On the Value of Stochastic Side Information in Online Learning

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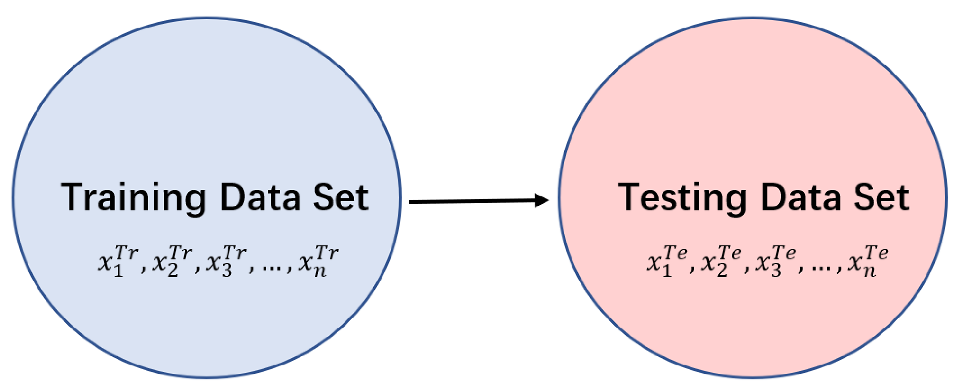
Introduction to Online learning

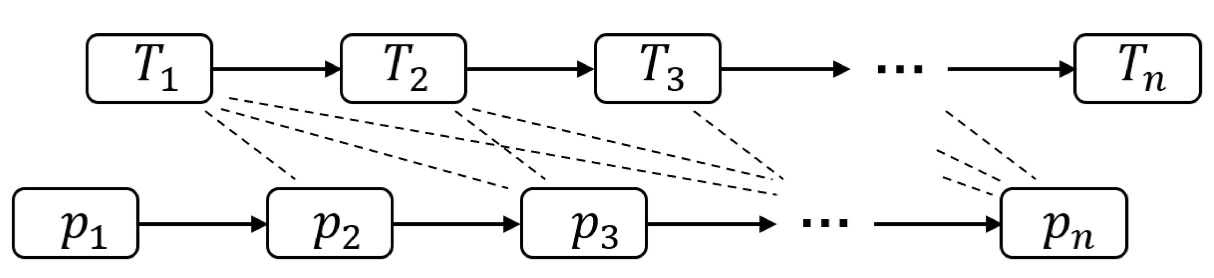
Traditional Machine Learning:

- All training data are known initially.



Online Learning:

- The data arrive sequentially.



Experts and Side Information

Target		x_1^T	x_2^T	x_3^T	$\cdots$	x_n^T
Experts	$\theta = 1$	f_1^1	f_2^1	f_3^1	$\cdots$	f_n^1
	$\theta = 2$	f_1^2	f_2^2	f_3^2	$\cdots$	f_n^2
	$\vdots$					$\vdots$
	$\theta = N$	f_1^N	f_2^N	f_3^N	$\cdots$	f_n^N
Sequential Side Information		s_1	s_2	s_3	$\cdots$	s_n

A Simple Motivation
Weather Forecaster System

Target:
Local weather

Experts:
Predictions from different models

Side information:
The weather from a nearby area

Minimax Expected Regret

Maximum over all possible target sequences

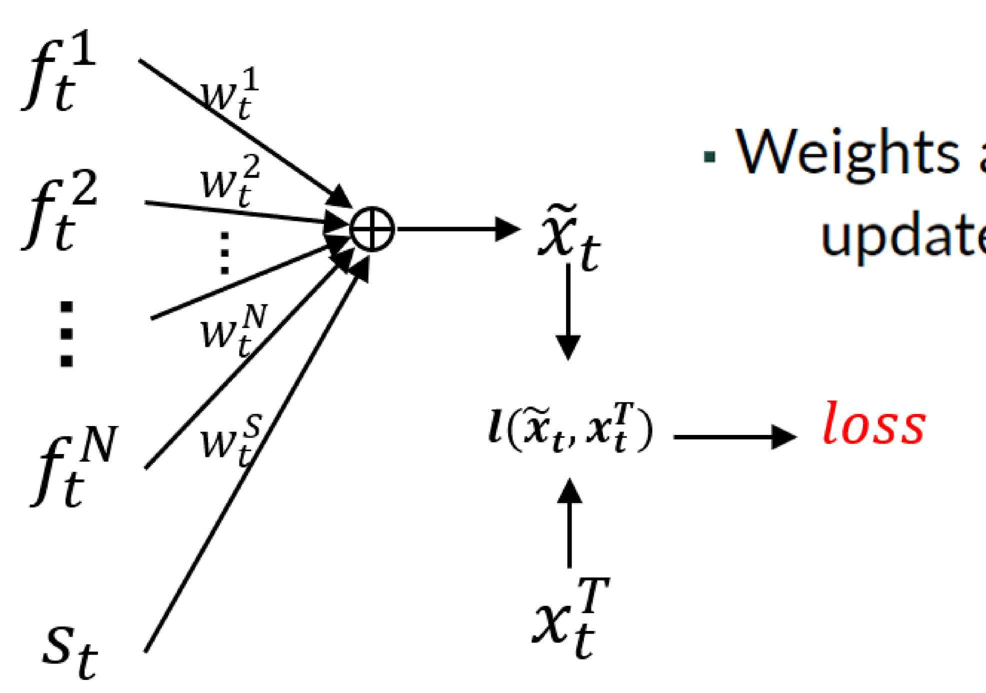
Comparing to the best expert

$$R := \min_{\tilde{x}_1,\ldots,\tilde{x}_n} \max_{x_1^T,\ldots,x_n^T} \mathbb{E}_{s_1,\ldots,s_n} \left\{ \sum_{t=1}^{n} \ell(\tilde{x}_n(s_1,\ldots,s_t), x_t^T) - \min_{\theta} \sum_{t=1}^{n} \ell(f_t^\theta, x_t^T) \right\}$$

Expectation over the stochastic side information

Minimum over all possible algorithms

Weighted Average Algorithm



- Weights are exponentially updated by their induced loss.

Main Contribution

This work mainly contributes to three aspects:

- **A Novel Setup:** We proposed a novel online learning problem formulation with stochastic side information.
- **Negative Regret:** We showed that introducing SSI can improve the typical learning rate under some possible cases.

Results

Theorem 1(Upper Bound): The minimax expected regret for predicting the target sequence with length n:

$$R \leq \sqrt{\frac{n}{2}\log(N+1)} + \min\{F(n), 0\},$$

where $F(n)$ depends on the problems' specifications.

Theorem 2(Lower Bound): Consider $\mathcal{X} = \{0,1\}$ and $\mathcal{D} = [0,1]$, and ℓ is the absolute loss $\ell(x,y) = |x-y|$, then

$$R \geq \sqrt{\frac{n}{2}\log(N+1)} + \left(\xi^* - \frac{1}{2}\right) n,$$

where ξ^* is a constant related to side information.

Both upper and lower bound could be negative!

Examples

SSI via a binary symmetric channel

$$R \leq \sqrt{\frac{n}{2}\log(N+1)} + \min\{0, nc_1\}$$
$$R \geq \sqrt{\frac{n}{2}\log(N+1)} + nc_2.$$

SSI via a Zero-mean Gaussian Channel

$$R \leq \sqrt{\frac{n}{2}\log(N+1)} + \min\{0, nc_3\}$$
$$R \geq \sqrt{\frac{n}{2}\log(N+1)} + nc_4.$$