Self-Sustainable OFDM Transmissions with Smooth Energy Delivery

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***Abstract*—Transmitting energy and information simultaneously using orthogonal frequency-division multiplexing (OFDM) to obtain receiver energy self-sufficiency has recently been proposed. To obtain smooth energy delivery, this project analyzes a new architecture for self-sustainable OFDM transmissions that reduces the peak-to-average power ratio (PAPR) in the cyclic prefix through a frame-theoretic operation that modifies the erasure pattern selection technique. Numerical analysis demonstrates this new approach outperforms prior self-sustainable OFDM techniques in terms of PAPR while maintaining nearly the same self-sustainability.**

***Index Terms*—energy harvesting, frame theory, green communications, OFDM, self-sustainability**

I. Introduction

Due to energy constraints faced by low-power communication receivers, there has been growing interest in transmitting information along with the energy needed to process it [1], [2]. Though information-theoretic limits assume receiver circuitry can exploit all information and energy in a signal simultaneously, whether considering frequency-selective wireless [3] or wireline [4] channels, this is not possible in current practice.

Traditional OFDM systems discard the cyclic prefix (CP) at the receiver, but this is energetically wasteful. Recent work proposes using the CP for energy and the remainder of the received signal for information (as usual) [5]–[7]. This is called *self-sustainable OFDM* and has straightforward circuit implementations, though prior work essentially assumes an infinite receiver battery.

OFDM, however, suffers from large peak-to-average power ratio (PAPR) [8], which is particularly problematic for energy harvesting where smooth energy delivery is required to ensure no energy outage [9]. If the receiver has limited battery, bursts of energy cannot be buffered for later use in times of low energy content; this battery-size limitation has been widely noted in energy-harvesting communication, see [10] for a review that also discusses certain practical circuit implementation issues.

This leads to the following research question that has not been addressed in the literature and is important, e.g. for application to the internet of things [11].

Research Problem 1: Is it possible to have small PAPR *in the cyclic prefix* of an OFDM signal, while maintaining self-sustainability?

Many methods have been proposed to reduce PAPR in OFDM signals [8], but as far as we know, there are no methods for reducing PAPR in the cyclic prefix specifically. As such, we propose a technique that uses a frame expansion to add redundancy to the data and then erases some of the resulting signal to yield the lowest PAPR in the CP. The receiver reconstructs erased samples by standard frame-theoretic methods [12]. This approach is a modification to the erasure pattern selection (EPS) technique developed for the typical OFDM PAPR reduction problem [13]. Our new approach, which we call EPS-CP, is analyzed in terms of the performance metrics important for self-sustainable OFDM.

This project demonstrates that it is possible to significantly reduce PAPR of the CP in self-sustainable OFDM, without much loss in self-sustainability via EPS-CP. Note that instead of considering the mean power of the signal [5], we consider the actual power realizations of each transmission.

II. Self-Sustainable OFDM Transmission

For conventional self-sustainable OFDM transmission, we follow the notation in [5]. Self-sustainable transmissions require the following constraints.

$$\beta q^H q - \delta P_d \geq 0 \tag{1}$$

$$\frac{1}{\mu}\sum_{i=1}^{N} p_i \leq P_M \tag{2}$$

$$p_i \geq 0, \text{ for all } i \in [1, N] \tag{3}$$

$$L \geq \ell, \tag{4}$$

with $\mu = N/(N+L)$ and $\delta \in \mathbb{R}_0^+$ as an adaptive scaling factor that flexibly models energy-harvesting requirements, whether full ($\delta \geq 1$) or partial ($\delta < 1$) self-sustainability. This is roughly the model of [5], but we consider the actual power realizations of the received signal rather than the mean power.

Conventional OFDM transmissions of this type suffer from large peak-to-average power ratio (PAPR) [8], [14]; the next section discusses an approach to reduce PAPR of the CP.

III. Reducing PAPR of Cyclic Prefix

Let us first define the PAPR of the CP, using the L-length vector $X = QAF^{-1}s = [X_1, \ldots, X_L]^T \in \mathbb{C}^L$ which is CP of the transmit signal. The PAPR of the CP is:

$$\text{PAPR}_{\text{CP}} = \frac{\max_{1\leq k\leq L} |X_k|^2}{\mathbb{E}\left[|X_k|^2\right]}. \tag{5}$$

A. Erasure Pattern Selection for Cyclic Prefix (EPS-CP)

To reduce PAPR_{CP}, we will use our EPS-CP method which is inspired by prior work on PAPR control [13] and draws on frame theory [12]. In particular, we select the pattern which gives the least PAPR_{CP} of the transmit signal.

