Image Source Method Based On the Directional Impulse Responses

Jiarui Wang, Prasanga Samarasinghe, Thushara Abhayapala, Jihui Aimee Zhang

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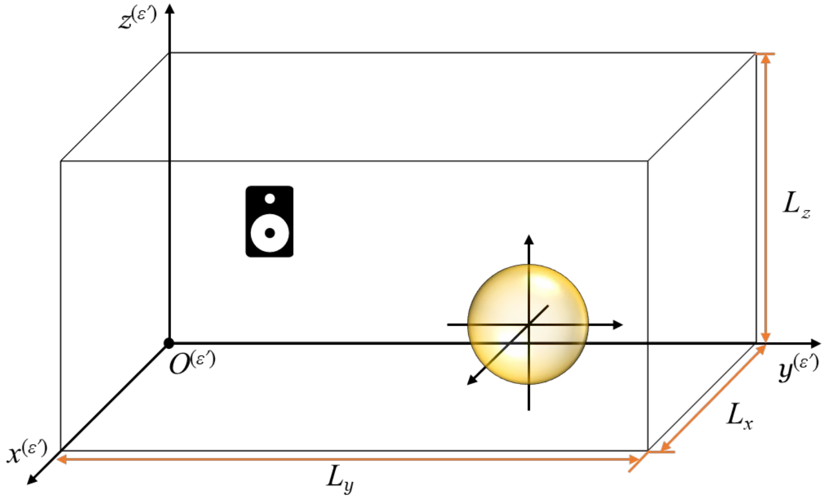
Summary

This paper presents **a wideband image source method** to simulate the time-domain signal on the boundary of the spherical listening region.

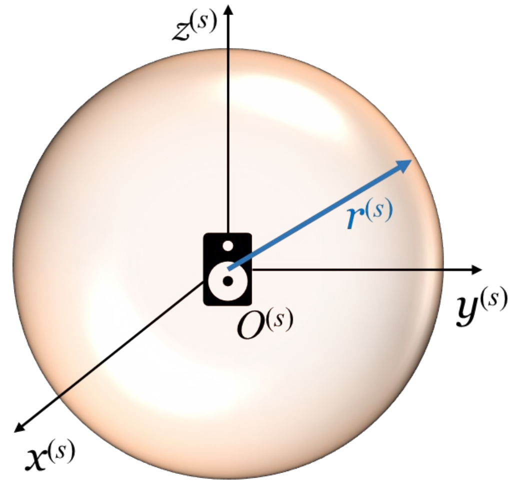
Key features:

- Direct time-domain solution, no FFT or IFFT.
- Uses spherical harmonics.
- Incorporates loudspeaker directional impulse responses.



Loudspeaker DIRs

- Loudspeaker directional impulse responses (DIRs) are treated as a sequence of propagating spherical wave fronts with direction-dependent amplitude.



- Far-field DIRs measured on a spherical surface of radius $r^{(s)}$

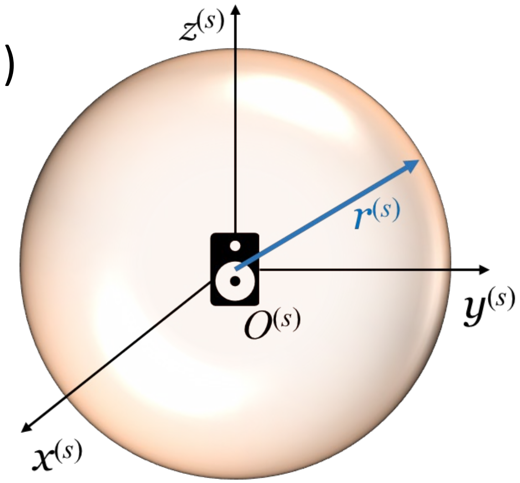
$$h(t, r^{(s)}, \theta^{(s)}, \phi^{(s)}) = \int_{\tau} h(\tau, r^{(s)}, \theta^{(s)}, \phi^{(s)}) \delta(t-\tau) d\tau$$

  - ➢ The integrand – a spherical wave front with amplitude $h(\tau, r^{(s)}, \theta^{(s)}, \phi^{(s)})$ captured at $t = \tau$ seconds.
  - ➢ The integral – the sequence of spherical wave fronts captured at successive time instances.
  - ➢ In free-field, the radius of each spherical wave front expands at c m/s. The amplitude of each spherical wave front experiences uniform attenuation related to the travelled distance (far-field assumption).

