SCIENCE
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Variational Message Passing-Based Respiratory Motion Estimation and Detection Using Radar Signals

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Problem Statement

- Detect people (especially infants) in parked cars using UWB-radar.
- Considering strong clutter as well as multipath propagation.

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- Focus on detecting respiratory chest motion to separate target from clutter.
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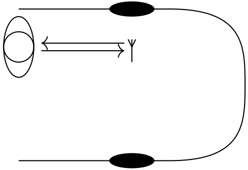
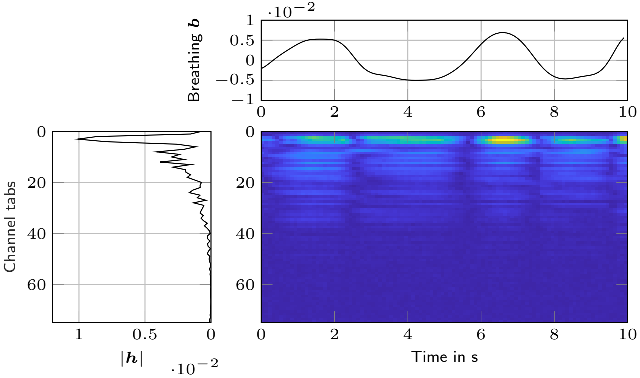
- Multipath propagation can be used to our advantage if the channel can be estimated.
- Use variational message passing to estimate (an approximation) of the model evidence.

Signal Model



The received signal can be expressed as outer product of the respiratory motion $\boldsymbol{b}$ and channel vector $\boldsymbol{h}$ [1]

$$\boldsymbol{R} = \boldsymbol{h}\boldsymbol{b}^{\mathsf{T}} + \text{noise}$$



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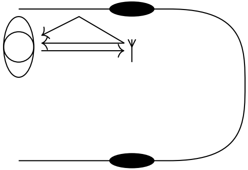

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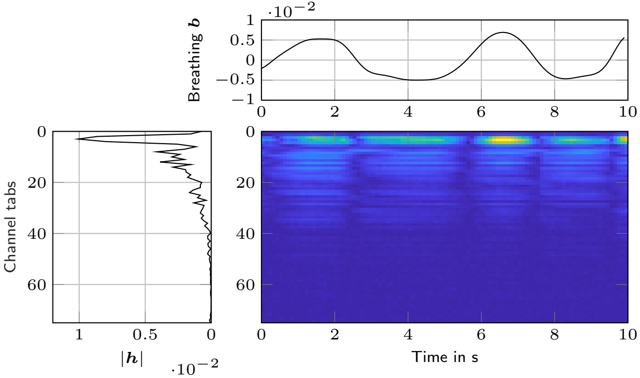
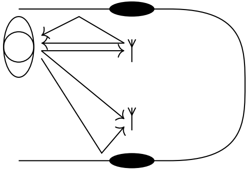
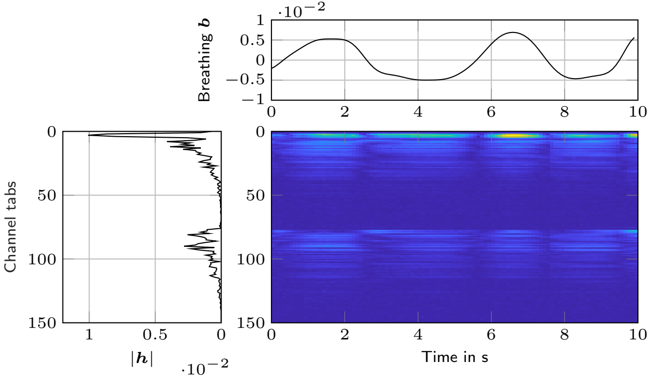


Signal Model (Multistatic)

For multistatic radar, we just stack the received signals.



$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_1 \\ \boldsymbol{R}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \end{bmatrix}}_{\boldsymbol{h}} \boldsymbol{b}^\mathsf{T} + \text{noise}$$



Bayesian Detection Problem

Nested decision problem

Empty car ($\mathcal{H}_0$): $\boldsymbol{b} = \boldsymbol{h} = \mathbf{0}$

$$p(\boldsymbol{R}|\boldsymbol{b}, \boldsymbol{h}, \lambda, \mathcal{H}_0) = \mathcal{CN}(\boldsymbol{R}|\mathbf{0},\ \lambda^{-1}\boldsymbol{I})$$

Occupied Car ($\mathcal{H}_1$):

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Model Evidence ($k \in \{0,\, 1\}$):

$$p(\mathcal{H}_k|\boldsymbol{R}) = \int \underbrace{p(\boldsymbol{R}|\boldsymbol{b}, \boldsymbol{h}, \lambda, \mathcal{H}_k)p(\boldsymbol{b})p(\boldsymbol{h})p(\lambda)p(\mathcal{H}_k)}_{p(\boldsymbol{b},\boldsymbol{h},\lambda,\mathcal{H}_k|\boldsymbol{R})}\, d\boldsymbol{b}\, d\boldsymbol{h}\, d\lambda$$

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Not feasible to solve integrals!

