Model-free Learning of Optimal Beamformers for Passive IRS-Assisted Sumrate Maximization

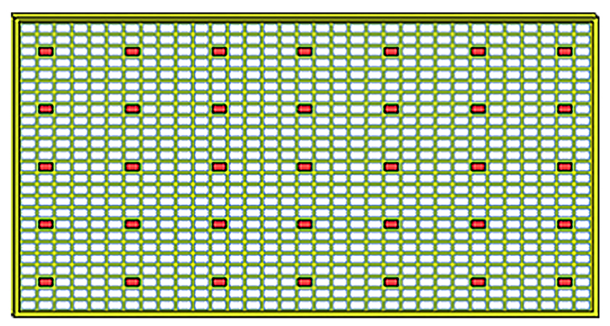
Hassaan Hashmi, Spyridon Pougkakiotis, Dionysios S. Kalogerias

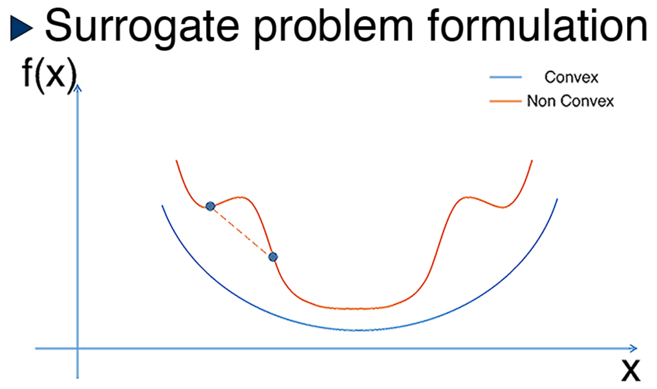
Department of Electrical Engineering

Yale School of Engineering & Applied Sciences

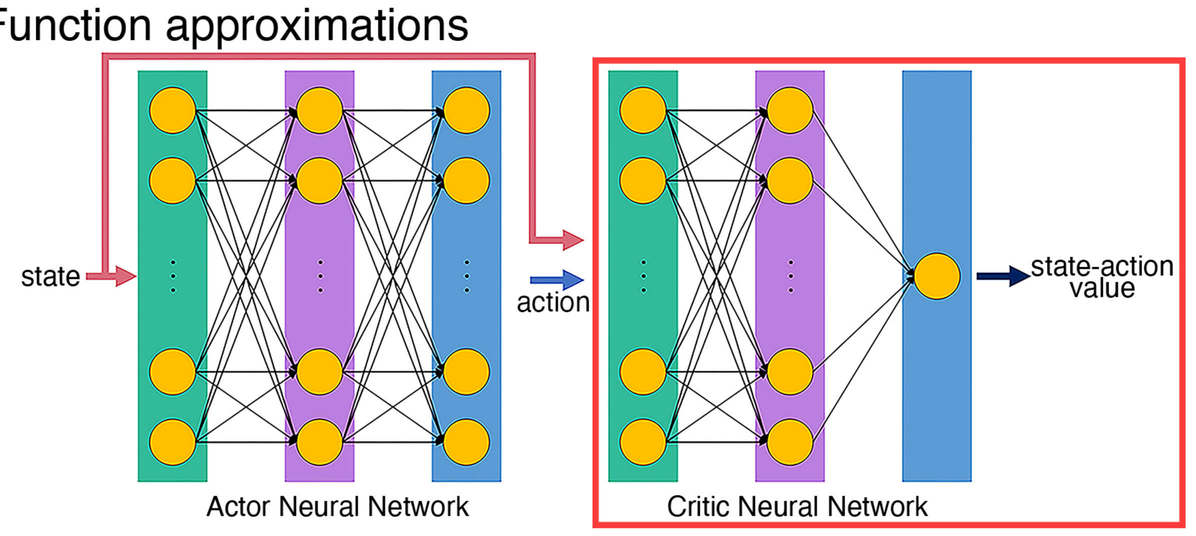
Overview

- Current IRS-aided beamforming methods rely on
  - Intermediate CSI estimation
  - Surrogate problem formulation





  - Function approximations



- We optimally beamform relying **only** on effective channel observations at the user end.
- In order to achieve this, we propose **ZoSGA**, a truly data-driven learning method for IRS-aided beamforming.
- We principally establish convergence of ZoSGA.
- ZoSGA delivers state-of-the-art performance on MISO downlink Sumrate Maximization.

