

# Kernel interpolation of acoustic transfer functions with adaptive kernel for directed and residual reverberations

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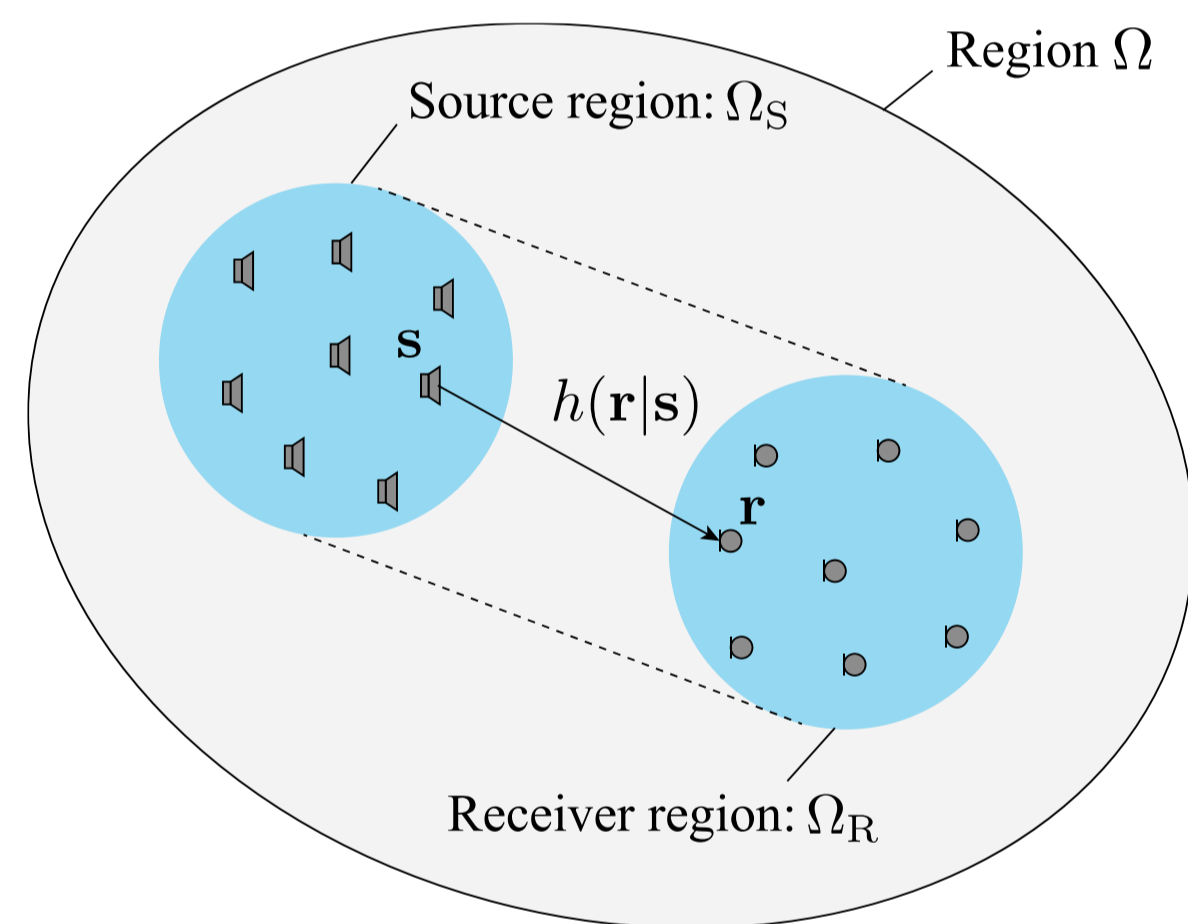
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## Background

- Sound wave propagation is unpredictable.  
⇒ Physical phenomena such as reflection and diffraction.
- **Acoustic transfer function (ATF)**:  
⇒ Describes space's influence on sound waves
- **Region-to-region**:  
⇒ Interpolate ATF continuously for source/receiver.
- **Sound field analysis**:  
⇒ Deeply related problem we can draw solutions from

## Problem statement

- Objective: interpolate ATF values from measurements.