Loudspeaker DIRs

- Far-field DIRs measured on a spherical surface of radius $r^{(s)}$

$$h(t, r^{(s)}, \theta^{(s)}, \phi^{(s)}) = \int_{\tau} h(\tau, r^{(s)}, \theta^{(s)}, \phi^{(s)}) \delta(t - \tau) d\tau \qquad (1)$$



- Replace with an ideal source at $O^{(s)}$ that emits

$$d(t, \theta^{(s)}, \phi^{(s)}) = \int_{\tau} d(\tau, \theta^{(s)}, \phi^{(s)}) \delta(t - \tau) d\tau$$

- Let

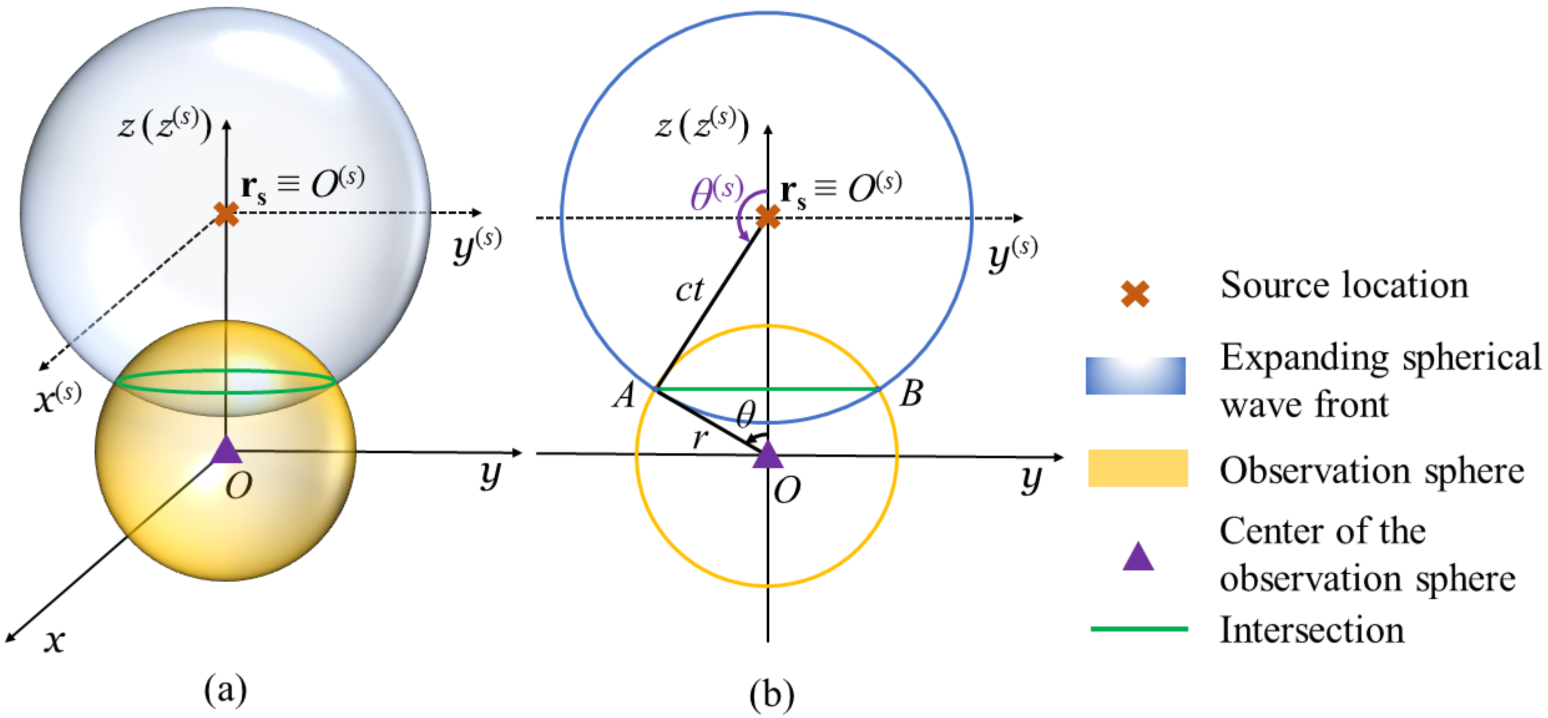
$$d(\tau, \theta^{(s)}, \phi^{(s)}) = 4\pi r^{(s)} h(\tau + r^{(s)}/c, r^{(s)}, \theta^{(s)}, \phi^{(s)})$$

Assume each spherical wave front only experiences uniform attenuation related to the travelled distance. The DIRs of this ideal source measured on the spherical surface of radius $r^{(s)}$should follow exactly (1).

Observed Signal in Free-Field

Single Spherical Wave Front

- The source emits the spherical wave front at t = 0 seconds.
- When $t \in [(r_s - r)/c\,, (r_s + r)/c]$, the intersection of the expanding spherical wave front and the observation sphere is a circle.