Variational Bayesian Inference

Variational Bayesian inference maximize a lower bound $\mathcal{L}(q_k)$ on the model evidence, based on a *factorized approximation* q_k, $k \in \{0, 1\}$ of the posterior [2]

$p(\boldsymbol{b}, \boldsymbol{h}, \lambda | \boldsymbol{R})$ is approximated by

$$q_1(\boldsymbol{b}, \boldsymbol{h}, \lambda) = q_\mathsf{b}(\boldsymbol{b}) q_\mathsf{h}(\boldsymbol{h}) q_\lambda(\lambda) \qquad q_0(\boldsymbol{b}=0, \boldsymbol{h}=0, \lambda) = q_0(\lambda)$$

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$$\ln p(\mathcal{H}_k | \boldsymbol{R}) \geq \mathcal{L}(q_k)$$

Equivalent to minimizing the KL-Divergence of $p(\boldsymbol{b}, \boldsymbol{h}, \lambda, \mathcal{H}_k | \boldsymbol{R})$ from q.

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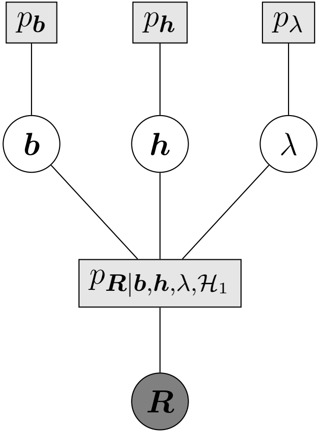
Equivalent to minimizing the KL-Divergence of $p(\boldsymbol{b}, \boldsymbol{h}, \lambda, \mathcal{H}_k|\boldsymbol{R})$ from q.
Can be implemented as message passing on a factor graph.

Probabilistic Model

Using conjugate priors results in well known shapes for the factors of q_1 [3]

$$q_{\mathsf{b}}(\boldsymbol{b}) = \mathcal{N}(\boldsymbol{b} \,|\, \hat{\boldsymbol{b}},\, \hat{\boldsymbol{C}}_{\mathsf{b}})$$
$$q_{\mathsf{h}}(\boldsymbol{h}) = \mathcal{CN}(\boldsymbol{h} \,|\, \hat{\boldsymbol{h}},\, \hat{\boldsymbol{C}}_{\mathsf{h}})$$
$$q_{\lambda}(\lambda) = \mathsf{Ga}(\lambda \,|\, NM,\, NM/\hat{\lambda})$$

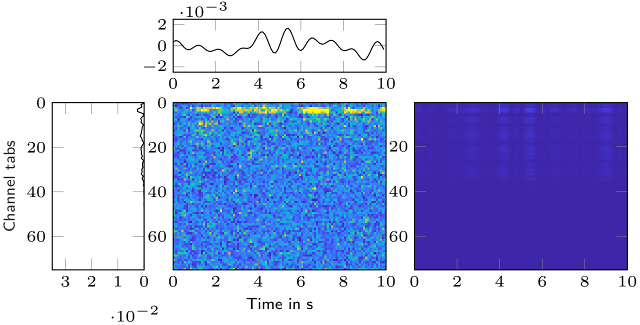
Only the means and covariances need to be passed as prarameters between iterations.



Interpretation

Using the estimate of $\boldsymbol{h}$ we (try to) coherently sum the channel in order to estimate $\boldsymbol{b}$ and vice versa.

The estimates are refined in each iteration until they are converged.



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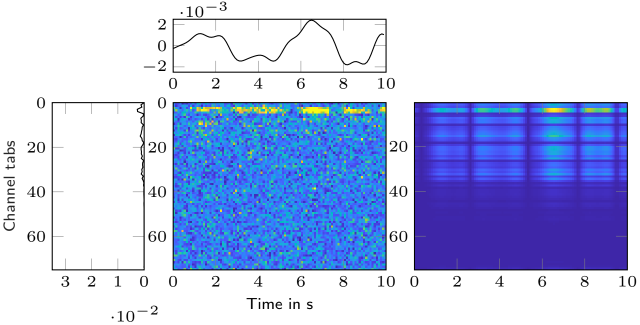

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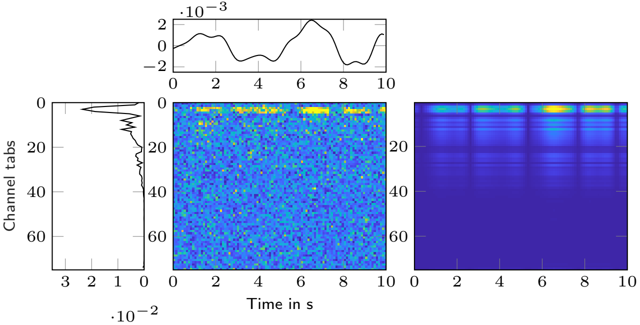

