# SIGNAL PROCESSING AND QUANTUM STATE TOMOGRAPHY ON NOISY DEVICES

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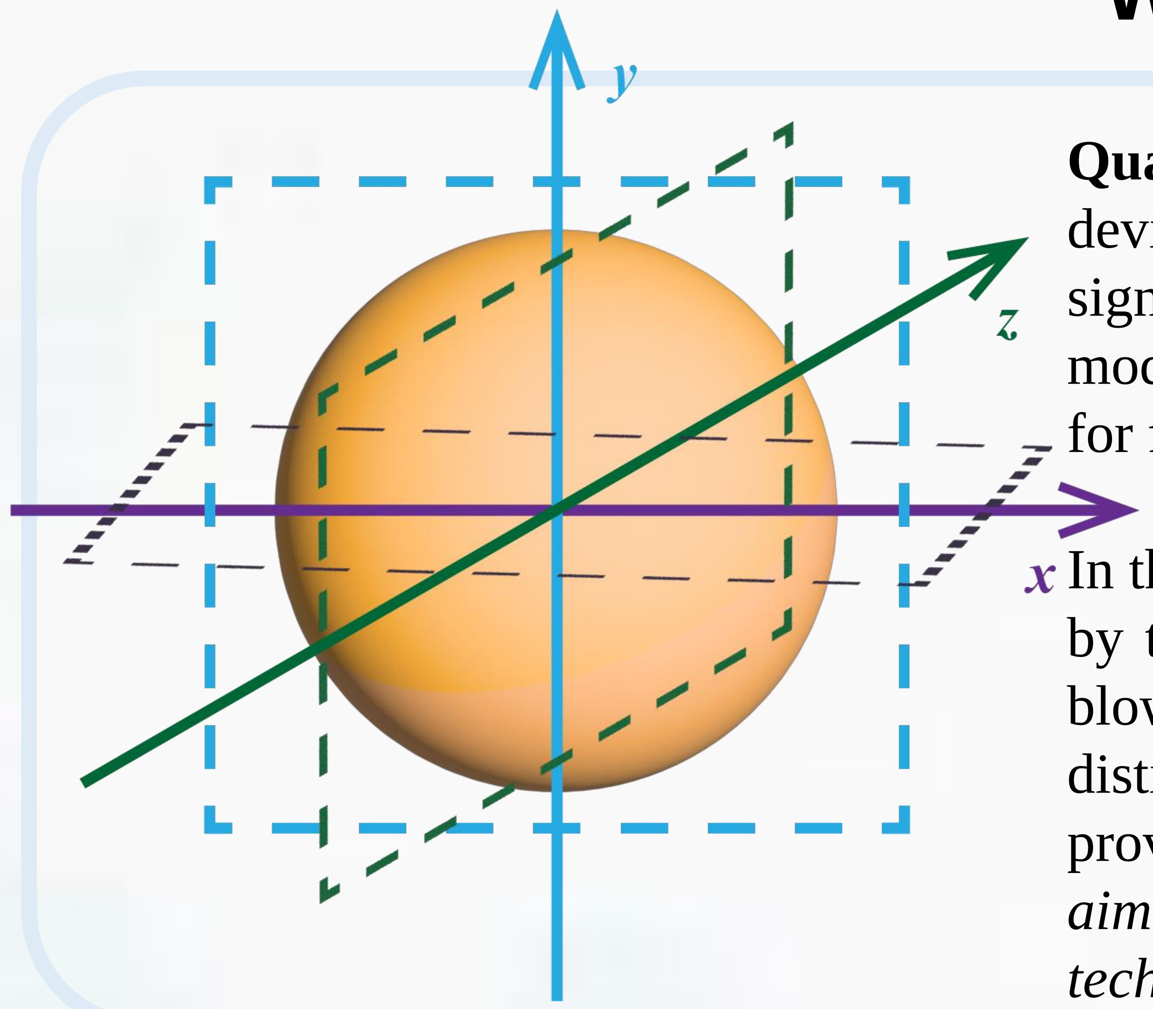



**Quantum State Tomography (QST)** is a fundamental tool for quantum signal processing. However, in real noisy quantum devices construction of the state's density matrix via QST can utilize a large amount of resources. Here, we discuss some signal processing techniques that are currently applied to this resource issue and implement on current quantum chips a modification that can assist in reducing resources. An application of QST to quantum entanglement distillation is provided for further insight.

