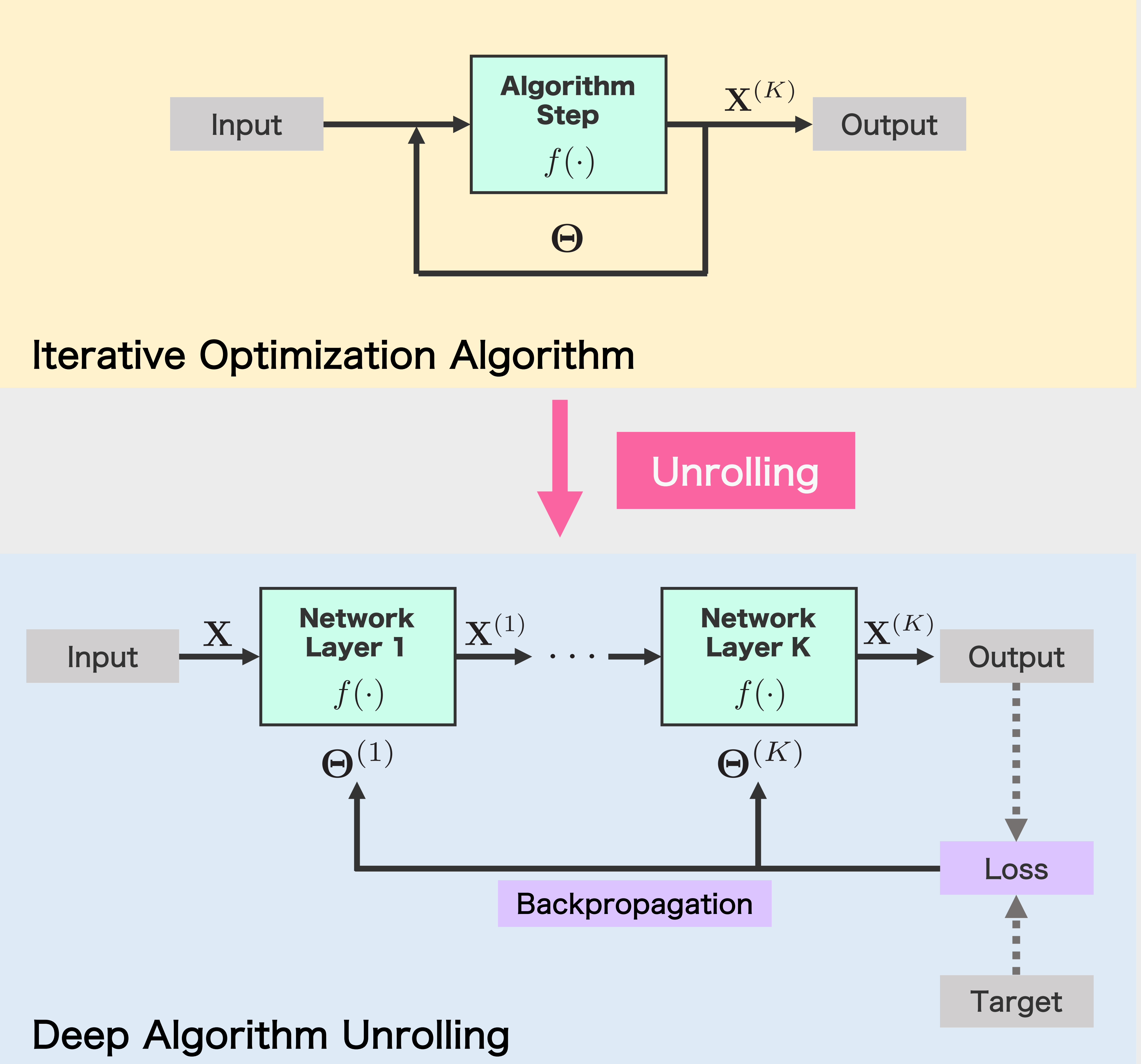
Proposed Method : Application of Deep Algorithm Unrolling