link to the paper

Two-stage Beamforming

$$\max_{\boldsymbol{\theta}\in\mathcal{K}} \mathbb{E}\left\{ \max_{\mathbf{W}:\|\mathbf{W}\|_F^2\leq P} F\left(\mathbf{W},\mathbf{H}(\boldsymbol{\theta},\omega)\right)\right\}$$

First stage — Second stage

Weighted Sumrate utility for a MISO downlink network

$$F(\mathbf{W},\mathbf{H}(\boldsymbol{\theta},\omega)) \triangleq \sum_{k=1}^{K} \alpha_k \log_2\left(1+\mathrm{SINR}_k\left(\mathbf{W},\mathbf{h}_k(\boldsymbol{\theta},\omega)\right)\right)$$

Zeroth-order Stochastic Gradient Ascent

Tackling the second-stage problem

We employ the well-known WMMSE algorithm

$$\boldsymbol{W}^*(\boldsymbol{\theta},\omega) \in \arg\max_{\mathbf{W}:\ \|\mathbf{W}\|_F^2\leq P} F(\mathbf{W},\mathbf{H}(\boldsymbol{\theta},\omega))$$

Why?
- Demonstrate our approach can be employed with standard precoding optimization methods
- Show the performance gain relative to a well-established baseline

Tackling the first-stage problem

$$\max_{\boldsymbol{\theta}\in\mathcal{K}} \mathbb{E}\left\{F\left(\mathbf{W}^*(\boldsymbol{\theta},\omega),\mathbf{H}(\boldsymbol{\theta},\omega)\right)\right\}$$

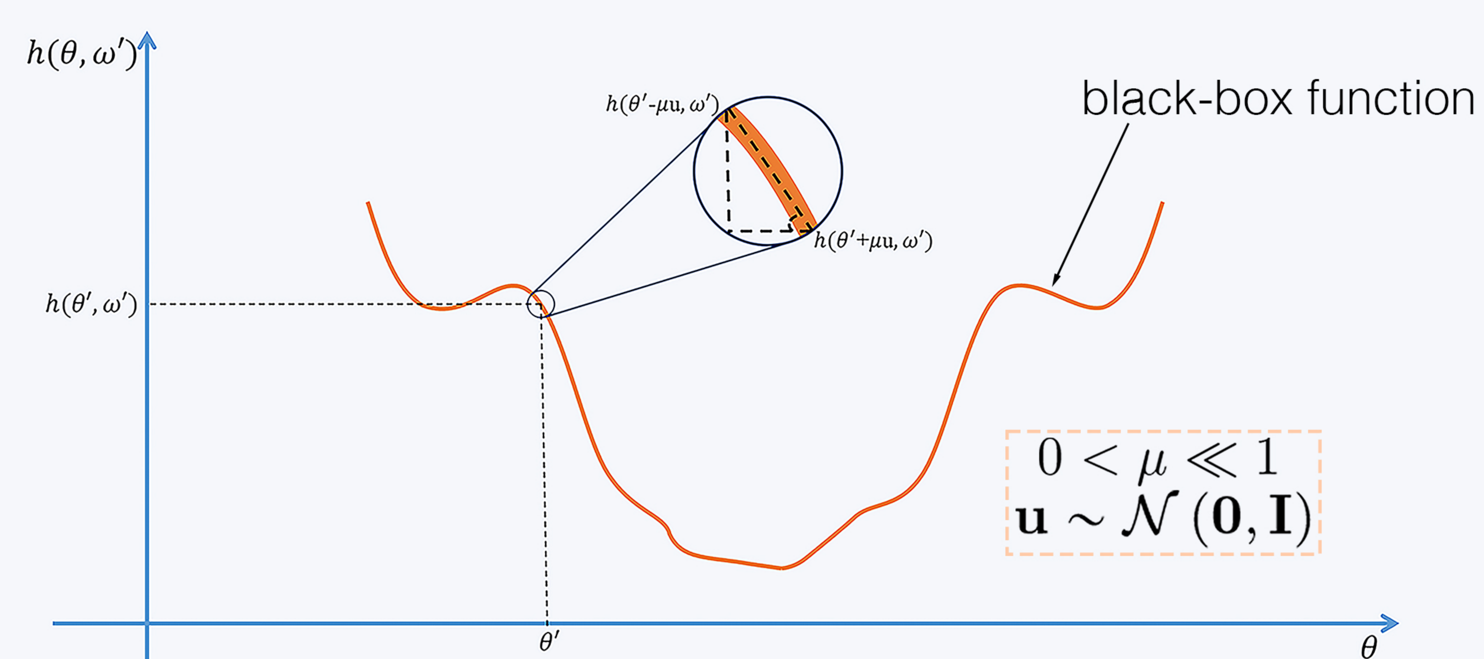
Gradient-ascent like scheme with a black-box channel function