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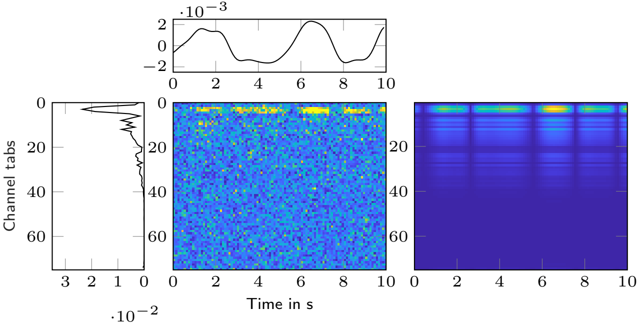

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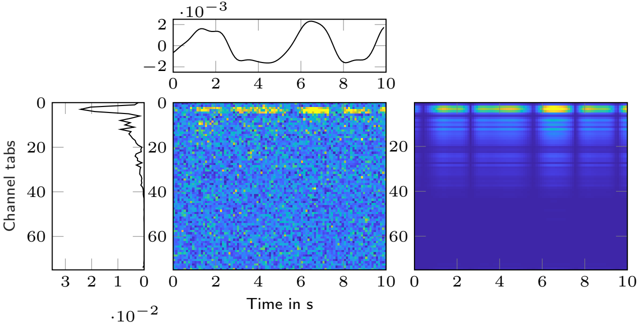

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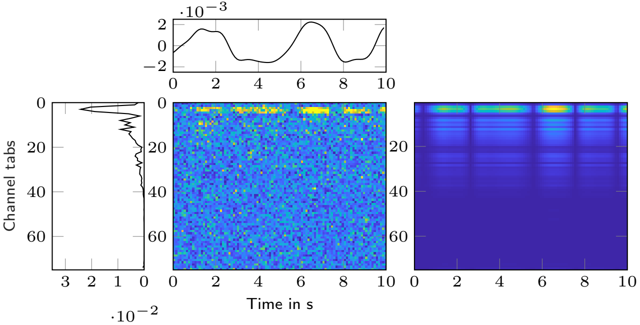

Measurements

- Measurements of 34 participants inside 2 different cars.
- 2 minutes per person, split into 10 seconds frames.

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- Compared against FFT detector (maximum of FFT over slow time).
- And against Estimator correlator [1].
  - Based on covariance $\boldsymbol{C}_{\mathsf{r}} = \boldsymbol{C}_{\mathsf{b}} \otimes \boldsymbol{C}_{\mathsf{h}}$ of the received signal but not the "hard" signal model $\boldsymbol{R} = \boldsymbol{h}\boldsymbol{b}^{\mathsf{T}} + \text{noise}$.

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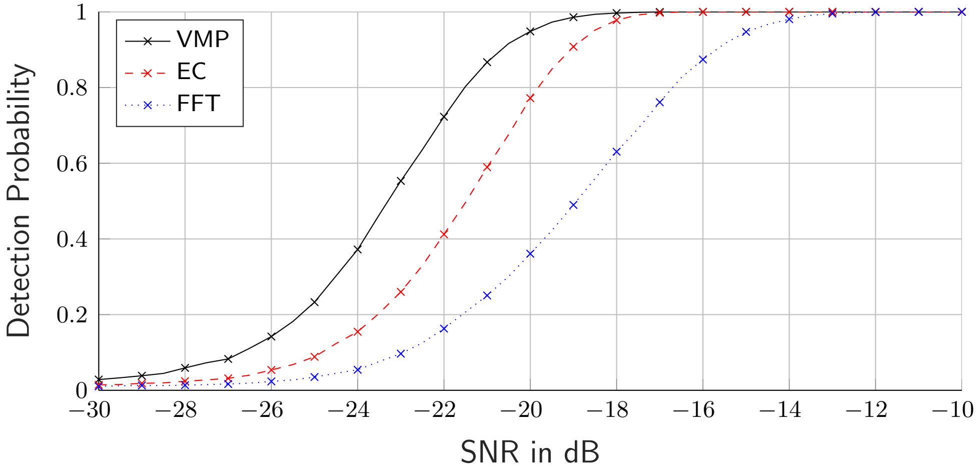

Conclusion

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- Variational message passing is a useful tool to detect and estimate vital signs in UWB radar.
- However, performance suffers if model assumptions (target is sitting still) are violated.

References

[1] J. Möderl, F. Pernkopf, and K. Witrisal, "Car occupancy detection using UWB radar," in *2021 18th Eur. Radar Conf.*, London, U.K., Apr. 5–7, 2022, pp. 313–316.

[2] C. M. Bishop, "Approximate inference," in *Pattern recognition and machine learning*, 8th ed., M. Jordan, J. Kleinberg, and B. Schölkopf, Eds. New York, NY, USA: Springer Science+Business Media, LLC, 2009, ch. 10, pp. 461–522.

[3] G. E. Kirkelund, C. N. Manchon, L. P. B. Christensen, E. Riegler, and B. H. Fleury, "Variational message-passing for joint channel estimation and decoding in MIMO-OFDM," in *2010 IEEE Global Telecommun. Conf.*, London, U.K., Dec. 6–10, 2010, pp. 1–6.