Framework Design of optimization problems + **Unrolled iterations**

Unrolled iterations

$$\min_{\mathbf{X}} \underbrace{\frac{1}{2}||\mathbf{J} \circ \mathbf{X} - \mathbf{Y}||_F^2}_{\text{Data fidelity}} + \frac{\lambda}{2} \underbrace{\mathrm{tr}\left((\mathbf{X}\mathbf{D}_L)^\top \mathbf{L}(\mathbf{X}\mathbf{D}_L)\right)}_{\text{Spatiotemporal smoothness of L-steps graph signals}}$$

: Training Parameter



Replacing **Iterative Algorithms** to **Neural Networks**

- ✔ Automatically learns parameters that fit the training data
- ✔ Different parameters can be trained for each layer

Improved **performance** and **convergence speed**

Experiments : Setup

Experimental method

- Performance comparison for missing
- Performance comparison for noise

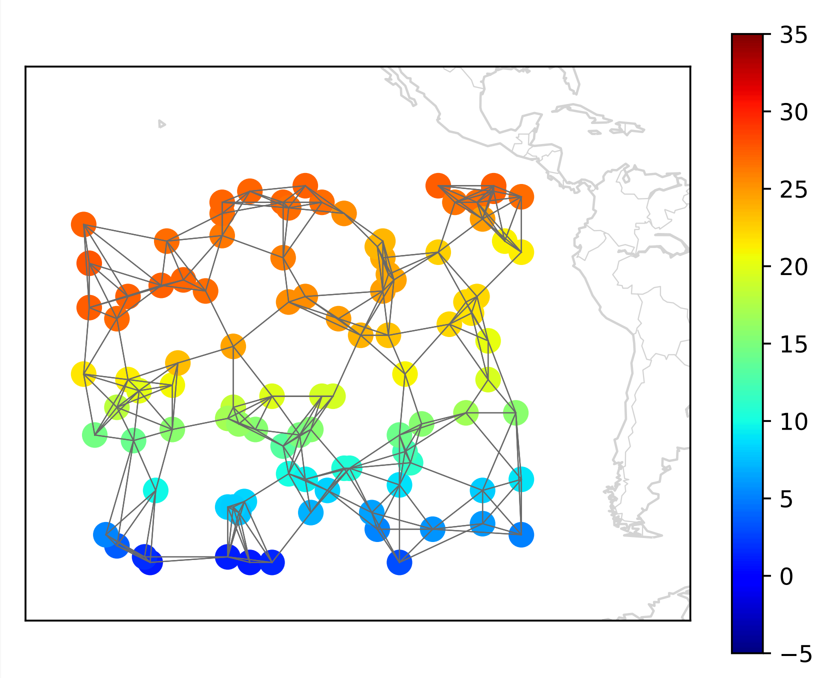
sampling operator noise tarm

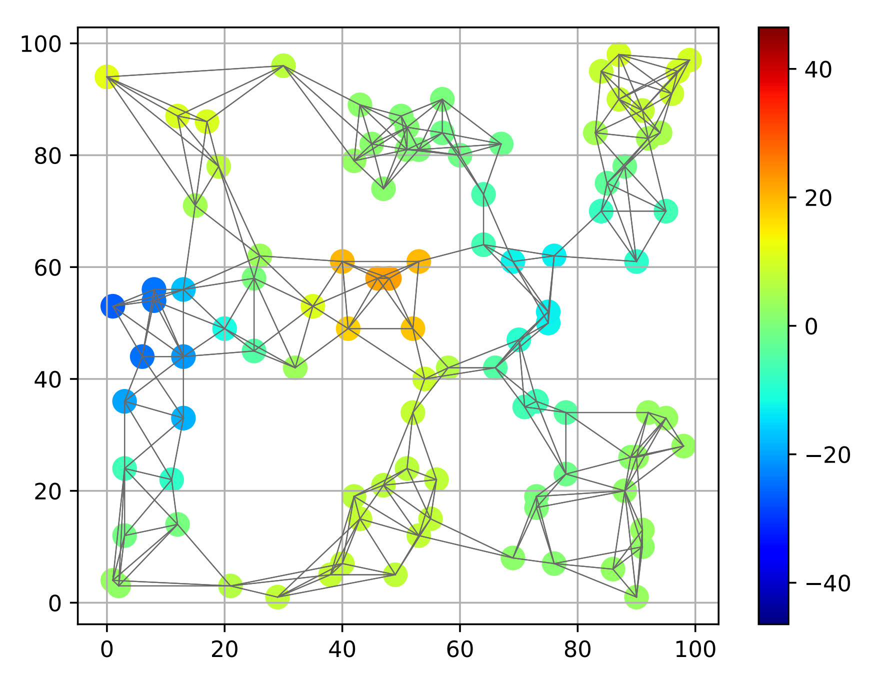
$$\mathbf{Y} = \mathbf{J} \circ (\mathbf{X} + \mathbf{V})$$

