DTU Compute
Department of Applied Mathematics and Computer Science

Angular Central Gaussian and Watson mixture models for assessing dynamic functional brain connectivity during a motor task

anders_s_olsen
a-s-olsen
ansol@dtu.dk

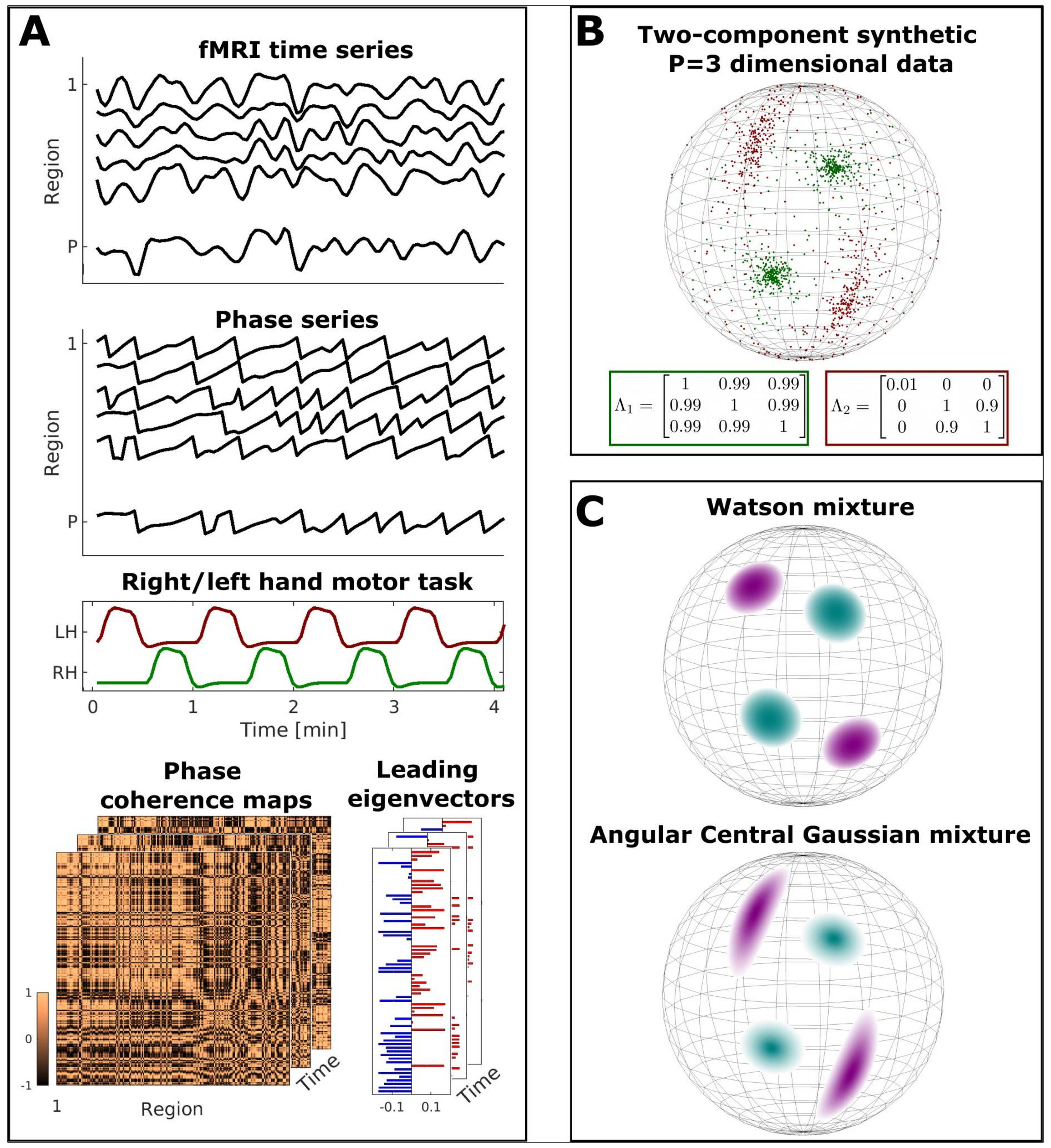
Anders S. Olsen[1], Emil Ortvald[1], Kristoffer H. Madsen[1,2], Mikkel N. Schmidt[1] Morten Mørup[1]

DTU

1 Summary

- Leading eigenvector dynamics analysis (**LEiDA**) [1] is among the favored methods for assessing instantaneous **dynamic functional brain connectivity**.
- Eigenvectors, e.g., those produced by LEiDA, are distributed on the **sign-symmetric unit hypersphere**, which is typically disregarded during modeling [2].
- Here we develop **mixture model** (MM) and **Hidden Markov model** (HMM) formulations for two sign-symmetric spherical distributions.
- We display their performance on synthetic data and functional magnetic resonance imaging (**fMRI**) data involving a **finger-tapping task**.