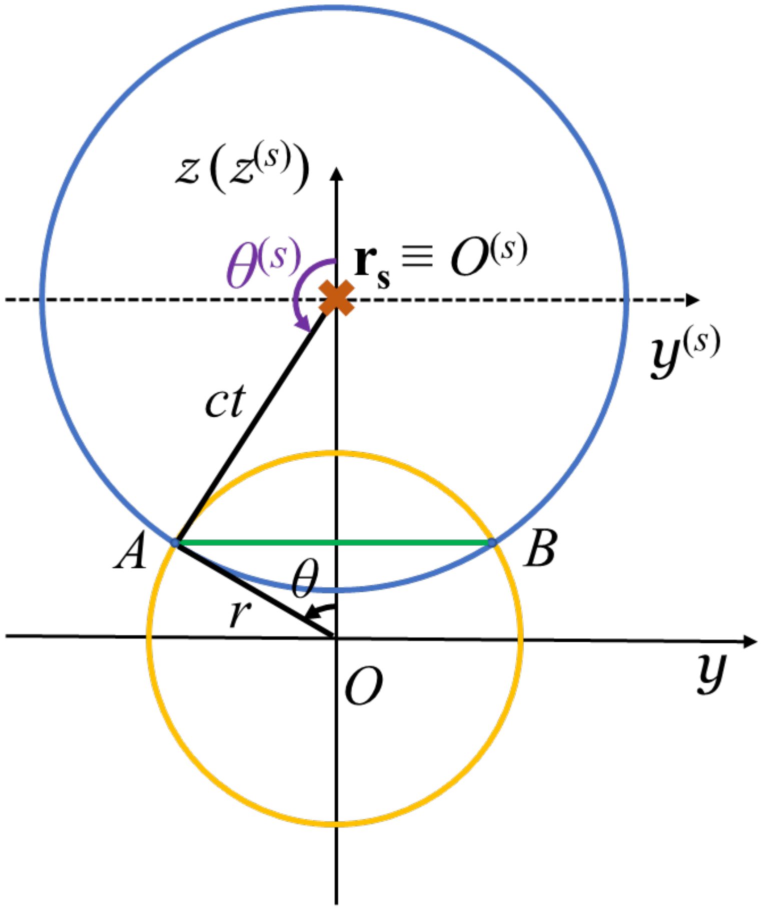


Single Spherical Wave Front

- The source signal $d(t, \theta^{(s)}, \phi^{(s)}) = Y_v^u(\theta^{(s)}, \phi^{(s)})\delta(t)$.
- At $t = t_0$, the colatitude of the circle (i.e., the intersection) $\theta = \theta_0$.
- The observed signal

$$g(t_0, r, \theta, \phi) = \frac{c}{4\pi r r_s}\delta(\cos\theta - \cos\theta_0)Y_v^u(\theta_0^{(s)}, \phi)$$

in which $\phi = \phi^{(s)}$and $\theta_0^{(s)}$ is expressed w.r.t. the $x^{(s)}y^{(s)}z^{(s)}$ coordinate system.



J. Wang, T. Abhayapala, P. Samarasinghe, J. A. Zhang; Spherical harmonic representation of the observed directional wave front in the time domain. JASA Express Lett 1 November 2022; 2 (11): 114801.

Single Spherical Wave Front

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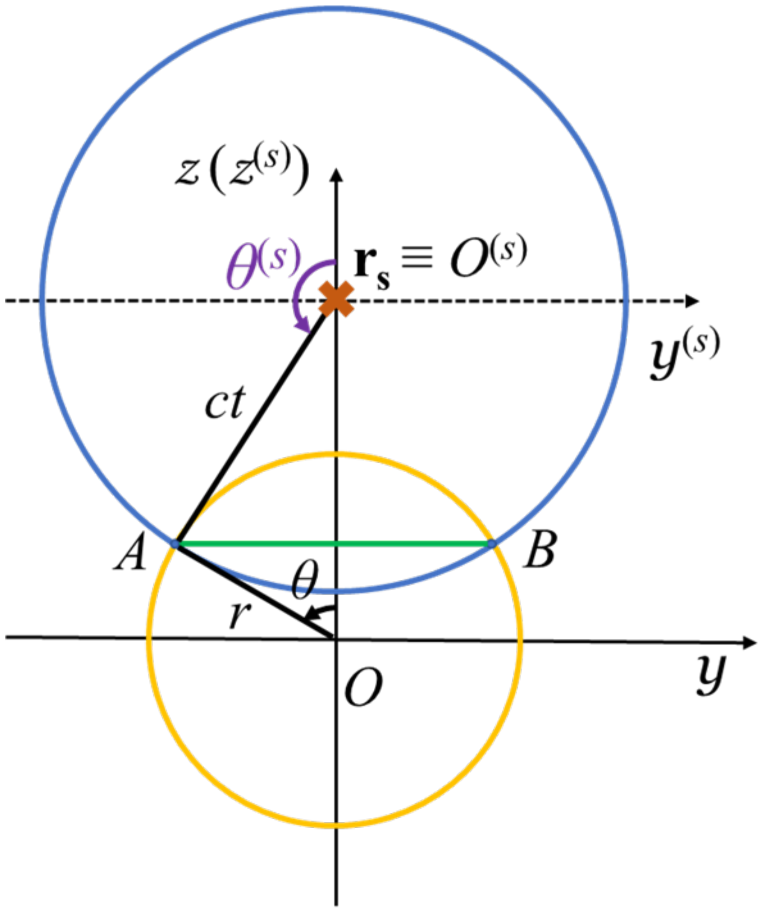
in which $\phi = \phi^{(s)}$ and $\theta_0^{(s)}$ is expressed w.r.t. the $x^{(s)}y^{(s)}z^{(s)}$ coordinate system.

