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Structural Optimization of Factor Graphs for Symbol Detection via Continuous Clustering and Machine Learning

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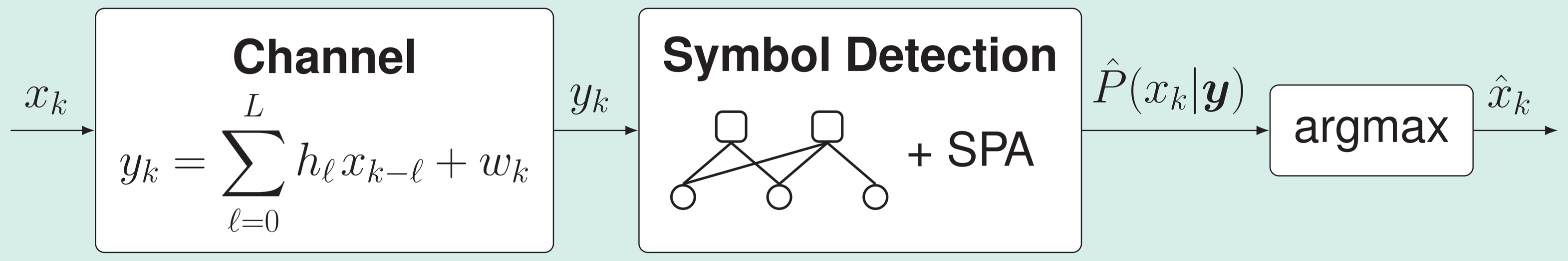
1. Introduction

- Inference tasks can be efficiently calculated via the sum-product algorithm (SPA) on a corresponding factor graph
- **Problem:** For factor graphs with cycles, the SPA performance heavily relies on the factor graph structure
- **Idea:** Optimize SPA performance by learning the factor graph structure

2. Symbol Detection

- **Example inference task:** Transmission of independent uniformly distributed BPSK symbols over an inter-symbol interference channel



- Impulse response: $\boldsymbol{h} = [0.407, 0.100, 0.815, 0.100, 0.407] \in \mathbb{R}^{L+1}$
- Additive white Gaussian noise (AWGN): $w_k \sim \mathcal{N}(0, \sigma^2)$

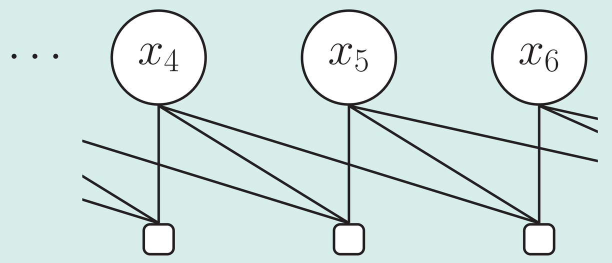
- Symbol detection via marginalization of $P(\boldsymbol{x} \mid \boldsymbol{y})$ using the SPA:

$$P(x_k \mid \boldsymbol{y}) = \sum_{\sim\{x_k\}} P(\boldsymbol{x} \mid \boldsymbol{y}), \quad P(\boldsymbol{x} \mid \boldsymbol{y}) \propto \prod_{k=1}^{K} \exp\left(-\frac{1}{2\sigma^2}\left|y_k - \sum_{\ell=0}^{L} h_\ell x_{k-\ell}\right|^2\right)$$

3. Factor Graph Models for Symbol Detection

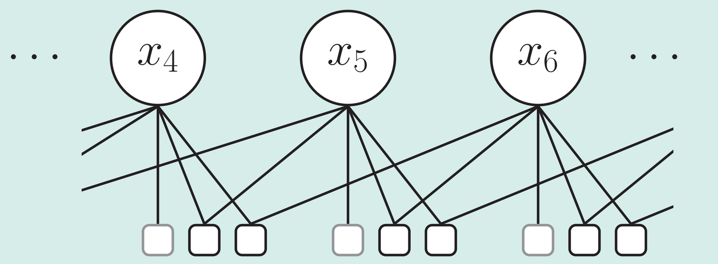
- **Problem:** SPA complexity increases exponentially with factor node (FN) degree
- Common factor graph models:

Forney Factor Graph (FFG)