2 Methods



*Methodological pipeline. (**A**): LEiDA constructs leading eigenvectors of instantaneous phase coherence maps estimated using the Hilbert transform. (**B**): Synthetic data on the sign-symmetric unit hypersphere generated by a two-component angular central Gaussian (ACG) mixture. (**C**): Two-component Watson and ACG mixture model fits on the synthetic data in (**B**).*

3 Sign-symmetric spherical distributions

- **Watson** distribution density [3]:

$$f_W(\pm \boldsymbol{x};\boldsymbol{\mu},\kappa) = \frac{\Gamma\left(\frac{p}{2}\right)}{2\pi^{p/2} M\left(\frac{1}{2},\frac{p}{2},\kappa\right)} e^{\kappa(\boldsymbol{\mu}^\top \boldsymbol{x})^2}, \boldsymbol{x} \in \mathbb{S}^{p-1},$$

$M(a,b,\kappa)$ is Kummer's confluent hypergeometric function, and $\Gamma(\cdot)$ is the Gamma function. The density is parameterized by a mean sign-symmetric direction $\boldsymbol{\mu}$ and scalar precision parameter κ.

- **Angular central gaussian** distribution density [4]:

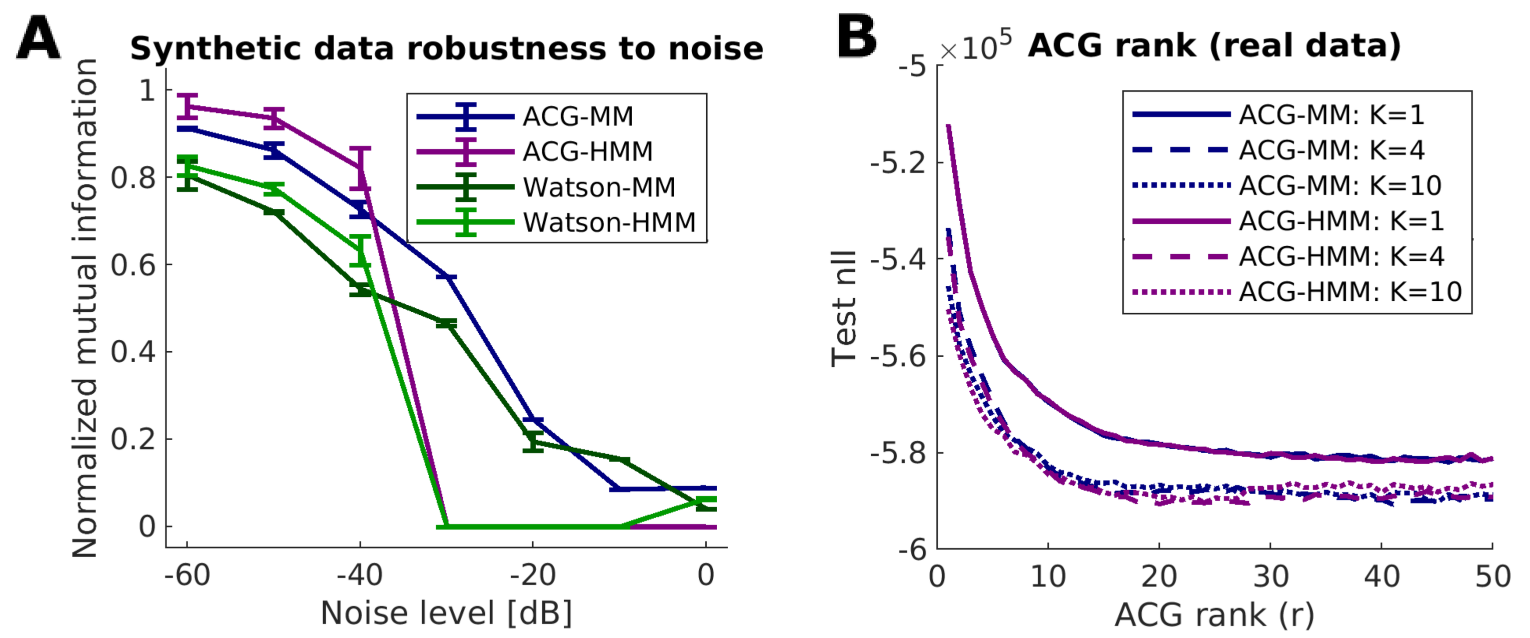
$$f_{ACG}(\pm \boldsymbol{x};\boldsymbol{\Lambda}) = \frac{\Gamma\left(\frac{p}{2}\right)}{2\pi^{p/2}|\boldsymbol{\Lambda}|^{1/2}} (\boldsymbol{x}^\top \boldsymbol{\Lambda}^{-1} \boldsymbol{x})^{-\frac{p}{2}}, \boldsymbol{x} \in \mathbb{S}^{p-1}$$