$$\nabla_{\boldsymbol{\theta}} F\left(\mathbf{W},\mathbf{H}(\boldsymbol{\theta},\omega)\right) = 2\nabla_{\boldsymbol{\theta}}\Re\left(\mathbf{H}(\boldsymbol{\theta},\omega)\right)\left(\Re\left(\boldsymbol{D}\left(\mathbf{W},\mathbf{H}(\boldsymbol{\theta},\omega)\right)\right)\right)^{\mathsf{T}} + 2\nabla_{\boldsymbol{\theta}}\Im\left(\mathbf{H}(\boldsymbol{\theta},\omega)\right)\left(\Re\left(j\boldsymbol{D}\left(\mathbf{W},\mathbf{H}(\boldsymbol{\theta},\omega)\right)\right)\right)^{\mathsf{T}}$$

unknown

Model-free Gradient Approximation

We approximate the unknown gradient using function evaluations



$$\nabla^{\mu}_{\boldsymbol{\theta}}\mathbf{H}(\boldsymbol{\theta},\omega) \triangleq \mathbb{E}\left\{\frac{\mathbf{H}(\boldsymbol{\theta}+\mu\mathbf{u},\omega)-\mathbf{H}(\boldsymbol{\theta}-\mu\mathbf{u},\omega)}{2\mu}\mathbf{u}^{\mathsf{T}}\right\}^{\mathsf{T}}$$

Model-free gradient approximation:

$$\frac{\mathbf{G}_\mu(\boldsymbol{\theta},\omega,\mathbf{u})}{2} \triangleq \left(\frac{\Re\left(\mathbf{H}(\boldsymbol{\theta}+\mu\mathbf{u},\omega)-\mathbf{H}(\boldsymbol{\theta}-\mu\mathbf{u},\omega)\right)}{2\mu}\mathbf{u}^{\mathsf{T}}\right)^{\mathsf{T}}\left(\Re\left(\boldsymbol{D}(\mathbf{W}^*,\mathbf{H}(\boldsymbol{\theta},\omega))\right)\right)^{\mathsf{T}} + \left(\frac{\Im\left(\mathbf{H}(\boldsymbol{\theta}+\mu\mathbf{u},\omega)-\mathbf{H}(\boldsymbol{\theta}-\mu\mathbf{u},\omega)\right)}{2\mu}\mathbf{u}^{\mathsf{T}}\right)^{\mathsf{T}}\left(\Re\left(j\boldsymbol{D}(\mathbf{W}^*,\mathbf{H}(\boldsymbol{\theta},\omega))\right)\right)^{\mathsf{T}}$$

ZoSGA: Simple update rule for IRS parameters

$$\boldsymbol{\theta}^{t+1} = \Pi_{\mathcal{K}}\left(\boldsymbol{\theta}^{t} + \eta^{t+1}\mathbf{G}_\mu(\boldsymbol{\theta}^{t},\omega^{t+1},\mathbf{u}^{t+1})\right)$$

Projection — Learning rate

Convergence Analysis

Does ZoSGA converge? *Yes, and we can prove it!*

- IRS having S parameters and setting $\mu = \mathcal{O}(1/\sqrt{(MKT)})$
- The gradient relative to the optimal IRS parameters is bounded by ϵ after at most $\mathcal{O}(\sqrt{S}\epsilon^{-4})$ iterations.