− FN degree $L+1$
→ High complexity
+ Near-MAP performance

Ungerboeck Factor Graph (UFG)

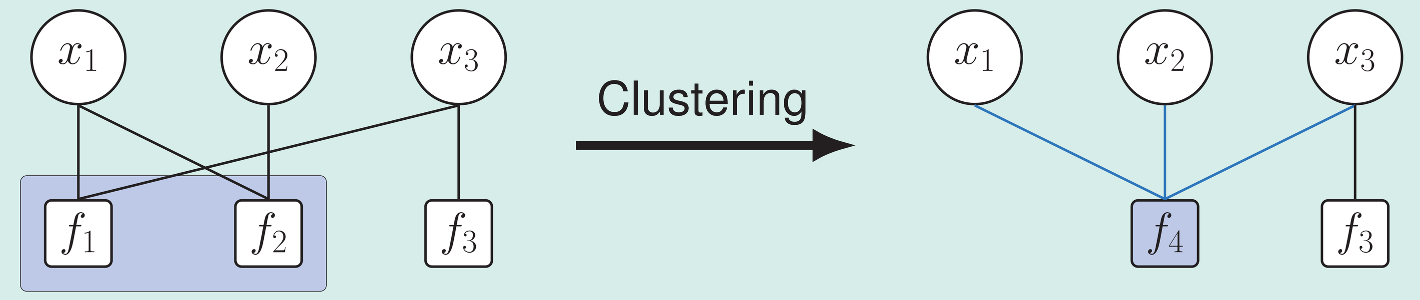


+ FN degree 1 and 2
→ Low complexity
− Poor performance

- **Idea:** Create new factor graphs by **clustering** FNs of the UFG
→ Interpolation between UFG and FFG

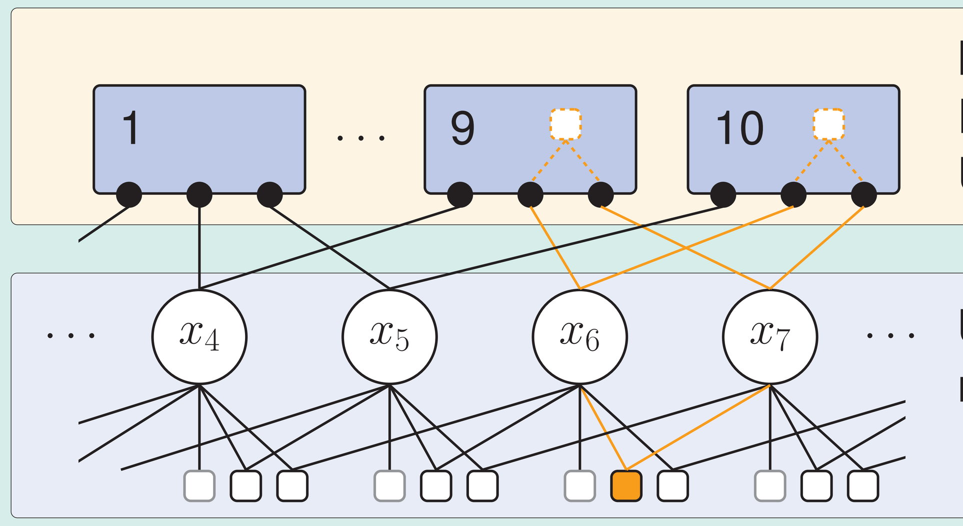
4. Factor Node Clustering

- Clustering of FNs: Combine FNs by multiplying their factors. Example:



$$f(x_1, x_2, x_3) = \underbrace{f_1(x_1, x_3) f_2(x_1, x_2)}_{f_4(x_1, x_2, x_3)} f_3(x_3)$$