- The spherical harmonic coefficients

$$\zeta_n^m(t_0, r) = \frac{c}{2 r r_s} \mathcal{P}_v^u(\cos\theta_0^{(s)}) \mathcal{P}_n^m(\cos\theta_0)\, \delta_{m,u}\, \Xi(t_0)$$

  - $\mathcal{P}_n^m(\cdot) = \sqrt{(2n+1)/4\pi}\sqrt{(n-m)!/(n+m)!}P_n^m(\cdot)$
  - $\Xi(t_0) = 1$ if $r_s - r \leq ct_0 \leq r_s + r$; else, $\Xi(t_0) = 0$.
  - $\zeta_n^m(t_0, r) \neq 0$ only when $n \geq |u|$ and $m = u$.
  - $\cos\theta_0 = (r^2 + r_s^2 - c^2 t_0^2)/(2 r r_s)$.
  - $\cos\theta_0^{(s)} = -(c^2 t_0^2 + r_s^2 - r^2)/(2 c t_0\, r_s)$.



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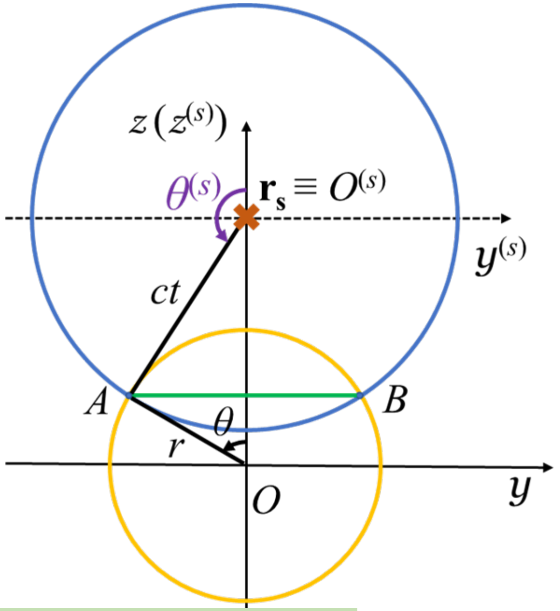
A Sequence of Spherical Wave Fronts



- The source signal

$$d(t,\theta^{(s)},\phi^{(s)}) = \int_\tau \underbrace{\sum_{v=0}^{V}\sum_{u=-v}^{v} \gamma_v^u(\tau) Y_v^u(\theta^{(s)},\phi^{(s)})}_{d(\tau,\theta^{(s)},\phi^{(s)})}\, \delta(t-\tau)d\tau$$

- Apply superposition principle, the SH coefficients of the observed signal

$$\zeta_n^m(t_0,r) = \frac{c}{2rr_s}\int_\tau \sum_{v=0}^{V}\sum_{u=-v}^{v} \gamma_v^u(\tau)\, \mathcal{P}_v^u[\cos\theta_0^{(s)}(\tau)]$$
$$\mathcal{P}_n^m[\cos\theta_0(\tau)]\delta_{m,u}\, \Xi(t_0,\tau)\, d\tau.$$

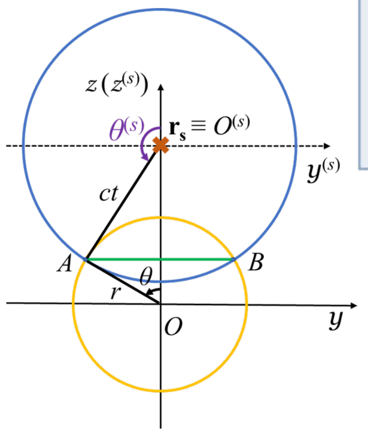
  - $\Xi(t_0,\tau) = 1$ if $r_s - r \le c(t_0-\tau) \le r_s + r$; else, $\Xi(t_0,\tau) = 0$.
  - $\cos\theta_0(\tau) = [r^2 + r_s^2 - c^2(t_0-\tau)^2]/[2rr_s]$.
  - $\cos\theta_0^{(s)}(\tau) = -[c^2(t_0-\tau)^2 + r_s^2 - r^2]/[2c(t_0-\tau)\,r_s]$.