Simulations

(averaged over 2000 unique simulations)

- IRS-aided MISO downlink wireless network with Rician fading channels

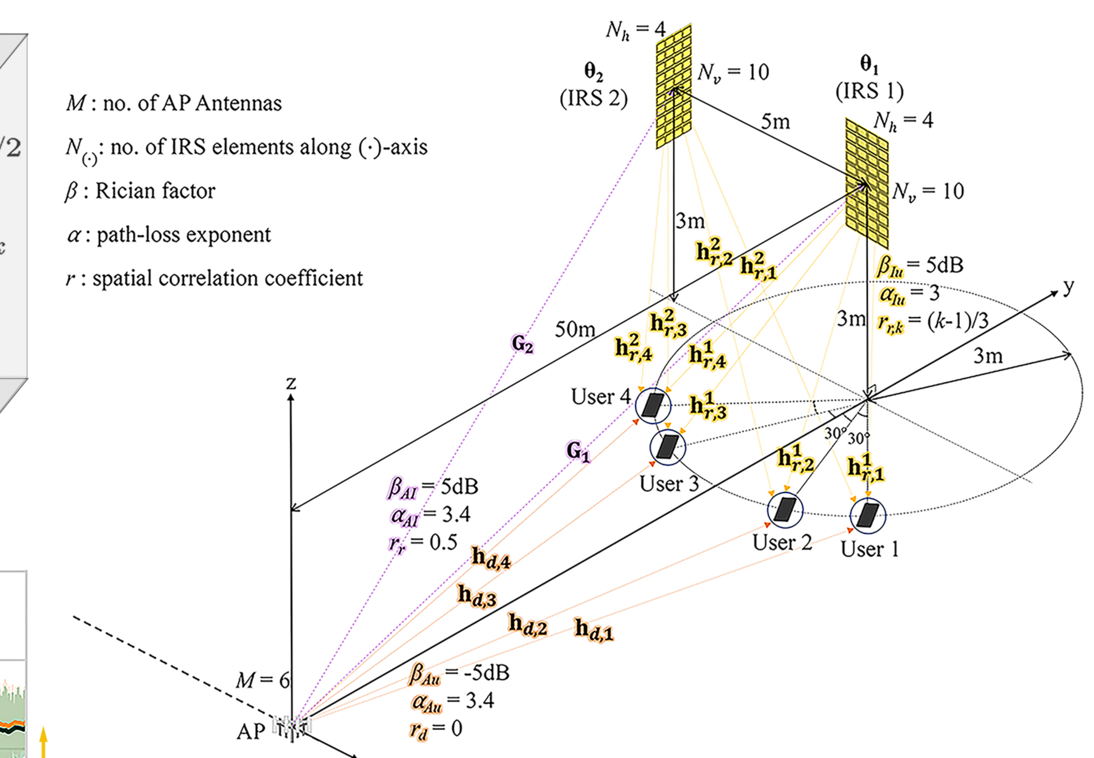
$$\mathbf{h}^i_{r,k} \triangleq \sqrt{\beta_{Iu}/(1+\beta_{Iu})}\bar{\mathbf{v}}^i_{r,k} + \sqrt{1/(1+\beta_{Iu})}\boldsymbol{\Phi}^{1/2}_{r,k}\mathbf{v}^i_{r,k}$$