In this work, we experimentally apply QST to several quantum states on a current superconducting quantum device provided by the IBM Quantum Experience (IBM Q), a cloud platform available to the researcher community. We implement full-blown QST, as well as simplifications, and we further apply QST to a practical quantum application, an entanglement distillation protocol - a protocol designed to convert a set of noisy entangled states to a smaller set of less-noisy states. This provides additional focus on the actual use of QST as well as additional performance insights. *Beyond this; we note another aim of this work is to encourage those in the classical signal processing community to consider further optimization techniques in the context of QST.*

$X$-basis states: $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$, $|-\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}}$

$Y$-basis states: $|R\rangle = (|0\rangle + j|1\rangle)/\sqrt{2}$, $|L\rangle = (|0\rangle - j|1\rangle)/\sqrt{2}$

$Z$-basis states: $|0\rangle$, $|1\rangle$

$$\hat{\sigma}_0 = I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \qquad \hat{\sigma}_1 = X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

$$\hat{\sigma}_2 = Y = \begin{bmatrix}0 & -j\\j & 0\end{bmatrix} \qquad \hat{\sigma}_3 = Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

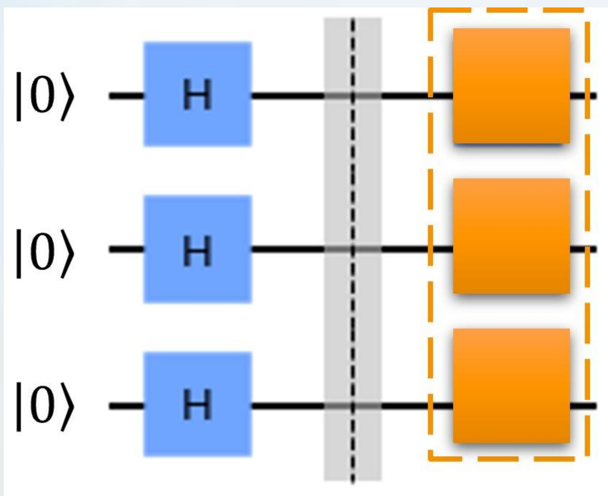
1-qubit state: $\rho = \frac{1}{2}\sum_{i=0}^{3} S_i\hat{\sigma}_i = \frac{1}{2}(\hat{\sigma}_0 + S_1\hat{\sigma}_1 + S_2\hat{\sigma}_2 + S_3\hat{\sigma}_3)$

$S_i = \mathrm{Tr}[\hat{\sigma}_i\rho]$ $\quad S_0 = P_0 + P_1$ $\quad S_1 = P_+ - P_-$ $\quad S_2 = P_R - P_L$ $\quad S_3 = P_0 - P_1$

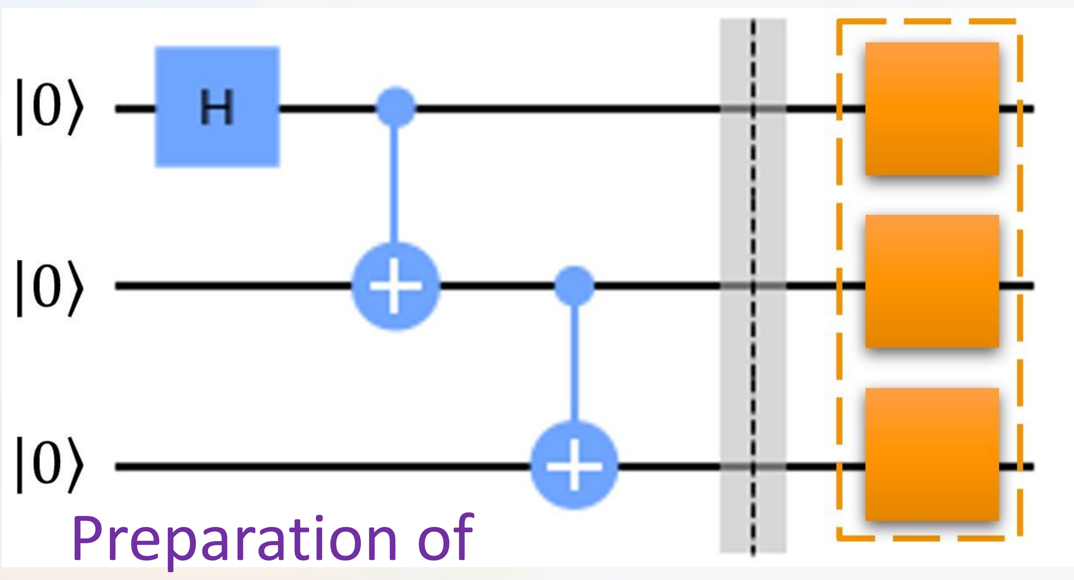
$n$-qubit state: $\hat{\rho} = \frac{1}{2^n}\sum_{i_1,i_2,\cdots i_n=0}^{3} S_{i_1,i_2,\cdots i_n}(\hat{\sigma}_{i_1}\otimes\hat{\sigma}_{i_2}\otimes\cdots\otimes\hat{\sigma}_{i_n})$