- ATF components  $h(\mathbf{r}|\mathbf{s}) = h_R(\mathbf{r}|\mathbf{s}) + h_D(\mathbf{r}|\mathbf{s})$ .
- Direct is known:  $h_D(\mathbf{r}|\mathbf{s}, k) = \frac{e^{ik\|\mathbf{r}-\mathbf{s}\|}}{4\pi\|\mathbf{r}-\mathbf{s}\|}$ .
- Reverberant:  $h_R(\mathbf{r}|\mathbf{s}, k)$  is more involved.  
⇒ [Ribeiro+, 2020]: physical constraint in **kernel ridge regression** with ATF kernel.  
⇒ [Ribeiro+, 2022]: directionality improved performance.
- Generalized representation: Herglotz wave function.

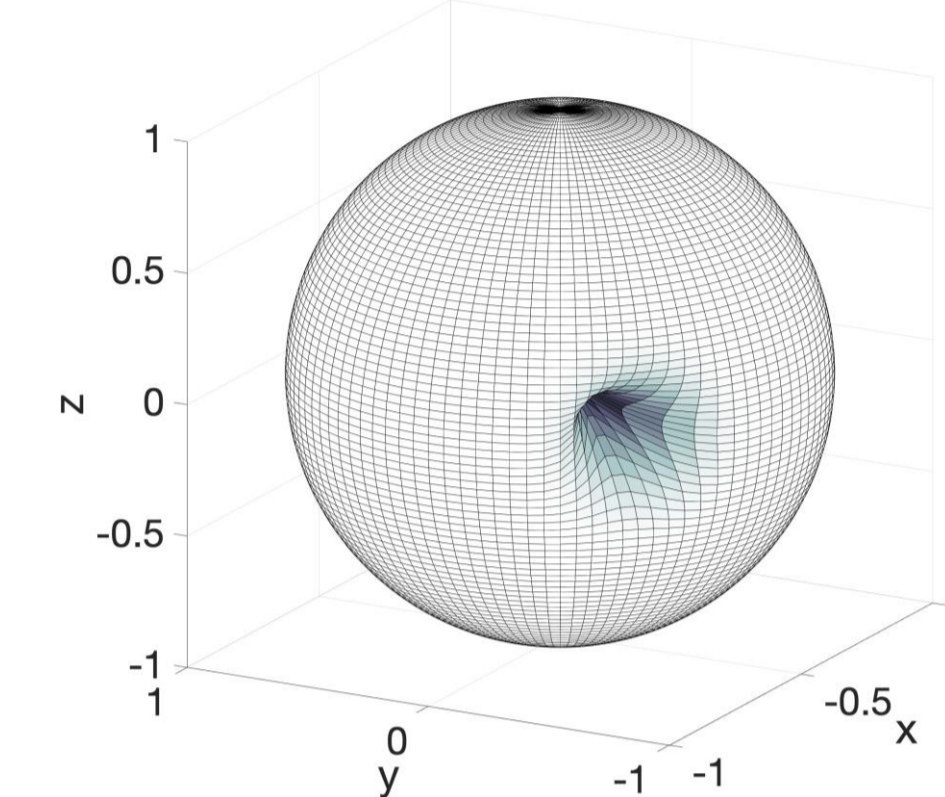
$$h_R(\mathbf{r}|\mathbf{s}) = \mathcal{T}(\tilde{h}_R; \mathbf{r}|\mathbf{s})$$

$$\text{Plane wave superposition} \Rightarrow \int_{\mathbb{S}^2 \times \mathbb{S}^2} e^{ik(\hat{\mathbf{r}} \cdot \mathbf{r} + \hat{\mathbf{s}} \cdot \mathbf{s})} \tilde{h}_R(\hat{\mathbf{r}}, \hat{\mathbf{s}}) d\hat{\mathbf{r}} d\hat{\mathbf{s}}$$

- Representation that guarantees physics of the problem.  
⇒ Kernel function that learns weight function as data model.

$$\kappa(\mathbf{r}|\mathbf{s}, \mathbf{r}'|\mathbf{s}') = \mathcal{T} \left( w(\hat{\mathbf{r}}, \hat{\mathbf{s}}) \frac{e^{-ik(\hat{\mathbf{r}} \cdot \mathbf{r}' + \hat{\mathbf{s}} \cdot \mathbf{s}')} + e^{-ik(\hat{\mathbf{r}} \cdot \mathbf{s}' + \hat{\mathbf{s}} \cdot \mathbf{r}')}}{2}; \mathbf{r}|\mathbf{s} \right)$$

- Previous kernels can be expressed with Herglotz wave function.
- [Ribeiro+, 2020]: kernel function equivalent to a **uniform** weight.  
⇒ Satisfies basic physical properties.
- [Ribeiro+, 2022]: **sunken sphere** weight function.  
⇒ Introduces directionality to the estimation.  
⇒ Physical model limited, assumes uniform gain on the sides.
- Data models for the weights are inflexible.
- We propose an **adaptive kernel** that learns the weight as a more general data model.



