DCT-based Air Interface Design for Function Computation

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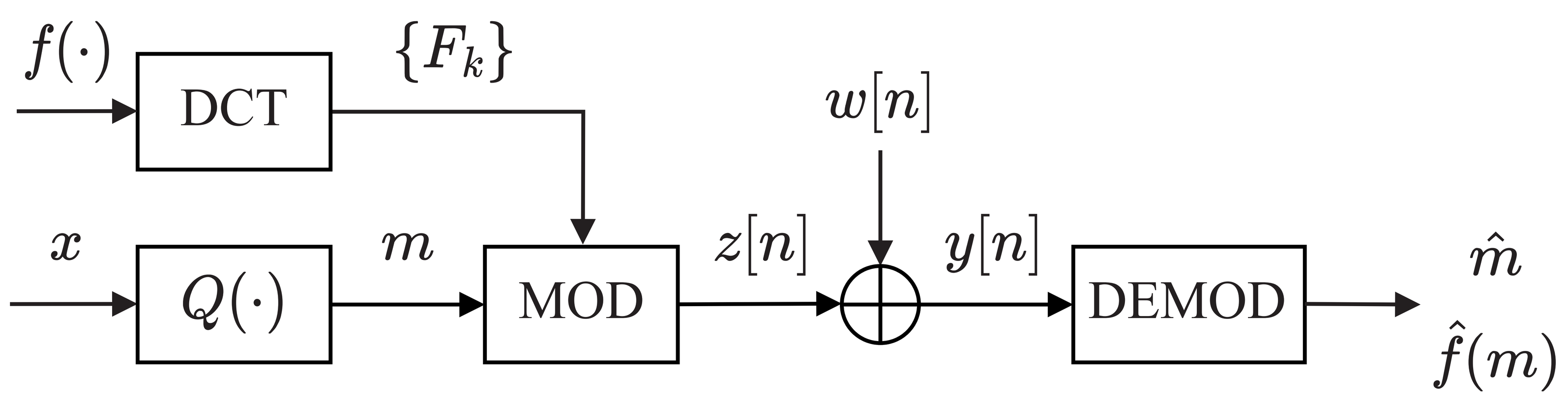
1. Introduction

Future networks are expected to integrate computing capabilities, in which terrestrial an non-terrestrial wireless sensor networks (WSN) will play a key role [1]. The emergence of joint communication and computing schemes can be traced back to Gastpar [2], where he shows that when the goal of the communication is not to transmit information reliably, but to assist the function computation of the data, the source-channel separation theorem does not hold in general. In this work we deal with the general problem of frequency modulation (FM) for function approximation through a communication channel.

2. Signal model

Consider a signal $x(t)$, which remains constant for an observation time T. We consider a Single-Input Single-Output (SISO) system where x is quantized into $m \in [0, N-1]$ and a function $f(m)$ is to be computed. The information is modulated using the Discrete Cosine Transform (DCT) with N samples per measurement. The channel is ideal with Gaussian noise of spectral density $N_o/2$. At the receiver side, an estimate of the function $\hat{f}(m)$ is obtained. The figure shows the joint communication and computing scheme.



The performance of the system is measured in terms of the average MSE, where K parameterizes the DCT-based approximation quality of the function.

$$MSE(f, K) = \frac{1}{N}\sum_{m=0}^{N-1} E\left\{\left|f(m) - \hat{f}(m)\right|^2\right\}$$