$S_{i_1,i_2,\cdots i_n} = \mathrm{Tr}[(\hat{\sigma}_{i_1}\otimes\hat{\sigma}_{i_2}\otimes\cdots\otimes\hat{\sigma}_{i_n})\hat{\rho}]$ $\quad i_k \in \{0,1,2,3\}$ $\quad k = 1,2,\cdots n$

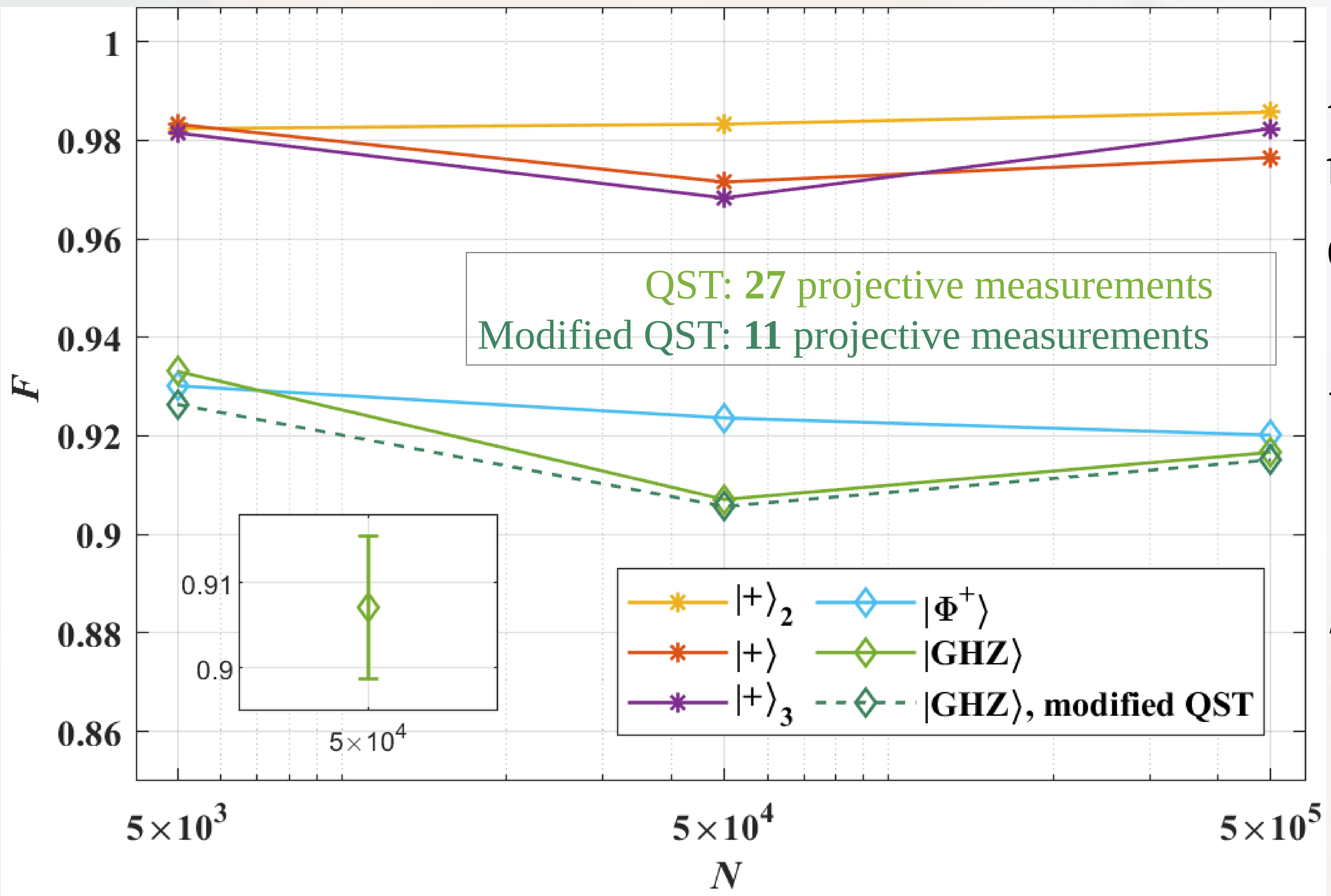
## EXPERIMENT: QST ON A REAL DEVICE




Preparation of $|+\rangle_3 = |+\rangle|+\rangle|+\rangle$




Preparation of $|\mathrm{GHZ}\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$