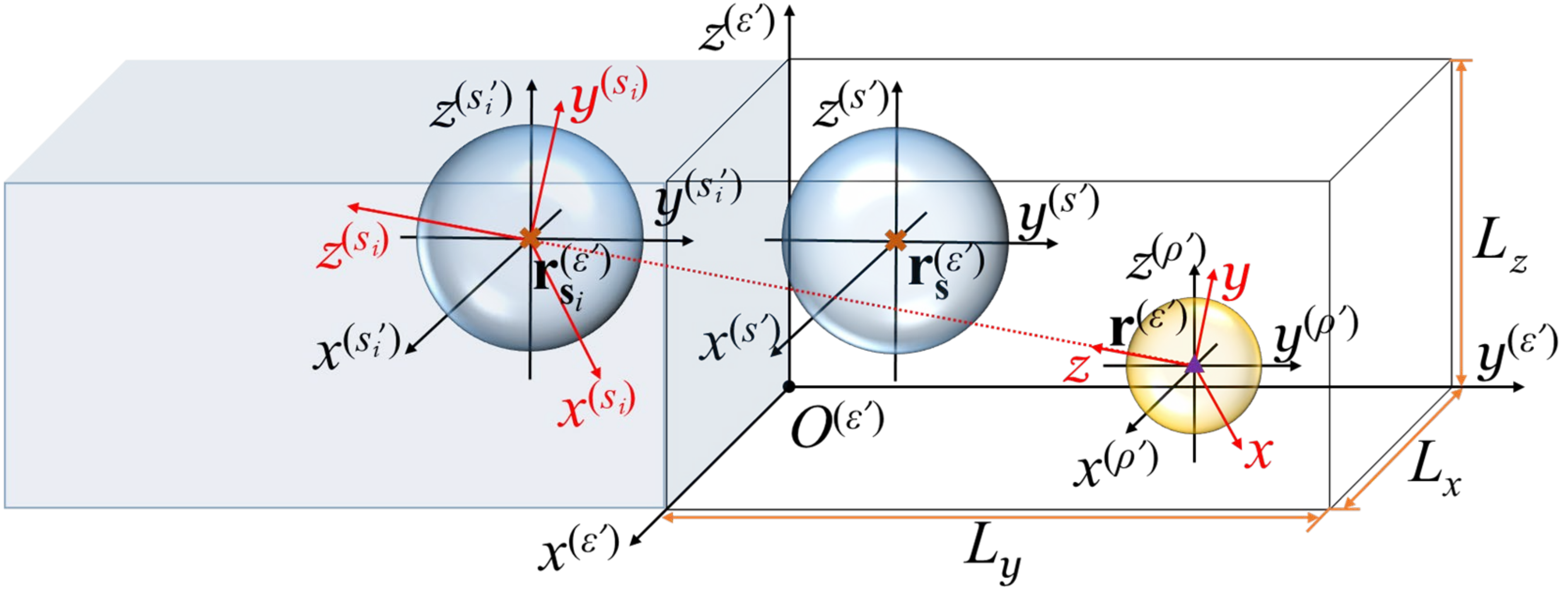
Single spherical wave front

$$\zeta_n^m(t_0,r) = \frac{c}{2rr_s}\mathcal{P}_v^u(\cos\theta_0^{(s)})\mathcal{P}_n^m(\cos\theta_0)\,\delta_{m,u}\,\Xi(t_0)$$

- $\mathcal{P}_n^m(\cdot) = \sqrt{(2n+1)/4\pi}\sqrt{(n-m)!/(n+m)!}P_n^m(\cdot)$
- $\Xi(t_0) = 1$ if $r_s - r \le ct_0 \le r_s + r$; else, $\Xi(t_0) = 0$.
- $\zeta_n^m(t_0,r) \neq 0$ only when $n \ge |u|$ and $m = u$.
- $\cos\theta_0 = (r^2 + r_s^2 - c^2t_0^2)/(2rr_s)$.
- $\cos\theta_0^{(s)} = -(c^2t_0^2 + r_s^2 - r^2)/(2ct_0\,r_s)$.