The EPS-CP method can be divided into transmission and reception parts. At the transmitter, a K-element input vector is expanded into an N-element vector by introducing $N-K$ zeros (which can be placed anywhere). Then, the N-element vector is applied to an IDFT operation denoted by matrix F^H, the Hermitian transpose of DFT matrix F. The cascade of data expansion and IDFT is called the *DFT frame*.

Multiplying a K-element vector with an (N, K) DFT frame is equivalent to inserting zeros in the locations of the $N-K$ columns erased from the DFT matrix, and then multiplying the resulting N-length vector with $N \times N$ matrix F^H. The elements of the expanded N-length vector that are zeros are referred to as syndrome coefficients.

The IDFT output is subjected to P different erasure patterns $E_1, E_2, \ldots, E_P$ to reduce the redundancy a little bit (remaining redundancy is used for error correction). Each erasure pattern produces a different OFDM symbol, and the one yielding the least PAPR_{CP} is chosen. We should always equally space the erasure pattern indices to get minimum reconstruction error [12]. Increasing the number N_E of erasures in each pattern reduces PAPR, but perhaps at the cost of error probability.

We demonstrate below that EPS-CP is effective in reducing PAPR_{CP}, much like EPS is for OFDM signals [13].

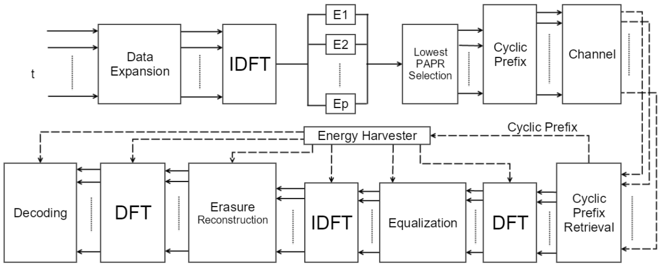

Fig. 1. A self-sustainable OFDM system with EPS-CP for $\mathrm{PAPR_{CP}}$ reduction.

IV. New Self-sustainable OFDM Architecture

We construct a novel self-sustainable OFDM architecture with reduced $\mathrm{PAPR_{CP}}$ according to the EPS-CP method, see Fig. 1. Let the K-length input vector $t = [t_1, \ldots, t_K]^T$ have covariance matrix $P_e = \mathrm{diag}(p_e) = \mathbb{E}[tt^H] \in \mathbb{R}^{K \times K}$, where $p_e = [p_{e,1}, \ldots, p_{e,k}]^T$ is a K-length vector carrying the mean power; we use water-filling. We get the expanding vector $s = [s_1, \ldots, s_N]^T$ by frame expansion. The reduced $\mathrm{PAPR_{CP}}$ vector $e = [e_1, \ldots, e_{N-N_E}]^T$ is obtained from the expanding vector s by using the operation of lowest $\mathrm{PAPR_{CP}}$ selection. Then, we can write the transmitted signal at the transmitter as

$$x_e = [x_{e,1}, \ldots, x_{e,N-N_E+L}]^T = A_e e \in \mathbb{C}^{N-N_E+L}, \quad (6)$$

with CP insertion matrix

$$A_e = \begin{bmatrix} 0_{L,N-N_E-L} & I_L \\ I_{N-N_E} & \end{bmatrix} \in \mathbb{R}^{(N-N_E+L)\times(N-N_E)}. \quad (7)$$

Like [5], h is the channel vector and its effect can implemented with a Toeplitz matrix $H_e \in \mathbb{C}^{(N-N_E+L)\times(N-N_E+L)}$:

$$H_e = \begin{bmatrix} h_0 & 0 & \ldots & 0 & h_{\ell-1} & h_{\ell-2} & \ldots & h_1 \\ h_1 & h_0 & 0 & \ldots & 0 & h_{\ell-1} & \ldots & h_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & 0 & h_{\ell-1} & h_{\ell-2} & \ldots & h_1 & h_0 \end{bmatrix}. \quad (8)$$