3. DCT-FM: Modulation for Computing

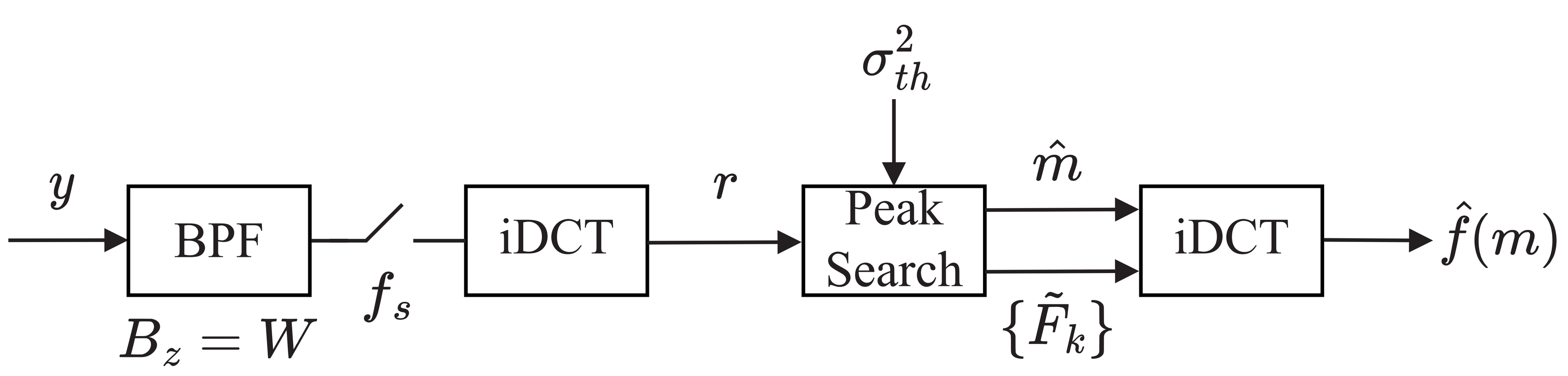
The DCT is used to approximate $f(m)$ with K coefficients, namely F_k. Simultaneously, the DCT is used as a waveform by incorporating a time index:

$$z[n] = A_c\sqrt{\frac{2}{N}}\sum_{k\in\mathcal{K}} F_k \cos\left(\frac{\pi k(2m+1)}{2N}n\right) \quad \text{for } n = 0, \ldots, N-1,$$

where A_c is the amplitude of the carrier. Notice that $z[n]$ is built with K FM tones whose frequency depends on k and m. Since the frequencies in (1) are orthogonal, the DCT applied over the received signal $y[n]$ results in a noisy version of the coefficients, namely $\tilde{F}_k$, due to the effect of the noise w.
The location of these coefficients can be found analytically by matching the frequency of a transmitted peak to the frequency bin of the DCT:

$$\frac{1}{2}\frac{1}{2N}k(2m+1) = \frac{\text{ifreq}(k,m) - 1/2}{2N} \quad \forall k \in \mathcal{K}',$$

where ifreq(k, m) is the location of the transmitted coefficient and $\mathcal{K}'$ is the set of indices for the detected coefficients at the receiver. The figure shows the receiver to recover both, the measurement and the function estimate. Notice that this modulation allows the receiver to be agnostic to the function, as it can recover all the information from the received signal $y[n]$ to reconstruct it.



- **Non-agnostic receiver**: As F_k decay fast, we propose replacing them by $\sqrt{2^{k-1}}$, i.e., decaying at 3 dB per coefficient. Nevertheless the receiver is no longer agnostic to the function, as it has to impose the coefficients upon reconstruction.