ISM

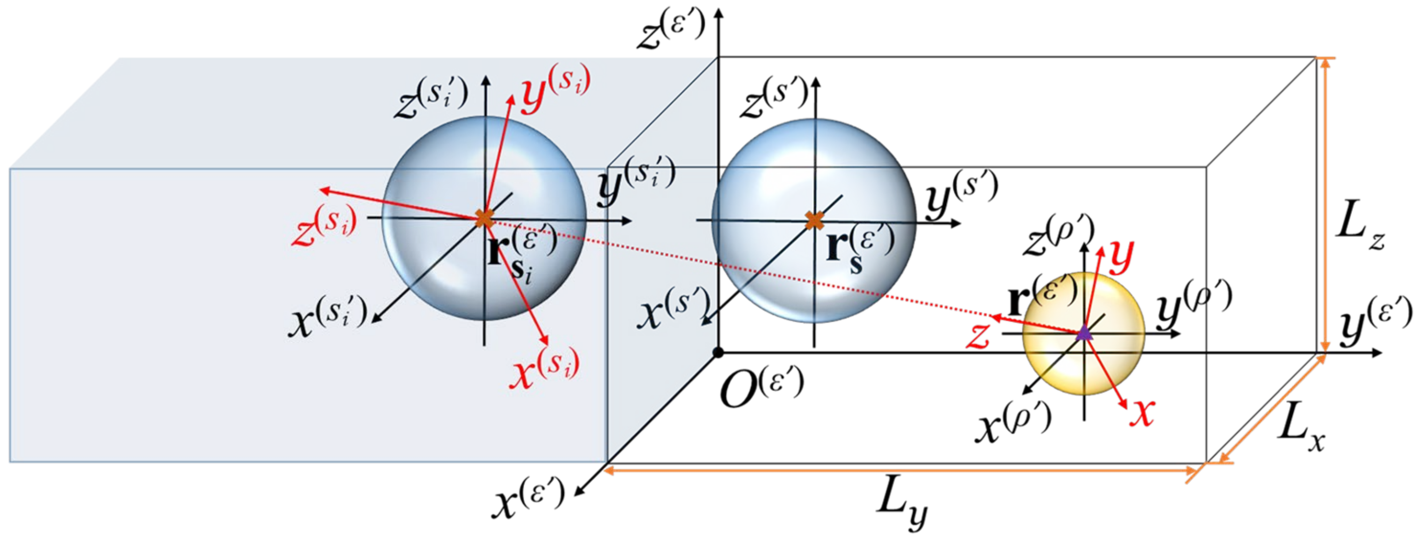


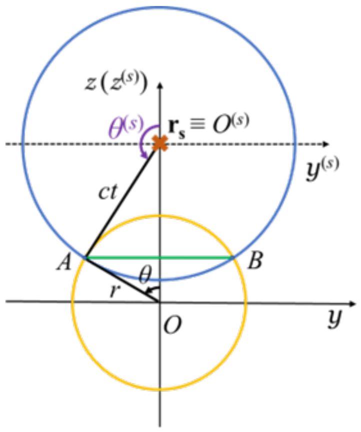


Setup of the ISM. The image source at $\mathbf{r}_{\mathbf{s}_i}^{(\varepsilon')}$ is the mirror reflection of the original source at $\mathbf{r}_{\mathbf{s}}^{(\varepsilon')}$. The observation sphere in yellow is centered at $\mathbf{r}^{(\varepsilon')}$.

- For each image source
  - The amplitudes of the emitted spherical wave fronts are the mirror reflections of those emitted by the original source.
  - The red coordinate systems are introduced so that the image source is on the positive z-axis.

ISM





The source at $\mathbf{r}_{\mathbf{s}}^{(\varepsilon')}$ emits a sequence of spherical wave fronts $d(t, \theta^{(s')}, \phi^{(s')})$.

Step 1 – Calculate the SH coefficients of the spherical wave fronts $d(t, \theta^{(s_i')}, \phi^{(s_i')})$ emitted by the image sources by using the parity properties of spherical harmonics.

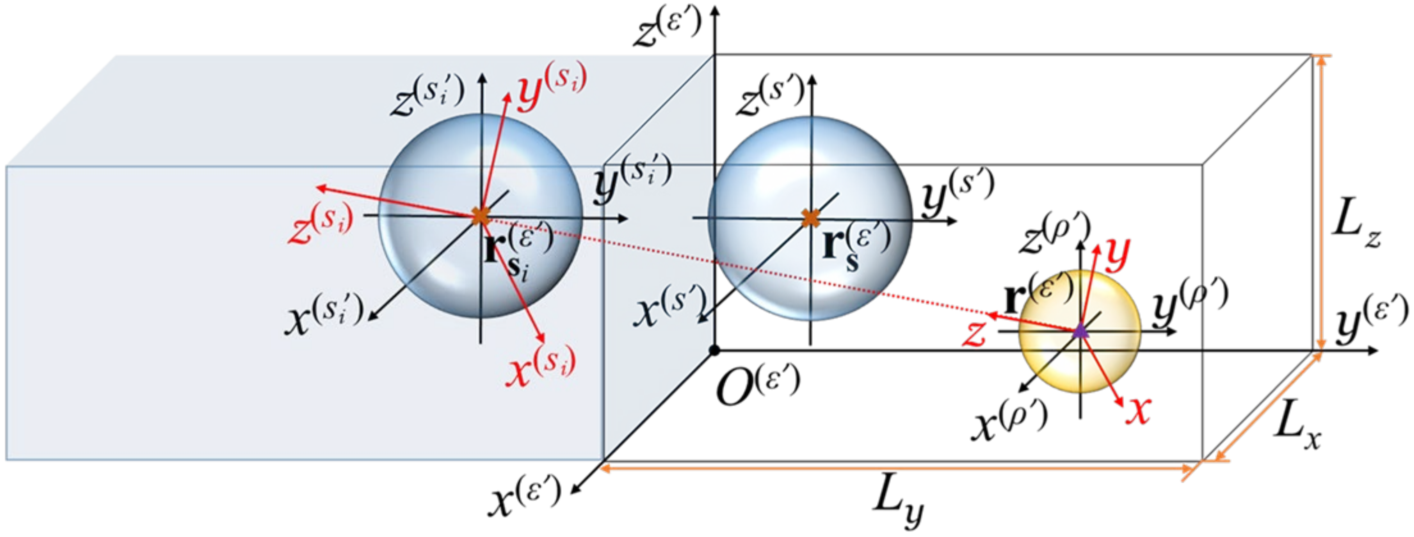
Step 2 – Calculate the SH coefficients of $d(t, \theta^{(s_i)}, \phi^{(s_i)})$, which is the spherical wave fronts emitted by the image sources expressed w.r.t. the $x^{(s_i)}y^{(s_i)}z^{(s_i)}$ coordinate system. The rotation of the coordinate system is achieved by using the Wigner *D*-matrix.