## Proposed method

- Weighting represents directed and residual reverberations as the sum  $w = w_{\text{dir}} + w_{\text{res}}$ .

### Directed weight $w_{\text{dir}}$

- High amplitudes on sparse set of directions
- Strong directionality.
- Combination of von Mises-Fisher distributions.

$$w_{\text{dir}}(\hat{\mathbf{r}}, \hat{\mathbf{s}}) = \varphi_{\text{dir}}(\hat{\mathbf{r}}) \varphi_{\text{dir}}(\hat{\mathbf{s}}),$$

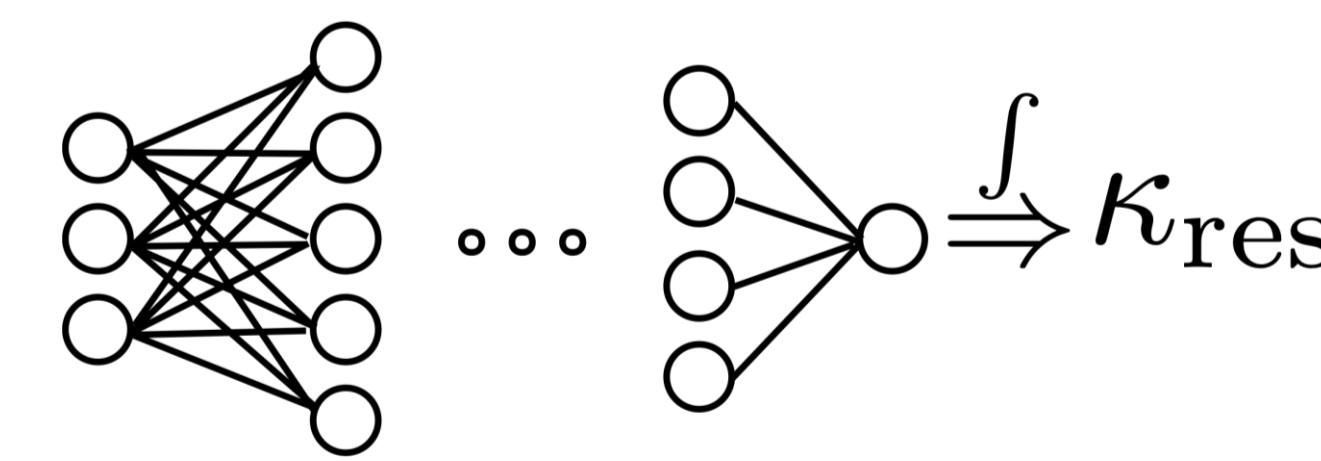
$$\varphi_{\text{dir}}(\hat{\mathbf{v}}) = \sum_{d=1}^D \alpha_d \frac{e^{\beta_d \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}_d}}{4\pi C(\beta_d)},$$

$$\|\alpha\|_1 = 1,$$

$$C(\beta_d) = \begin{cases} \frac{\sinh(\beta_d)}{\beta_d}, & \beta_d \neq 0 \\ 1, & \beta_d = 0 \end{cases}$$

### Residual weight $w_{\text{res}}$

- Low amplitude on a dense set of directions
- Unpredictable behavior.
- Represented by neural network with parameters  $\theta$ .
- Integral operator approximated numerically.



