Scaling Law Analysis for Covariance Based Activity Detection in Cooperative Multi-Cell Massive MIMO

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Background and Motivation

- Massive random access is a main challenge in massive machine-type communication (mMTC).
- A large number of devices with sporadic activities are connected to the multi-cell network.
- Active devices transmit their unique preassigned non-orthogonal signature sequences to the base-stations (BSs).
- The network identifies the active devices by detecting which sequences are transmitted based on the received signals.
- Covariance-based approach: formulate the detection problem as a maximum likelihood estimation (MLE) problem in the single-cell [1, 2, 3] and multi-cell [4] scenarios respectively.
- The scaling law of the covariance-based activity detection in the single-cell scenario has been thoroughly analyzed in [2, 3].

Main Contribution

- Characterize the scaling law of the covariance-based approach in the multi-cell massive MIMO system.
- Characterize the distribution of the estimation error.

System Model

- A multi-cell system consists of B cells, each of which contains
  - one base station (BS) equipped with M antennas;
  - N single-antenna devices, K of which are active during any coherence interval.
- Each device n in cell j is preassigned a unique signature sequence $\mathbf{s}_{jn} \in \mathbb{C}^L$ with L being the sequence length.
- Let a_{jn} be a binary variable with $a_{jn} = 1$ for active and $a_{jn} = 0$ for inactive devices.
- The channel between device n in cell j and BS b is denoted as $\sqrt{g_{bjn}}\mathbf{h}_{bjn}$, where
  - $g_{bjn} \geq 0$ is the large-scale fading coefficient depending on path-loss and shadowing;
  - $\mathbf{h}_{bjn} \in \mathbb{C}^M$ is the Rayleigh fading coefficient following $\mathcal{CN}(\mathbf{0}, \mathbf{I})$.
- The additive Gaussian noise $\mathbf{W}_b \in \mathbb{C}^{L\times M}$ follows $\mathcal{CN}(\mathbf{0}, \sigma_w^2\mathbf{I})$.
- Notations:
  - $\mathbf{S}_j = [\mathbf{s}_{j1}, \ldots, \mathbf{s}_{jN}] \in \mathbb{C}^{L\times N}$, and $\mathbf{S} = [\mathbf{S}_1, \ldots, \mathbf{S}_B] \in \mathbb{C}^{L\times BN}$;
  - $\mathbf{A}_j = \text{diag}(a_{j1}, \ldots, a_{jN}) \in \mathbb{R}^{N\times N}$, $\mathbf{A} = \text{diag}(\mathbf{A}_1, \ldots, \mathbf{A}_B) \in \mathbb{R}^{BN\times BN}$, and $\mathbf{a} \in \mathbb{R}^{BN}$ denotes the diagonal entries of $\mathbf{A}$;
  - $\mathbf{G}_{bj} = \text{diag}(g_{bj1}, \ldots, g_{bjN}) \in \mathbb{R}^{N\times N}$, and $\mathbf{G}_b = \text{diag}(\mathbf{G}_{b1}, \ldots, \mathbf{G}_{bB}) \in \mathbb{R}^{BN\times BN}$;
  - $\mathbf{H}_{bj} = [\mathbf{h}_{bj1}, \ldots, \mathbf{h}_{bjN}]^T \in \mathbb{C}^{N\times M}$.

System Model (Cont.)

- The received signal $\mathbf{Y}_b \in \mathbb{C}^{L\times M}$ at BS b can be expressed as

$$\mathbf{Y}_b = \sum_{n=1}^{N} a_{bn}\mathbf{s}_{bn}g_{bbn}^{\frac{1}{2}}\mathbf{h}_{bbn}^T + \sum_{j\neq b}\sum_{n=1}^{N} a_{jn}\mathbf{s}_{jn}g_{bjn}^{\frac{1}{2}}\mathbf{h}_{bjn}^T + \mathbf{W}_b$$
$$= \mathbf{S}_b\mathbf{A}_b\mathbf{G}_{bb}^{\frac{1}{2}}\mathbf{H}_{bb} + \sum_{j\neq b}\mathbf{S}_j\mathbf{A}_j\mathbf{G}_{bj}^{\frac{1}{2}}\mathbf{H}_{bj} + \mathbf{W}_b. \quad (1)$$