Step 3 – Calculate the SH coefficients of the observed signal on the observation sphere w.r.t. the xyz coordinate system. Should also incorporate attenuation due to wall reflection coefficients.

$$\zeta_n^m(t_0, r) = \frac{c}{2 r r_s} \int_\tau \sum_{v=0}^{V} \sum_{u=-v}^{v} \gamma_v^u(\tau)\, \mathcal{P}_v^u[\cos\theta_0^{(s)}(\tau)]\, \mathcal{P}_n^m[\cos\theta_0(\tau)] \delta_{m,u}\, \Xi(t_0, \tau)\, d\tau.$$

- $\Xi(t_0, \tau) = 1$ if $r_s - r \le c(t_0 - \tau) \le r_s + r$; else, $\Xi(t_0, \tau) = 0$.
- $\cos\theta_0(\tau) = [r^2 + r_s^2 - c^2(t_0 - \tau)^2]/[2 r r_s]$.
- $\cos\theta_0^{(s)}(\tau) = -[c^2(t_0 - \tau)^2 + r_s^2 - r^2]/[2c(t_0 - \tau) r_s]$.

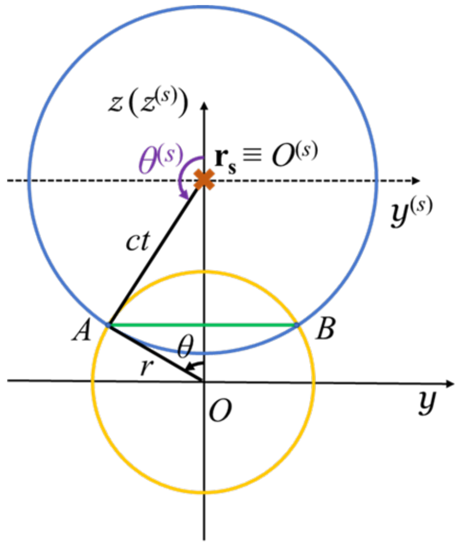
ISM



Step 4 – Calculate the SH coefficients of the observed signal expressed w.r.t. the $x^{(\rho')}y^{(\rho')}z^{(\rho')}$ coordinate system by using the Wigner *D*-matrix.

Step 5 – Add the contributions of all the image sources.

Simulations



- Room dimension [4, 6, 3] metres.
- Wall reflection coefficients [0.45, 0.7, 0.8, 0.5, 0.6, 0.75].
- Source location $\mathbf{r}_s^{(\varepsilon')}$ =[1.5, 3.4, 2.4] metres.
- Observation sphere of radius r=0.2 metres centred at $\mathbf{r}^{(\varepsilon')}$= [1.5, 3.4, 1] metres. The source is on the positive $z^{(\rho')}$ -axis.
- Source emits spherical wave fronts

$$d(t, \theta^{(s')}, \phi^{(s')}) = Y_0^0(\theta^{(s')}, \phi^{(s')})\delta(t) + Y_1^0(\theta^{(s')}, \phi^{(s')})\delta(t - 0.01).$$

- 24 image sources, 16 kHz sampling frequency, SH truncation of the observed signal is 10.
- Uniform sampling of the equation below results in aliasing. Here, the uniformly sampled version of the equation below is convolved with a low-pass filter with 257 samples and cut-off frequency at 2 kHz.

$$\zeta_n^m(t_0, r) = \frac{c}{2rr_s} \int_\tau \sum_{v=0}^{V} \sum_{u=-v}^{v} \gamma_v^u(\tau)\, \mathcal{P}_v^u[\cos\theta_0^{(s)}(\tau)]$$

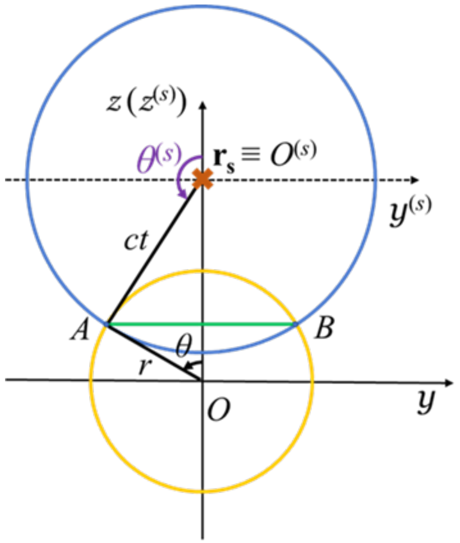
$$\mathcal{P}_n^m[\cos\theta_0(\tau)]\delta_{m,u}\, \boxed{\Xi(t_0, \tau)}\, d\tau.$$