$\boldsymbol{\Lambda} \in \mathbb{R}^{p\times p}$ is a positive definite matrix identifiable up to multiplication with a positive scalar (spherical covariance matrix).

4 Data and implementation

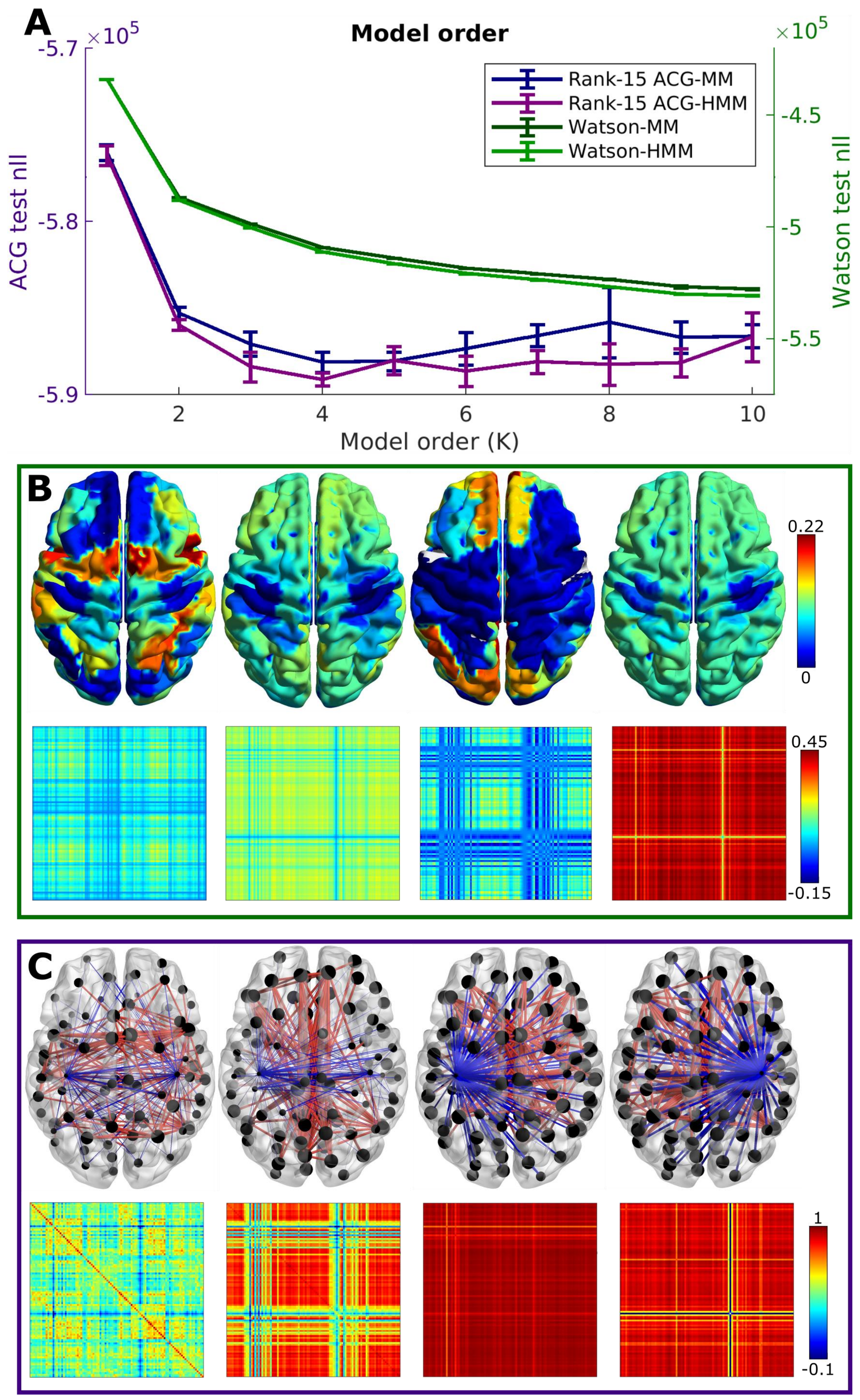
- 3T fMRI data (TR/TE 2490/30 ms, 240 volumes, 3mm isotropic voxels). 29 participants, block-design finger-tapping motor task [5].
- Schaefer-100 atlas for spatial downsampling [6].
- Split-half cross-validation for model performance investigations.
- Mixtures and HMMs implemented in PyTorch with the ADAM optimizer (lr=0.1). Constraints were handled by reparametrization, e.g., $\boldsymbol{\mu} = \tilde{\boldsymbol{\mu}}/||\tilde{\boldsymbol{\mu}}||$ and $\kappa = \log(1+e^{\tilde{\kappa}})$ while optimizing $\tilde{\boldsymbol{\mu}}$ and $\tilde{\kappa}$ unconstrained.
- Two ACG estimation schemes. For low-dimensional problems, $\boldsymbol{\Lambda}^{-1} = \boldsymbol{L}\boldsymbol{L}^T$. For high-dimensional problems, $\boldsymbol{\Lambda} = \boldsymbol{M}\boldsymbol{M}^T + \boldsymbol{I}$, where $\boldsymbol{M} \in \mathbb{R}^{p\times r}$is of rank r.