- **Idea:** FN containers transform clustering problem into systematic form



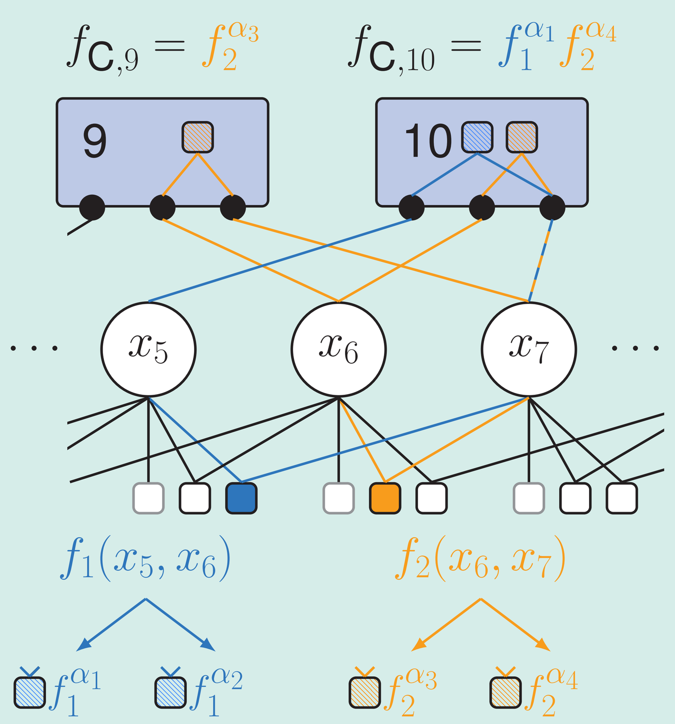
FN Containers: Empty FNs in which FNs of the UFG can be clustered

UFG: Basis to construct new FGs via clustering

- **Clustering options:** Each FN of the UFG can be clustered into the FN containers that are connected with the same variable nodes (VNs)
- Create new factor graphs by putting all FNs in one of their options
- FN container degree defines SPA complexity

5. Continuous Clustering (CC)

- **Idea:** Instead of clustering each FN in only one container, use every FN simultaneously in all of its clustering options:



Step 2: Cluster the factorized factors $f_i^{\alpha_j}$ into connected FN containers f_{C}

Step 1: Factorize all FNs f_i of the UFG via weights $\alpha_j \in [0, 1]$:

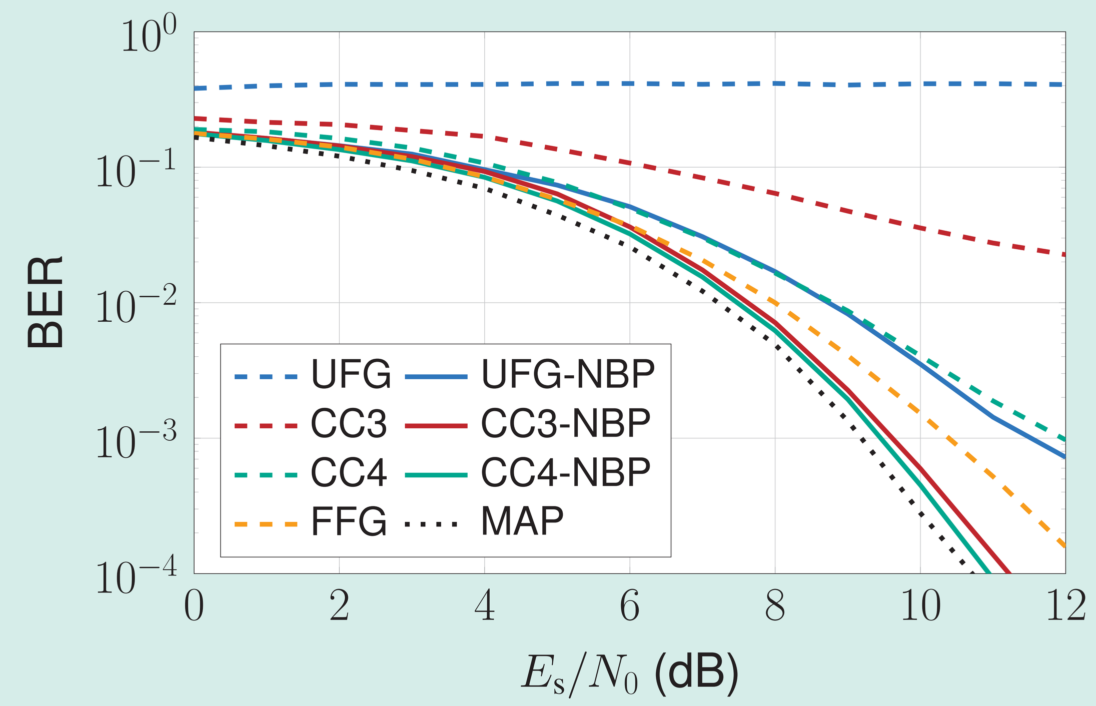
$$f_i(\cdot) = \prod_j f_i^{\alpha_j}(\cdot), \quad \sum_j \alpha_j = 1$$