- The covariance matrix $\mathbf{\Sigma}_b$ of $\mathbf{Y}_b$ is given by

$$\mathbf{\Sigma}_b = \frac{1}{M}\mathbb{E}\left[\mathbf{Y}_b\mathbf{Y}_b^H\right] = \mathbf{S}\mathbf{G}_b\mathbf{A}\mathbf{S}^H + \sigma_w^2\mathbf{I}. \quad (2)$$

- The MLE problem can be formulated as [4]

$$\underset{\mathbf{a}}{\text{minimize}} \quad \sum_{b=1}^{B}\left(\log|\mathbf{\Sigma}_b| + \text{tr}\left(\mathbf{\Sigma}_b^{-1}\widehat{\mathbf{\Sigma}}_b\right)\right) \quad (3a)$$
$$\text{subject to} \quad a_{bn} \in [0,1], \ \forall\, b, n. \quad (3b)$$

- The sample covariance matrix $\widehat{\mathbf{\Sigma}}_b = \mathbf{Y}_b\mathbf{Y}_b^H/M$ is computed by averaging over different antennas.
- We are interested in answering the following two theoretical questions:
  - given the system parameters $L, B,$ and N, how many active devices can be correctly detected via solving the MLE problem (3) as $M \to \infty$?
  - what is the asymptotic distribution of the MLE error?

Consistency of MLE [4]

Lemma 1. Consider the MLE problem (3) with given $\mathbf{S}$, $\{\mathbf{G}_b\}$, and σ_w^2. Define

$$\widetilde{\mathbf{S}} \triangleq [\mathbf{s}_{11}^* \otimes \mathbf{s}_{11}, \ldots, \mathbf{s}_{BN}^* \otimes \mathbf{s}_{BN}] \in \mathbb{C}^{L^2\times BN}, \quad (4)$$

where $(\cdot)^*$ is the conjugate operation and $\otimes$ is the Kronecker product. Let $\hat{\mathbf{a}}^{(M)}$ be the solution to (3) when the number of antennas M is given, and let $\mathbf{a}^\circ$ be the true activity indicator vector. Define $\mathcal{I} \triangleq \{i \mid a_i^\circ = 0\}$,

$$\mathcal{N} \triangleq \{\mathbf{x} \in \mathbb{R}^{BN} \mid \widetilde{\mathbf{S}}\mathbf{G}_b\mathbf{x} = \mathbf{0}, \ \forall\, b\}, \quad (5)$$
$$\mathcal{C} \triangleq \{\mathbf{x} \in \mathbb{R}^{BN} \mid x_i \geq 0 \text{ if } i \in \mathcal{I}, \ x_i \leq 0 \text{ if } i \notin \mathcal{I}\}, \quad (6)$$

then a necessary and sufficient condition for $\hat{\mathbf{a}}^{(M)} \to \mathbf{a}^\circ$ as $M \to \infty$ is that the intersection of $\mathcal{N}$ and $\mathcal{C}$ is the zero vector, i.e., $\mathcal{N} \cap \mathcal{C} = \{\mathbf{0}\}$.

- The signature sequence matrix and the large-scale fading coefficients play vital roles in the scaling law analysis (due to Eq. (5)).

Main Results

- The assumptions and main results:

Assumption 1. The columns of the signature sequence matrix $\mathbf{S}$ are uniformly drawn from the sphere of radius $\sqrt{L}$ in an i.i.d. fashion.

Assumption 2. The multi-cell system consists of B hexagonal cells with radius R. In this system, the large-scale fading components decrease exponentially with distance [5], i.e.,

$$g_{bjn} = P_0\left(\frac{d_0}{d_{bjn}}\right)^{\gamma}, \quad (7)$$

where P_0 is the received power at the point with distance d_0 from the transmitting antenna, d_{bjn} is the BS-device distance between device n in cell j and BS b, and γ is the path-loss exponent.

- Scaling law of the MLE problem (3):

Theorem 1. Under Assumption 1 and Assumption 2 with $\gamma > 2$, then there exist constants $c_1, c_2 > 0$ independent of system parameters K, L, N, and B, such that if

$$K \leq c_1 L^2/\log^2(eBN/L^2), \quad (8)$$

then the condition $\mathcal{N} \cap \mathcal{C} = \{\mathbf{0}\}$ in Lemma 1 holds with probability at least $1 - \exp(-c_2 L)$.

- The maximum number of active devices that can be correctly detected by solving the MLE problem (3) is in the order of L^2 shown in (8);
- The inter-cell interference is not a limiting factor of the detection performance because B affects K only through $\log B$;
- Scaling law in (8) in the multi-cell scenario is approximately the same as the single-cell scenario [2, 3].