The received signal at the receiver is

$$y_R = H_e x_e + n_e \in \mathbb{C}^{N-N_E+L}, \quad (9)$$

where $n_e = [n_{e,1}, \ldots, n_{e,N-N_E+L}]^T \sim \mathcal{N}(0, \sigma^2 I_{N-N_E+L})$ is additive white Gaussian noise with variance σ^2. The CP of the received signal will be retrieved and used for energy harvesting. The $(N-N_E)$-length vector of samples $r_e \in \mathbb{C}^N$ can be written as

$$r_e = B_e y_R = B_e H_e A_e e + n'_e, \quad (10)$$

where the CP removing matrix $B_e = \begin{bmatrix} 0_{(N-N_E)\times L} & I_{N-N_E} \end{bmatrix} \in \mathbb{R}^{(N-N_E)\times(N-N_E+L)}$. Also, n'_e is the noise vector after removing first L elements. Then, the subsequent DFT yields the frequency domain representation of received symbols

$$r_{e,F} = F r_e = F B_e H_e A_e e + F n'_e \in \mathbb{C}^{N-N_E}. \quad (11)$$

Notice that $F \in \mathbb{C}^{(N-N_E)\times(N-N_E)}$ is the DFT matrix

$$F_{(k+1,\ell+1)} = \frac{1}{\sqrt{N-N_E}} e^{-i2\pi \frac{k\ell}{N-N_E}} \text{ for } k, \ell = 0, \ldots, N-N_E-1, \quad (12)$$

which is different from Sec. II.

For one-tap equalization, $r_{e,F}$ can be written as [15]:

$$r_{e,F,n} = \sqrt{N-N_E}\tilde{h}_n \tilde{e}_n + \tilde{w}_n \text{ for } n = 1, \ldots, N-N_E. \quad (13)$$

Here, $\tilde{h} = [\tilde{h}_1, \ldots, \tilde{h}_{N-N_E}]$ is the $(N-N_E)$-point DFT of $h' = [h_0, \ldots, h_\ell, 0, 0, \ldots, 0]$ which is the channel vector h appended with $N - N_E - \ell - 1$ zeros. Also, $\tilde{e} = [\tilde{e}_1, \ldots, \tilde{e}_{N-N_E}]$ is the $(N-N_E)$-point DFT of the transmitted reduced $\mathrm{PAPR_{CP}}$ vector $e = [e_1, \ldots, e_{N-N_E}]^T$ and $\tilde{w} = [\tilde{w}_1, \ldots, \tilde{w}_{N-N_E}]$ is the resulting Gaussian noise vector. After channel equalization, the reduced $\mathrm{PAPR_{CP}}$ vector e is obtained by applying the IDFT matrix on $\tilde{e}$. To reconstruct the erased samples at the transmitter, we use frame-theoretic means in. Then, we reconstruct the expanding vector by using the known erasure pattern. Finally, the original input signal is obtained by the syndrome coefficients and decoder. The erased samples can be reconstructed by frame-theoretic means.

Now, let us focus on the retrieved CP, $q_e = Q_e y_R = Q_e H_e A_e e \in \mathbb{C}^L$, where matrix $Q_e = \begin{bmatrix} I_L & 0_{L,N-N_E} \end{bmatrix} \in \mathbb{R}^{L\times(N-N_E+L)}$ is the CP retrieval matrix. We model self-sustainability constraints with $\mathrm{PAPR_{CP}}$ reduction as:

$$\beta q_e^H q_e - \delta P'_d \geq 0 \quad (14)$$

$$\frac{1}{\epsilon} \sum_{i=1}^{k} p_{e,i} \leq P_M \quad (15)$$

$$p_{e,i} \geq 0, \text{ for all } i \in [1, k] \quad (16)$$

$$L \geq \ell, \quad (17)$$

where $\epsilon = (N-N_E)/(N-N_E+L)$. Also, let P'_d be the power consumed for digital signal processing at the receiver of the new architecture.

Note the difference in power constraint here in (15) as compared to (2). Notably there are different parameters $\mu = N/(N+L)$ and $\epsilon = (N-N_E)/(N-N_E+L)$ respectively, but the same transmit power constraint P_M. Of course, ϵ is less than μ due to N_E. This implies the new system has lower power budget at the transmitter. As we will see, $\mathrm{PAPR_{CP}}$ of this new architecture is however reduced significantly.