4. Results

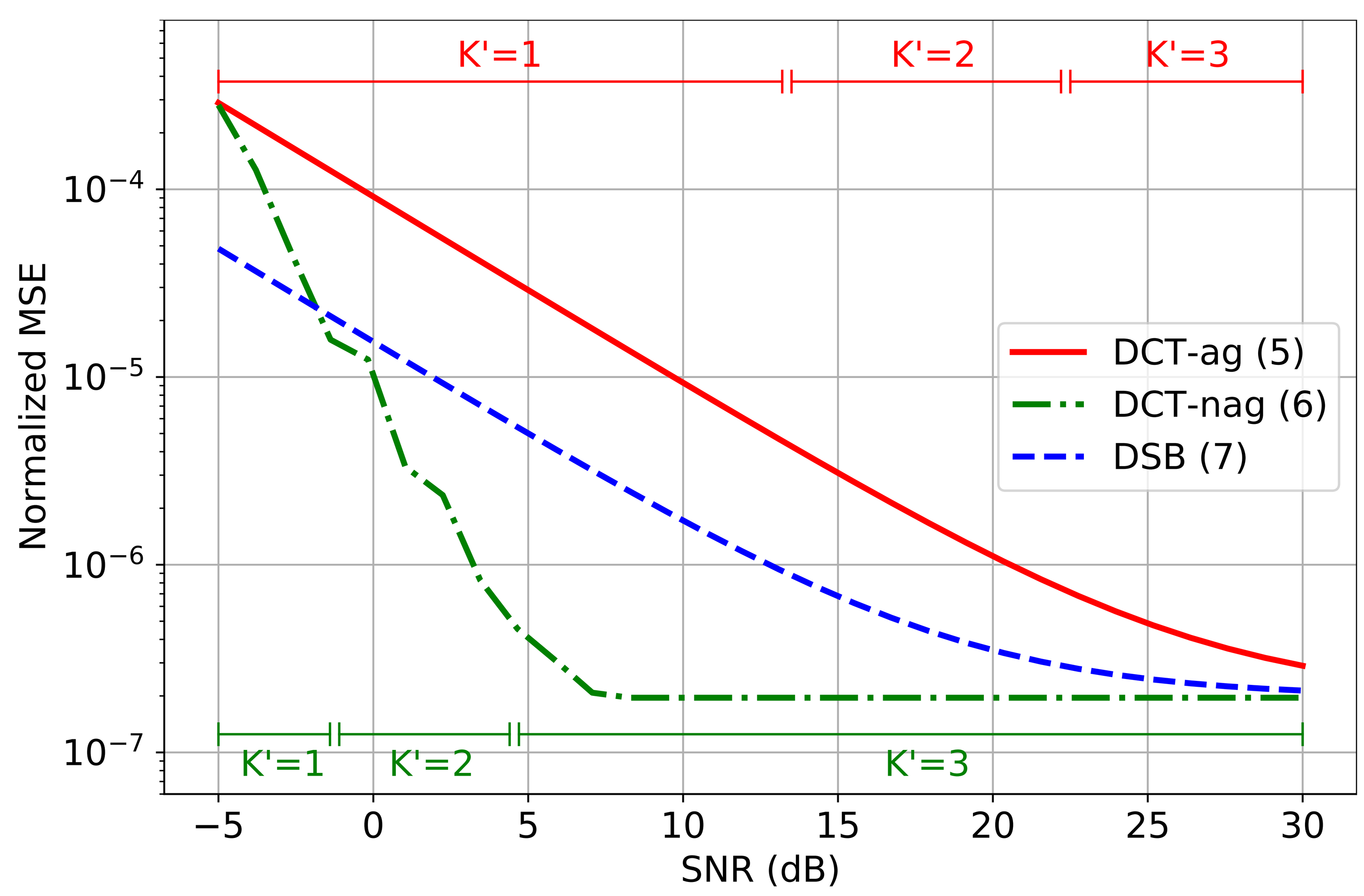
We consider the double sideband (DSB) modulation as a benchmark, since linear analog modulations are a standard choice for these scenarios. The performance of the agnostic DCT-FM, non-agnostic DCT-FM and DSB modulations respectively are

$$MSE_{ag}(f, K) = \sum_{k\notin\mathcal{K}'} F_k^2 + \frac{K'\sigma^2}{NA_c^2}, \tag{5}$$

$$MSE_{nag}(f, K) = \sum_{k\notin\mathcal{K}'} F_k^2, \tag{6}$$

$$MSE_{DSB}(f, K) = \sum_{k\notin\mathcal{K}} F_k^2 + \frac{\sigma^2}{2NA_c^2}, \tag{7}$$

where $\sigma^2 = 2N_oW$ and K' is the number of detected coefficients at the receiver. The sum of the non-used coefficients corresponds to the MSE due to the DCT approximation.
The figure shows the experimental $MSE(f, K)$ for the sigmoid function with $K = 3$. We assume an observation time of $T = 1$ s, $N = 256$ samples, $\alpha = 0.995$ and $A_c = 1$. The experiments are averaged over 10^2 Monte Carlo runs and the MSE is normalized by the energy of the function value.



5. Conclusions

In this paper we have presented a novel frequency modulation for joint communication and computing that relies on the DCT to approximate a mathematical function and transport the information simultaneously.

- Integration in existing IoT or M2M communication schemes.
- First system to provide an FM structure for distributed and AirComp systems.
- The measurement and the function are recovered in a single transmission.
- The receiver may be agnostic to the computed function.

6. References

[1] Chengwen Zhang, Liankai Wang, Libin Jiao, Shipeng Wang, Jun Shi, and Jia Yue. A novel orthogonal LoRa multiple access algorithm for satellite Internet of Things. *China Communications*, 19(3):279–289, 2022.

[2] M. Gastpar, M. Vetterli, and P.L. Dragotti. Sensing reality and communicating bits: a dangerous liaison. *IEEE Signal Processing Magazine*, 23(4):70–83, 2006.