$N$: Execution time of the quantum circuit

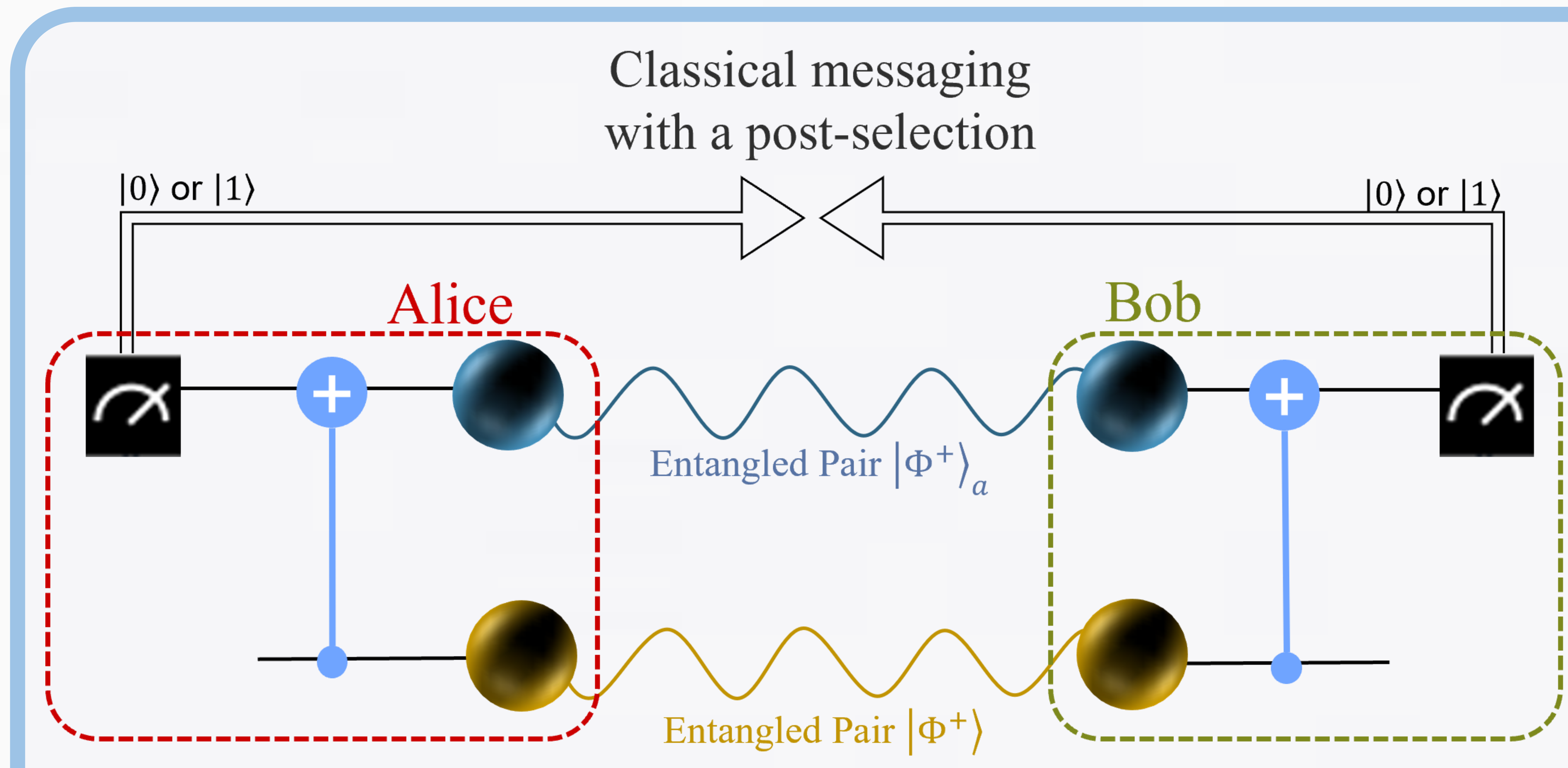
η: theoretical density matrix

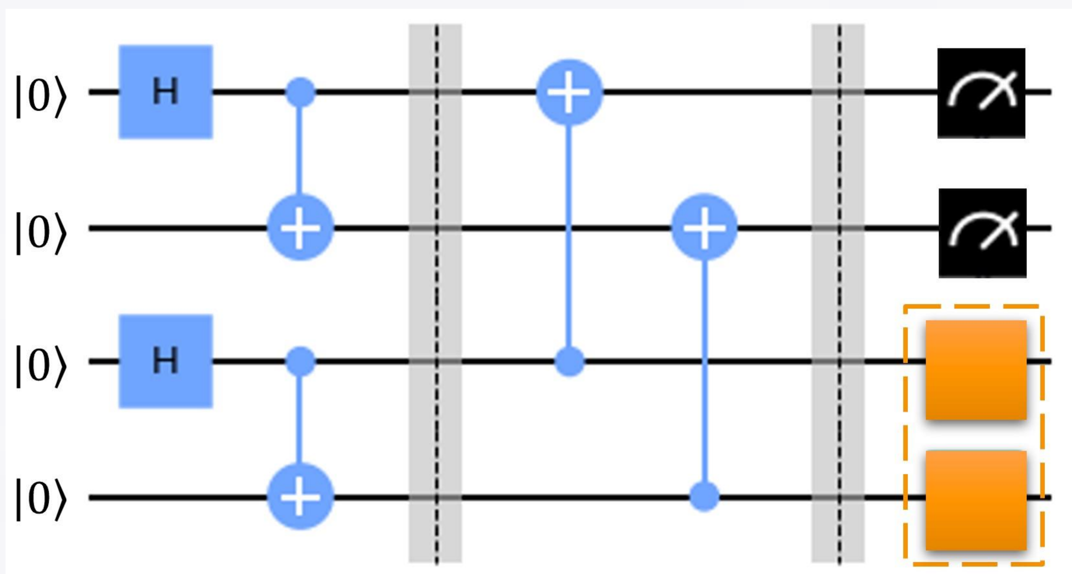
$\hat{\rho}$: experimental density matrix

$|+\rangle_2 = |+\rangle|+\rangle$

Fidelity: $F = \left(\mathrm{Tr}\sqrt{\sqrt{\eta}\,\hat{\rho}\sqrt{\eta}}\right)^2$ $\qquad |\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$

