THE CONVERSION OF ANALOG PHASE LOCKED LOOP

DESIGNS TO DIGITAL IMPLEMENTATIONS

Robert E. Rouquette
Offshore Navigation, Inc.
P.O. Box 23504
Harahan, LA 70183

Summary

A digital phase locked loop may have an advantage over an equivalent analog design. Sub-hertz bandwidths can be achieved without component problems. There is freedom from component sensitivity. A digital processor may already be present in the system. A phase locked receiver may have digital frequency synthesizers. Some disadvantages of a digital pll with respect to an analog pll exist. The phase information must be band limited to half the sampling frequency. The digital circuitry is more complex than the analog circuitry. The sampling rate cannot exceed the clock rate of the logic.

In a second order analog pll, the bandwidth, damping, and velocity error constant can be independently chosen. The finite sampling rate in a digital implementation causes one of the above parameters to degrade in a digital pll as the Nyquist sampling ratê is approached. The parameter that degrades is dependent upon the digital design method. If a digital pll is sampled at ten times the Nyquist rate, it is equivalent within one percent error to an analog pll.

2nd Order Analog PLL

Using established techniques, the bandwidth, damping, and velocity error constant are chosen.[1] The second order analog pll block diagram is in figure 1. A type 2 loop has been chosen.

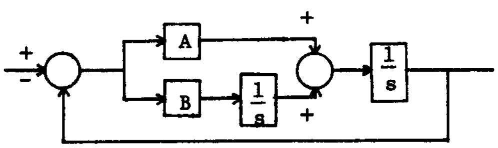

Figure 1 Second Order Analog PLL

The closed loop transfer function is

$$H(s) = \frac{A\left(s + \frac{B}{A}\right)}{s^2 + As + B} = \frac{2\zeta\omega_n\left(s + \frac{\omega_n}{2\zeta}\right)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The pole and zero locations are

$$p = -\zeta\omega_n \pm j\omega_d \qquad z = -\frac{\omega_n}{2\zeta}$$

Where ζ is the damping ratio, ω_n is the natural frequency, and $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the damped frequency.

$$A = 2\zeta\omega_n \qquad B = \omega_n^2$$

The open loop transfer function is

$$G(s) = \frac{As + B}{s^2} = \frac{2\zeta\omega_n\left(s + \frac{\omega_n}{2\zeta}\right)}{s^2}$$

The acceleration error constant is

$$K_a = \lim_{s\to 0} s^2\, G(s) = \omega_n^2$$

The velocity error constant is

$$K_v = \lim_{s\to 0} s\, G(s) = \infty$$

2nd Order Digital PLL

A digital pll may be derived from the analog pll by replacing the integrators with accumulators and ω by ωT where T is the sampling period. [2], [3] The second order digital pll block diagram is in figure 2.

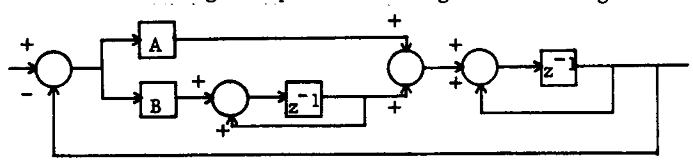

Figure 2 Second Order Digital PLL

The closed loop transfer function is

$$H(z) = \frac{Az^{-1}\left(1-\left(1-\frac{B}{A}\right)z^{-1}\right)}{1-(2-A)z^{-1}+(1-A+B)z^{-2}}$$

$$= \frac{2\zeta\omega_n T z^{-1}\left(1-\left(1-\frac{\omega_n T}{2\zeta}\right)z^{-1}\right)}{1-2(1-\zeta\omega_n T)z^{-1}+(1-2\zeta\omega_n T+(\omega_n T)^2)z^{-2}}$$

The pole and zero locations are

$$p = (1-\zeta\omega_n T) \pm j\omega_d T \qquad z = 0,\ 1-\frac{\omega_n T}{2\zeta}$$

Where ζ is the damping ratio, $\omega_n T$ is the natural frequency, and $\omega_d T = \omega_n T\sqrt{1-\zeta^2}$ is the damped frequency.

$$A = 2\zeta\omega_n T \qquad B = (\omega_n T)^2$$

The open loop transfer function is

$$G(z) = \frac{Az^{-1}\left(1-\left(1-\frac{B}{A}\right)z^{-1}\right)}{(1-z^{-1})^2} = \frac{2\zeta\omega_n T z^{-1}\left(1-\left(1-\frac{\omega_n T}{2\zeta}\right)z^{-1}\right)}{(1-z^{-1})^2}$$

The acceleration error constant is

$$K_a = \frac{1}{T^2}\lim_{z\to 1}\frac{(1-z^{-1})^2}{z^{-2}}\ G(z) = \omega_n^2$$

The velocity error constant is

$$K_v = \frac{1}{T}\lim_{z\to 1}\frac{(1-z^{-1})}{z^{-1}}\ G(z) = \infty$$

The quadratic in z for a pole pair having a damping ratio ζ and damped frequency $\omega_d T$ and its factored poles are

$$1 - 2e^{-\zeta\omega_n T}\cos\omega_d T\ z^{-1} + e^{-2\zeta\omega_n T}\ z^{-2}$$

$$p = e^{-\zeta\omega_n T}(\cos\omega_d T \pm j\sin\omega_d T)$$

In this analog to digital transformation the velocity and acceleration error constants are preserved but the damping ratio and bandwidth are not. Let's examine the case where $\omega_d T \ll 1$ and $\zeta = \sqrt{2}/2$. The following approximations are accurate to 1% error if $\omega_d T \le \pi/10$.