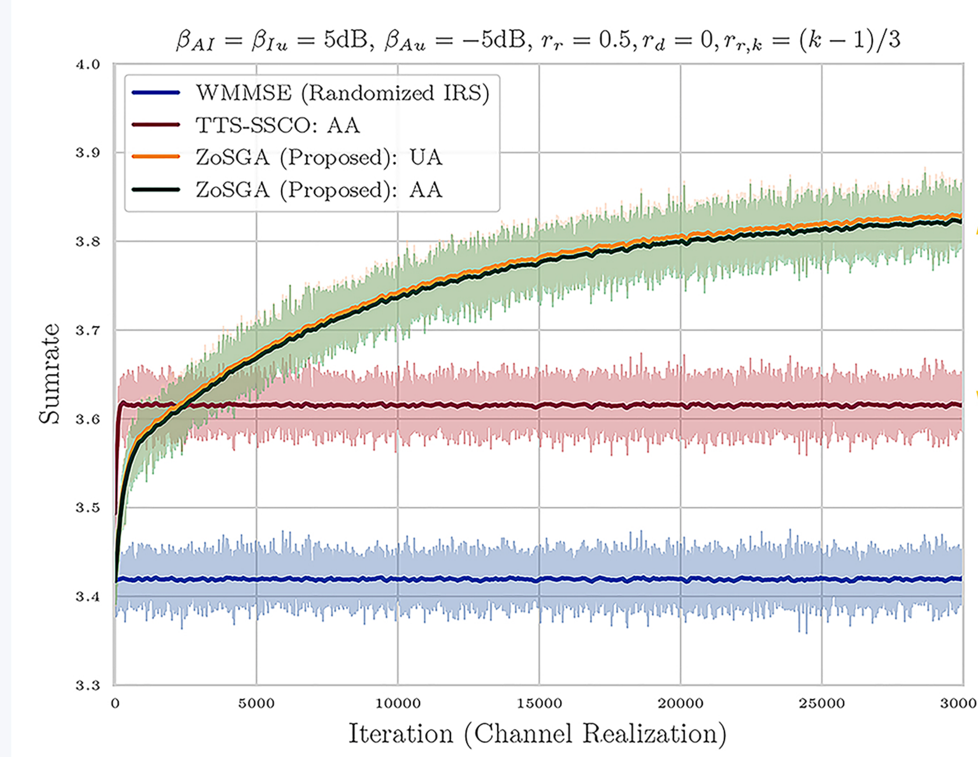
$$\mathbf{G}_i \triangleq \sqrt{\beta_{AI}/(1+\beta_{AI})}\bar{\mathbf{F}}^i + \sqrt{1/(1+\beta_{AI})}\boldsymbol{\Phi}^{1/2}_r\mathbf{F}^i\boldsymbol{\Phi}^{1/2}_d$$

$$\mathbf{h}_{d,k} \triangleq \sqrt{\beta_{Au}/(1+\beta_{Au})}\bar{\mathbf{v}}_{d,k} + \sqrt{1/(1+\beta_{Au})}\boldsymbol{\Phi}^{1/2}_d\mathbf{v}_{d,k}$$

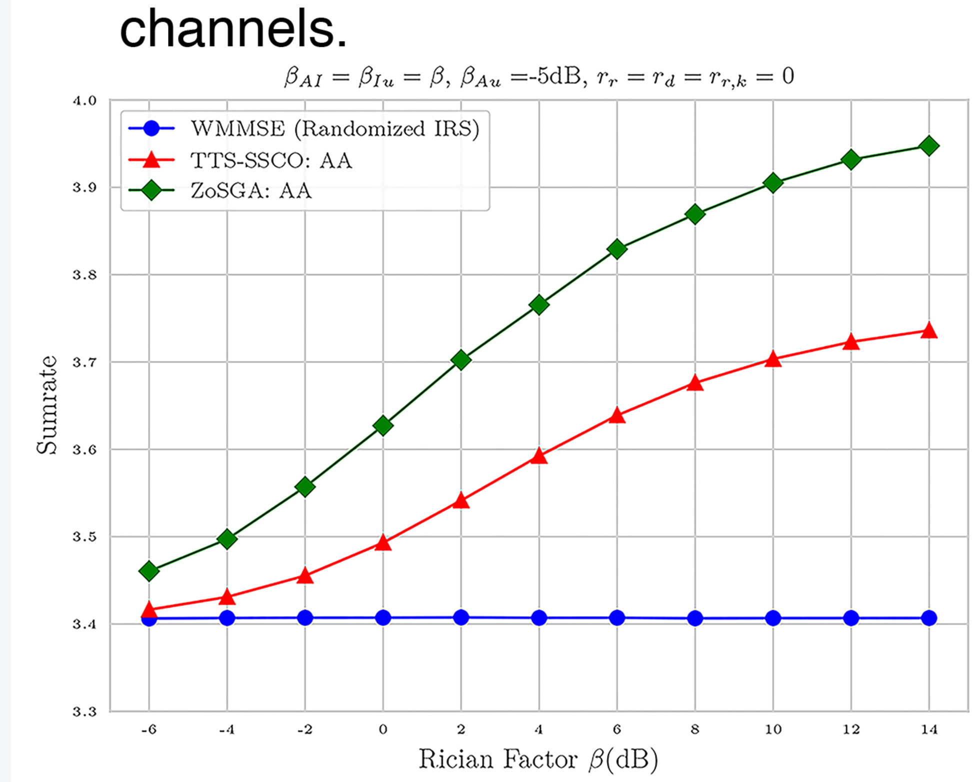
$$\mathbf{h}_k(\boldsymbol{\theta},\omega) = \underbrace{\sum_{i=1}^{2}\mathbf{G}_i^{\mathsf{H}}\mathrm{diag}(\mathbf{A}_i\circ e^{-j\boldsymbol{\phi}_i})\mathbf{h}^i_{r,k}}_{\boldsymbol{\theta}_i\text{-reflected link}} + \underbrace{\mathbf{h}_{d,k}}_{\text{LoS link}}$$