- A summary of phase transition and scaling law results on covariance-based activity detection:

	Single-Cell Scenario	Multi-Cell Scenario
Phase Transition	Theorem 2 in [3]	Theorem 3 in [4]
Scaling Law	Theorem 9 in [3]	**Theorem 1**

- Distribution of estimation error (see [6] for more details):

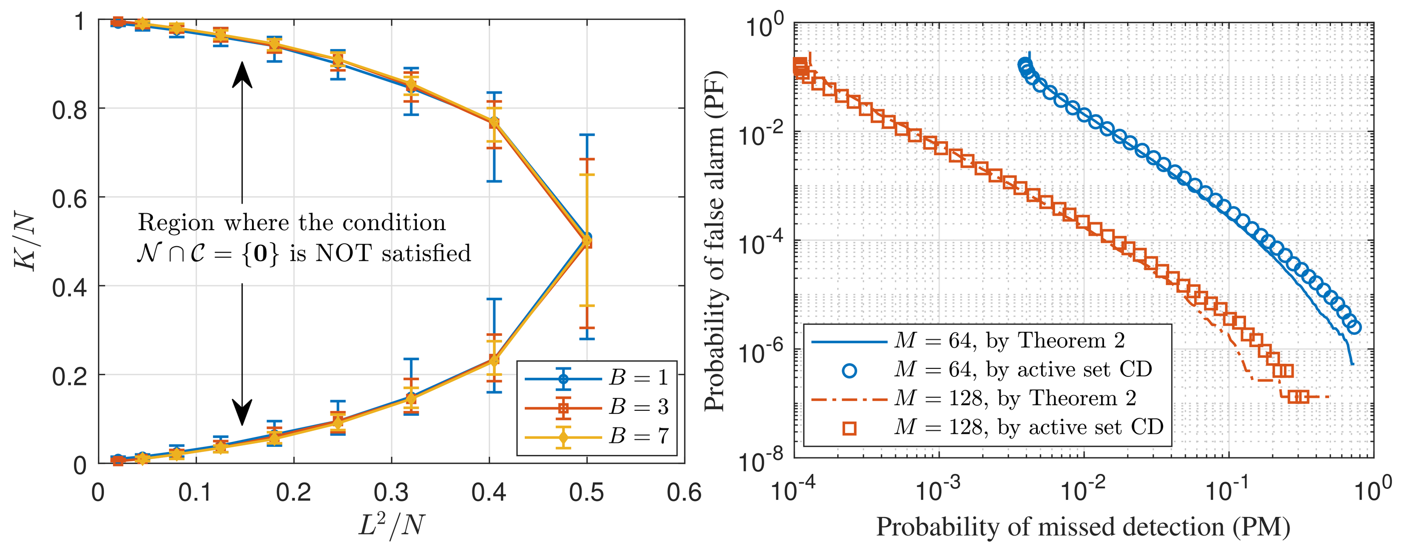
Theorem 2. Define a random vector $\hat{\boldsymbol{\mu}}$ whose distribution depends only on $\mathbf{S}$, $\{\mathbf{G}_b\}$, σ_w^2, and $\mathbf{a}^\circ$, then $\sqrt{M}(\hat{\mathbf{a}}^{(M)} - \mathbf{a}^\circ)$ converges in distribution to $\hat{\boldsymbol{\mu}}$ as $M \to \infty$.

Main Results (Cont.)

- The estimation error $\hat{\mathbf{a}}^{(M)} - \mathbf{a}^\circ$ can be approximated by $\frac{1}{\sqrt{M}}\hat{\boldsymbol{\mu}}$ for a sufficiently large M;
- We can numerically compute the error distribution by Theorem 2.

Simulation Results

- The channel path-loss is modeled as $128.1 + 37.6\log_{10}(d)$ in Assumption 2, where d is in km.
- All signature sequences are uniformly drawn from the sphere as in Assumption 1.



Left: Scaling law of covariance-based activity detection when $N = 200$;
Right: Comparison of the simulated results and the analysis in terms of PM and PF when $B = 7, N = 200, K = 20, L = 20$.

Left Figure:

- The curves with different B's overlap with each other, implying that the scaling law is almost independent of B.
- K is approximately proportional to L^2, which verifies Eq. (8).

Right Figure:

- The curves obtained from Theorem 2 match well with those obtained from the active set CD algorithm.

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