6. Optimization

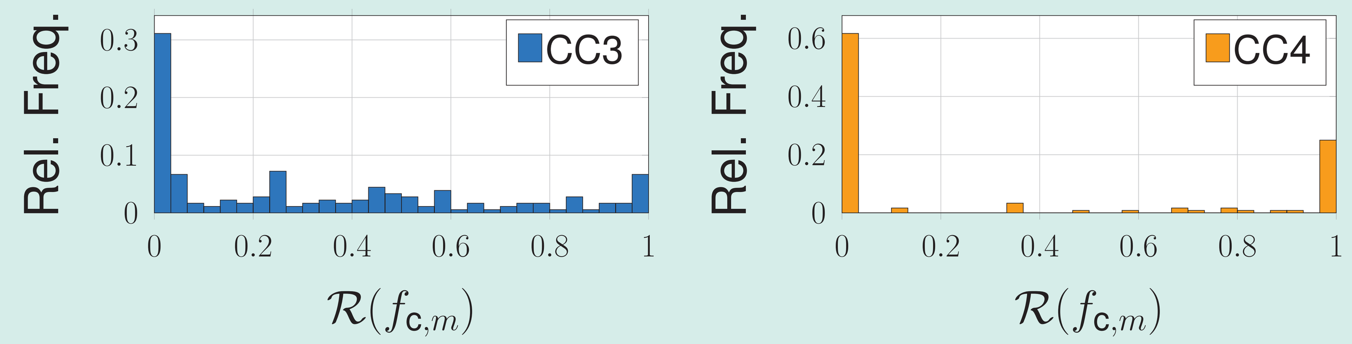
- Factor graph structure is parameterized by continuous weights α_{ij}
→ Enables optimization of SPA performance via **gradient descent**
- Direct combination with **neural belief propagation (NBP)** possible
- **Structure extraction:** Learned graph is extracted by pruning empty FN containers $f_{\mathsf{C},m}$ (i.e., $\mathcal{R}(f_{\mathsf{C},m}) \approx 0$)
  - Relevance $\mathcal{R}(f_{\mathsf{C},m})$: Maximum weight α_{ij} in container $f_{\mathsf{C},m}$

7. Results

- Bit error rate (BER) for factor graphs based on containers of degree 3 or 4 (CC3 or CC4) with and without NBP:

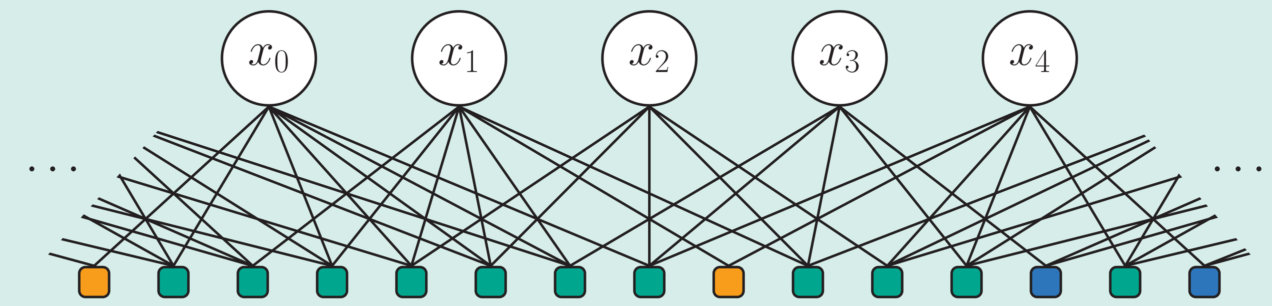


- Histogram of the relevance of the learned FN containers:



→ Large proportion of containers is irrelevant → Pruning possible

- Section of CC4 after pruning with $\alpha_{\text{thr}} = 0.01$:



FN Degree: 2 3 4