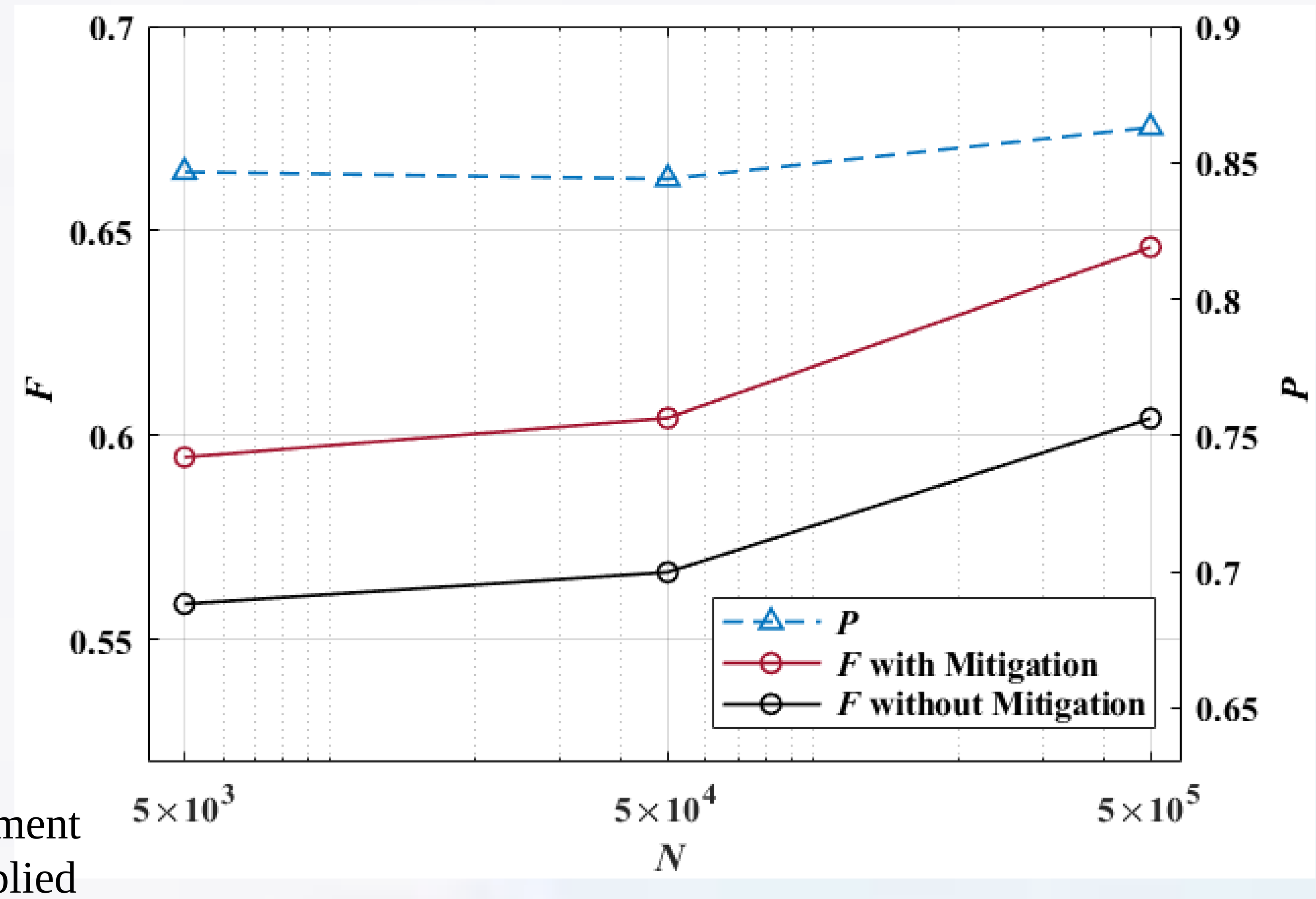
## APPLICATION: QST AND DISTILLATION




**Entanglement Distillation Protocol**

Mitigation: Measurement Error Mitigation (applied to all experiments)




$F$: Fidelity $\qquad P$: Success Probability of the Protocol

## Conclusions

In this work, we considered QST and its implementation on a Noisy Intermediate-Scale Quantum (NISQ) superconducting device, the *ibmq_jakarta*. The important issue regarding the number of repeated quantum measurements required to construct density matrices within some required tolerance was discussed and implemented. Our work highlights the importance of QST, the fact that it can be implemented on real NISQ devices, and that further optimizations that can save run-time on the devices are likely. We encourage further research by the signal processing community in the area of optimized QST.

SCAN ME

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