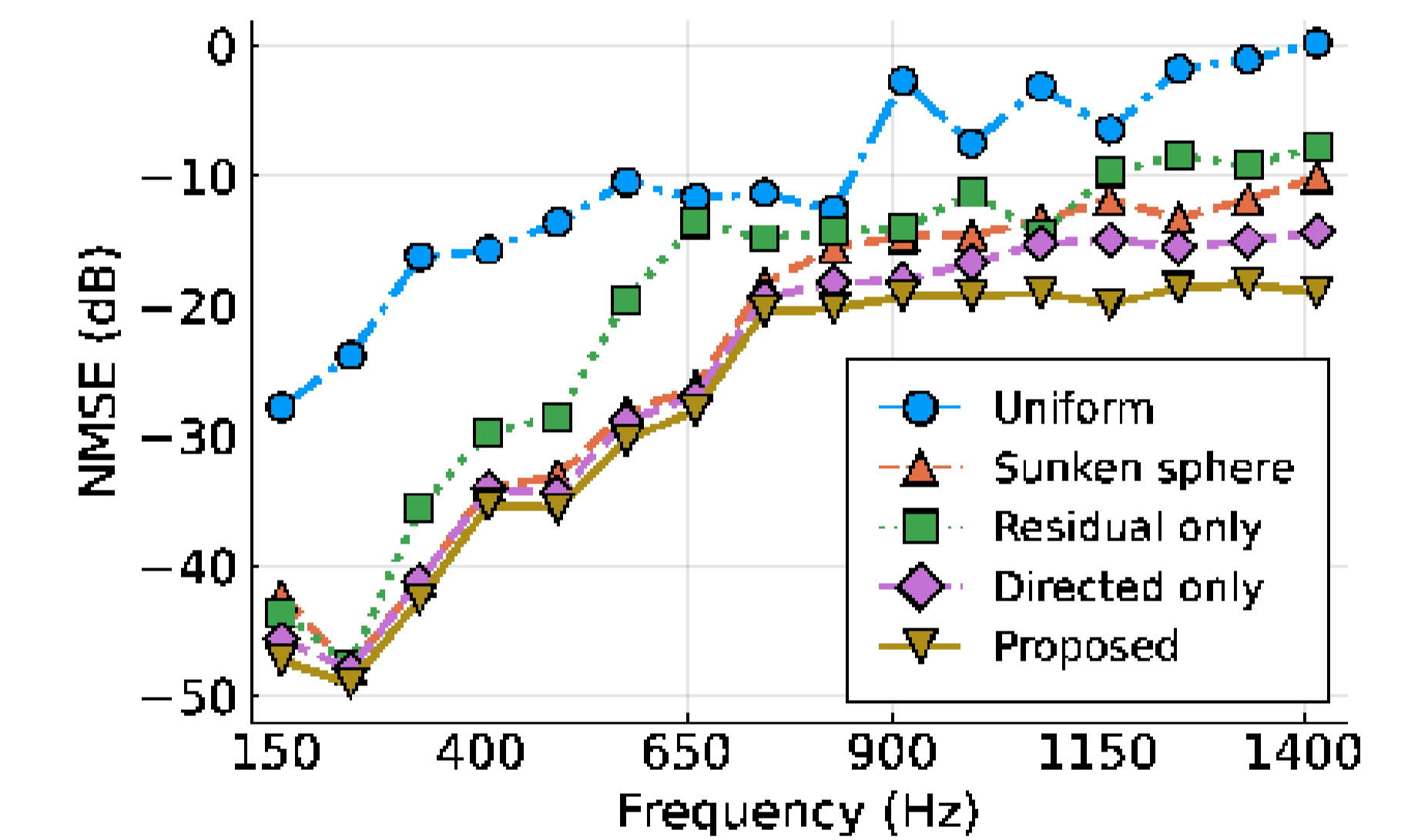
- The resulting adaptive kernel will be the superposition of the kernels.
- Kernel optimized as to minimize leave-one-out cross validation loss.
- Model parameters  $\beta$  and  $\theta$  optimized using gradient descent.
- Parameter  $\alpha$  optimized with reduced gradient descent to guarantee restrictions are upheld.
- The adaptive kernel learns particularities of the ATF in question without compromising model dynamics.

## Experiments

- Simulations with the image source method.
  - Room dimensions: 3.2 m × 4.0 m × 2.7 m.
  - Reverberation time:  $T_{60} = 0.45$  s.
  - Radius of both regions: 0.2 m.
  - Centers of  $\Omega_{S,R}: \pm[0.65, 0.8, 0.48]^T$ .
- **Proposed** compared to **uniform** and **sunken sphere** kernels.

### Normalized mean square error (NMSE)

$$\text{NMSE} = 10 \log_{10} \left( \frac{\sum_n |\hat{h}(q'_n) - h(q'_n)|^2}{\sum_n |h(q'_n)|^2} \right)$$



### Reconstruction of ATF (1150Hz)