$\sin\omega_d T = \omega_d T \qquad \cos\omega_d T = 1-(\omega_d T)^2/2$

$e^{-\zeta\omega_n T} = e^{-\omega_d T} = 1-\omega_d T + (\omega_d T)^2/2$

Then the pole and zero locations are

$p = 1-\omega_d T \pm j\omega_d T \qquad z = 0,\ 1-\omega_d T$

The quadratic for a pole pair with damping ratio $\zeta = \sqrt{2}/2$ and damped frequency $\omega_d T \leq \pi/10$ is

$1-2(1-\omega_d T)\, z^{-1} + (1 - 2\omega_d T + 2(\omega_d T)^2) z^{-2}$

$p = (1-\omega_d T)(1 \pm j\omega_d T)$

The actual and desired bandwidth and damping ratio are nearly equal if the loop is sampled above ten times the Nyquist rate

$f_s = \frac{1}{T} = \frac{20\omega_d}{2\pi} \qquad \omega_d T = \frac{\pi}{10}$

Where f_s is the sampling rate.

Alternate 2nd Order Digital PLL

The integrators are again replaced by accumulators, but the denominator of the closed loop transfer function is constrained to have the desired damping ratio and bandwidth. The block diagram is in figure 2. The closed loop transfer function is

$$H(z) = \frac{Az^{-1}(1-(1-\frac{B}{A})z^{-1})}{1-(2-A)z^{-1}+(1-A+B)z^{-2}}$$

$$= \frac{2(1-e^{-\zeta\omega_n T}\cos\omega_d T)z^{-1}\left(1- \frac{1-e^{-2\zeta\omega_n T}}{2(1-e^{-\zeta\omega_n T}\cos\omega_d T)}z^{-1}\right)}{1-2e^{-\zeta\omega_n T}\cos\omega_d T z^{-1}+e^{-2\zeta\omega_n T}z^{-2}}$$

The pole and zero locations and loop constants are

$p = e^{-\zeta\omega_n T}(\cos\omega_d T \pm j\sin\omega_d T)$

$z = 0,\ \frac{1-e^{-2\zeta\omega_n T}}{2(1-e^{-\zeta\omega_n T}\cos\omega_d T)}$

$A = 2(1-e^{-\zeta\omega_n T}\cos\omega_d T) \qquad B = 1-2e^{-\zeta\omega_n T}\cos\omega_d T+e^{-2\zeta\omega_n T}$

The open loop transfer function is

$$G(z) = \frac{Az^{-1}(1-(1-\frac{B}{A})z^{-1}}{(1-z^{-1})^2}$$

$$= \frac{2(1-e^{-\zeta\omega_n T}\cos\omega_d T)z^{-1}\left(1- \frac{1-e^{-2\zeta\omega_n T}}{2(1-e^{-\zeta\omega_n T}\cos\omega_d T)}z^{-1}\right)}{(1-z^{-1})^2}$$

The acceleration error constant is

$$K_a = \frac{B}{T^2} = \frac{1-2e^{-\zeta\omega_n T}\cos\omega_d T + e^{-2\zeta\omega_n T}}{T^2}$$

The velocity error constant $K_v = \infty$

In this analog to digital transformation the damping ratio and bandwidth are preserved but the acceleration error constant is not. The frequency response of this digital pll is high pass because the closed loop transfer function zero is near the zero frequency point on the unit circle. The location of the zero is determined by the pole locations and the type 2 constraint. Let $\omega_d T \leq \pi/10$. Then the pole and zero locations, acceleration error constant, and loop constants are

$p = (1-\omega_d T)(1 \pm j\omega_d T) \qquad z = 0,\ 1-\omega_d T$

$$K_a = \frac{1-2(1-\omega_d T)+(1-2\omega_d T + 2(\omega_d T)^2)}{T^2} = 2\omega_d^2 = \omega_n^2$$

$A = 2\omega_d T = 2\zeta\omega_n T \qquad B = (\omega_n T)^2$

This digital loop is equivalent to the previous digital loop and to the analog prototype if the sampling rate is above ten times the Nyquist rate.

Conclusion

The oversampled 2nd order digital pll will be useful for most applications. A sampling rate of ten times the Nyquist rate allows a simple band limiting filter to precede the digital pll. If operation near the Nyquist sampling rate is desired, a complex bandlimiting filter before the pll is required. This filter can be designed to compensate the high pass frequency response of the digital pll if required.

References

1. F.M. Gardner, "Phaselock Techniques." John Wiley and Sons: 1979

2. B.C. Kuo, "Digital Control Systems." Holt, Rinehart, and Winston: 1980

3. A.V. Oppenheim, R.W. Schafer, "Digital Signal Processing." Prentice-Hall: 1975