5 Results



*(**A**): Noise-dependent information overlap between two-component fit state probabilities (for MMs) or state sequence (for HMMs) with the true cluster identity. (**B**): Test negative log-likelihood depending on ACG rank for three model orders.*

6 Results (cont'd)



*Model fits to experimental data. (**A**): Evolution of test performance over model order. (**B**): Watson MM fit for $K = 4$ including connectivity map $\boldsymbol{\Lambda} = \sqrt{\kappa}\boldsymbol{\mu}\boldsymbol{\mu}^\top$ and a surface rendering of the diagonal of $\boldsymbol{\Lambda}$. (**C**): Brain graph rendering showing the top (red) and bottom (blue) 2.5% edges and connectivity map for the ACG MM fit for $K = 4$, where node size is the diagonal of $\boldsymbol{\Lambda}$.*

7 Discussion

- HMMs are more affected by noise than their corresponding MM formulations. Similarly, the ACG can be difficult to estimate for high p.
- Future studies could investigate the distribution of ACG rank indicating an optimal LEiDA-determined brain complexity level. Similarly, the HMM overlap with task information may be investigated.
- Likewise, the models deployed on resting-state data or more complicated tasks may reveal novel information on the brain.

References
[1] *Cognitive performance in healthy older adults relates to spontaneous switching between states of functional connectivity during rest*, Cabral J et al., SciRep (2017)
[2] *Psilocybin modulation of time-varying functional connectivity is associated with plasma psilocin and subjective effects*, Olsen AS et al., NeuroImage (2022)
[3] *The multivariate Watson distribution: Maximum-likelihood estimation and other aspects*, Sra & Karp, Journal of Multivariate Analysis (2013)
[4] *Statistical Analysis for the Angular Central Gaussian Distribution on the Sphere,* Tyler DE, Biometrika (1987)
[5] *Model sparsity and brain pattern interpretation of classification models in neuroimaging*, Rasmussen PM et al., Pattern Recognition (2012)
[6] *Local-Global Parcellation of the Human Cerebral Cortex from Intrinsic Functional Connectivity MRI*, Schaefer A et al., Cerebral Cortex (2018)

Affiliations
[1] DTU Compute, Technical University of Denmark
[2] Danish Research Centre for Magnetic Resonance, Copenhagen University Hospital Hvidovre