observed signal original signal

Datasets

Sea surface temperature[5] Synthetic





Alternative methods

Tikhonov [Perraudin+ IEEE ICASSP 2017]

OGTR [Qiu+ IEEE JSTSP 2017]

Sobolev [Giraldo+ IEEE TSIPN 2022]

DAU [Chen+ IEEE ICASSP 2022]

Parameter fix

Proposed method and DAU : Training

Alternative method : Grid search

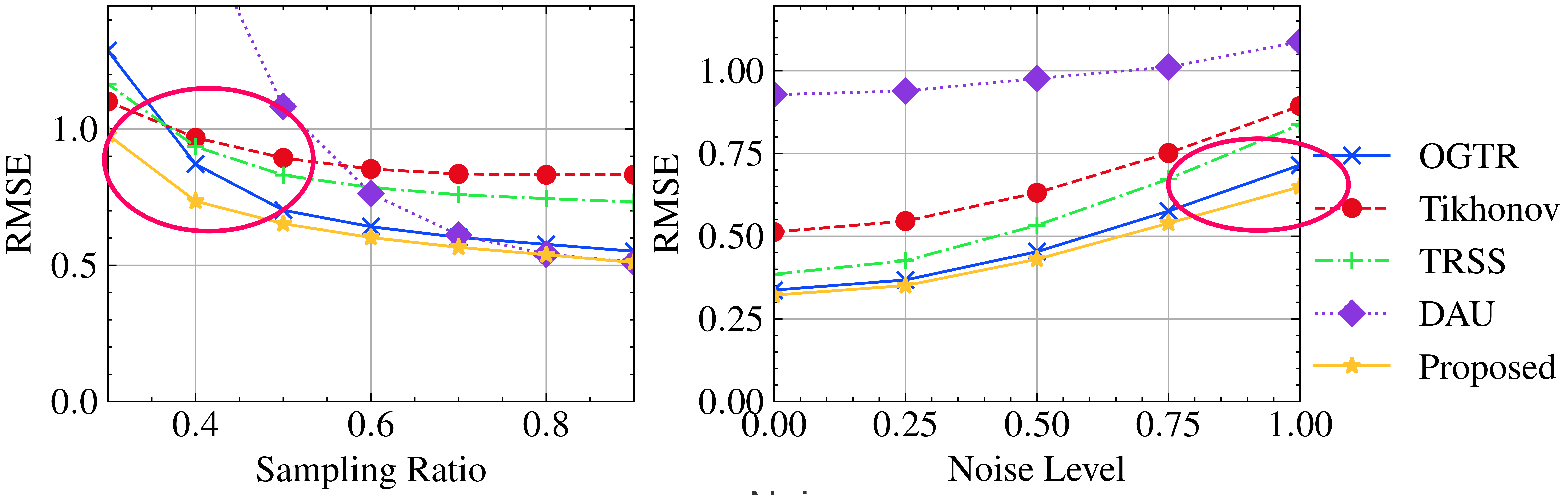
Evaluation Measure

$$loss_{\mathrm{RMSE}} = \sqrt{\frac{1}{NT}\sum_{n,t}(\tilde{X}_{n,t} - X^*_{n,t})^2}$$