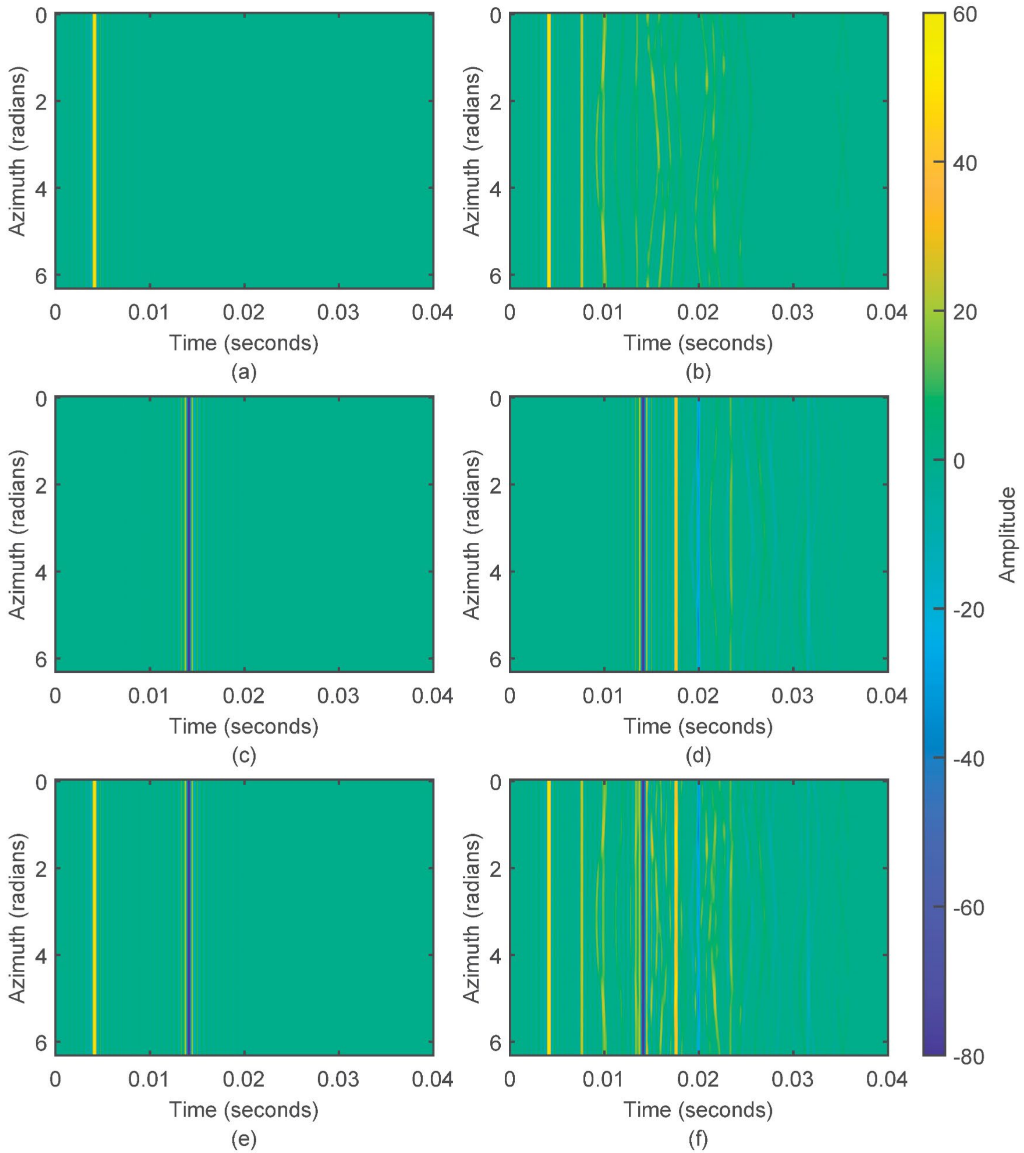
- $\Xi(t_0, \tau) = 1$ if $r_s - r \le c(t_0 - \tau) \le r_s + r$; else, $\Xi(t_0, \tau) = 0$.

Simulations

Observed signals along the equator of the observation sphere

In (a) and (b), $d(t,\theta^{(s')},\phi^{(s')}) = Y_0^0(\theta^{(s')},\phi^{(s')})\delta(t)$.
In (c) and (d), $d(t,\theta^{(s')},\phi^{(s')}) = Y_1^0(\theta^{(s')},\phi^{(s')})\delta(t-0.01)$.
In (e) and (f), $d(t,\theta^{(s')},\phi^{(s')}) = Y_0^0(\theta^{(s')},\phi^{(s')})\delta(t) + Y_1^0(\theta^{(s')},\phi^{(s')})\delta(t-0.01)$.
Moreover, (a), (c) and (e) are in anechoic condition; while (b), (d), and (f) are in reverberant condition.





Thanks for listening!