unknown to ZoSGA

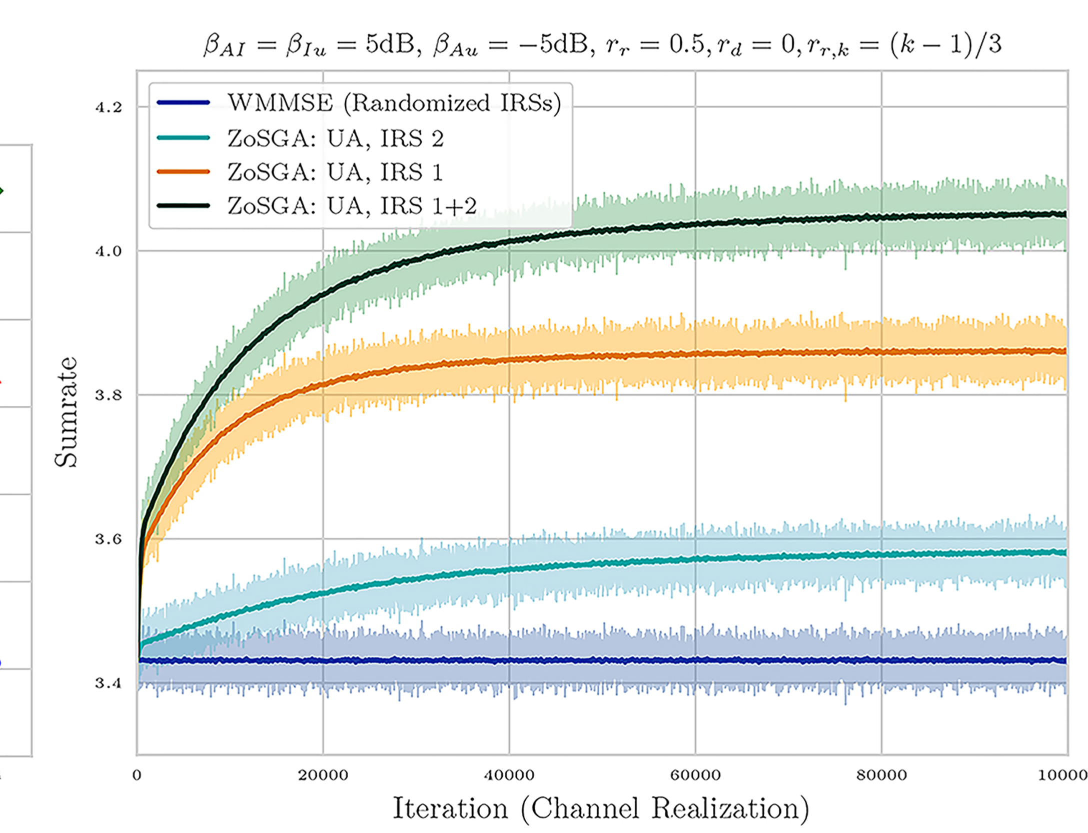




- Comparison of ZoSGA with a model-based state-of-the-art Two-Timescale SSCO method (IRS 1)
- The relative gain of ZoSGA increases more than TTS-SSCO as we move from I-CSI to S-CSI dominated channels.



- To show the model-free capability as well as robustness of ZoSGA, we optimize both IRSs without any changes to the algorithm hyper-parameters.



Future Work

- Policy function approximation instead of WMMSE to reduce time delay during operation.
- Evaluating ZoSGA on a multitude of problems related to IRS-aided Wireless Networks, including real-world and practical settings.

- We have extended this work to Physical IRSs, by directly tuning varactor capacitances using ZoSGA.
- You can find that and the detailed convergence analysis in our extended work

link to the extended paper