Experiments : Result (sea surface temperature dataset)

performance for **missing**

performance for **noise**



Missing **High** ⟷ **Low**

Noise Level **Low** ⟷ **High**

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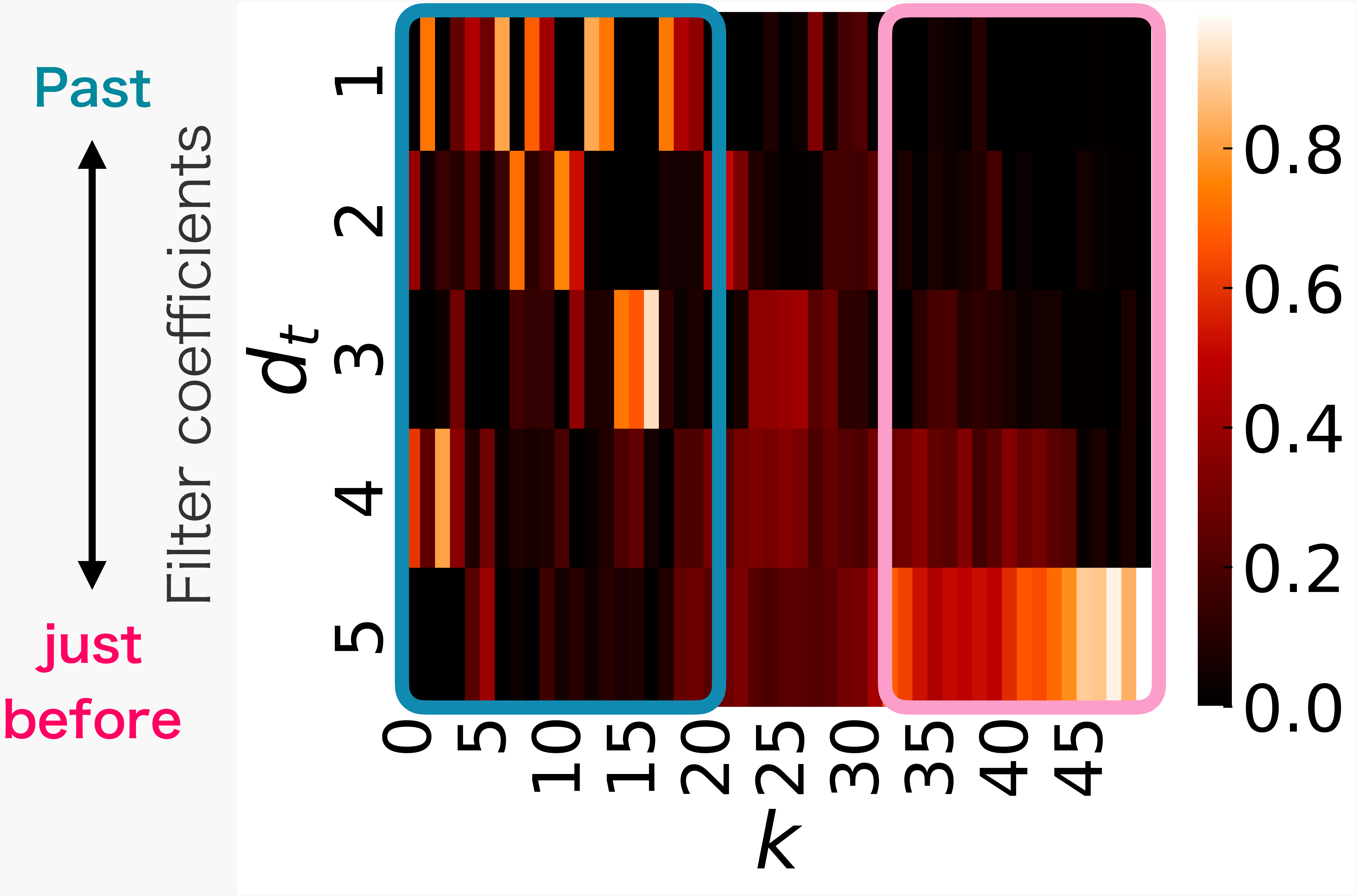
Discussion : Learned parameters



Layer of trained network

Beginning of the iterations

Extensive use of past signals

→ Captures global characteristics of signals

End of the iterations

Extensive use of just before signals

→ Brush up using detailed features

Appropriate parameters are learned

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Conclusion

Purpose

Restoration of **time-varying graph signals** from **observed signals**

Framework

Design of optimization problems + **Unrolled iterations**

Experiments

Sea surface temperature dataset・synthetic dataset
Performance is compared by sampling ratio and noise level.

Results

High restoration performance in
Low sampling ratio
High noise level

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