V. System Performance

Performance of the proposed scheme is assessed by a set of simulations that follow the framework of [5]. We let $N = 256$ and $L \in \{18, 19, \ldots, 64\}$ at the transmitter, and we let $h = [h_0, \ldots, h_\ell] \sim \mathcal{N}(0, I_{\ell+1}/(\ell+1))$ be the frequency-selective Rayleigh fading channel vector of size $\ell + 1$ taps, and take $L = \ell$. The efficiency of energy harvesting is taken to be $\beta = 0.5$. According to realistic implementations [5], we consider $P_M \in [4.5, 5]$ W and $P'_d = P_d = 500$mW.[1]

As a baseline for comparison, we consider conventional self-sustainable OFDM [5] with $N = 256$. For our approach with $\mathrm{PAPR_{CP}}$ reduction, we consider $K = 128$, and expansion to the N-element vector. Also, $P = 8$ erasure patterns were

[1]Note that the only operational difference between a conventional OFDM receiver and the proposed EPS-CP-based receiver is reconstructing erased samples. Due to energy-intensive operations such as symbol timing estimation, packet detection, CFO compensation, and channel equalization [16], the additional energy is negligible. Thus we let $P'_d \approx P_d$.

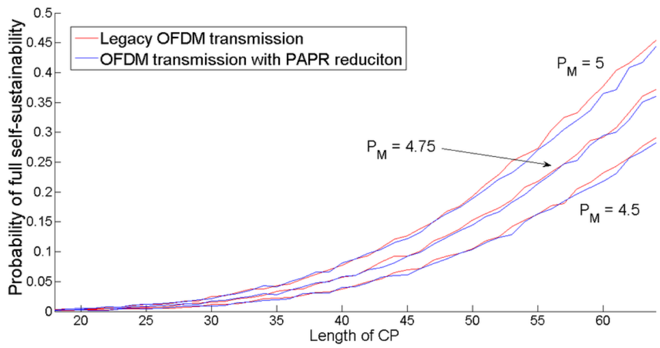

Fig. 2. Probability of full self-sustainability.

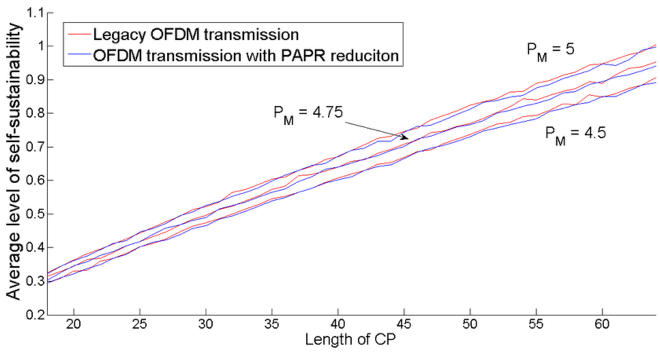

Fig. 3. Average level of self-sustainability.

used and each pattern composed of $N_E = 17$ erasures. Other parameters are the same as the baseline conventional system.

Let us compute Π_F, the probability of full self-sustainability, i.e. $\delta \geq 1$. Performance of the new system is only slightly worse than the conventional system, due to the difference between μ and ϵ noted above. If we focus on average behavior of self-sustainability δ, we obtain Fig. 3 for $N = 256$. Likewise, there is little loss in performance by the new system as compared to baseline.

Now let us demonstrate that introducing EPS-CP into self-sustainable OFDM does indeed reduce $\mathrm{PAPR_{CP}}$ significantly. Because the EPS-CP method is probabilistic, we consider the average of each transmission's $\mathrm{PAPR_{CP}}$. We also consider different numbers of erasures N_E with $P_M = 5$ and $L = 64$. To effectively reduce $\mathrm{PAPR_{CP}}$, we restrict attention to N_E values that yield at least 5 different erasure patterns. For these different systems which have different patterns, Fig. 4 shows Π_F as $\mathrm{PAPR_{CP}}$ changes. As we can see, the higher N_E we use, the lower $\mathrm{PAPR_{CP}}$ and Π_F we have. All the while, there is significant $\mathrm{PAPR_{CP}}$ improvement with little change in Π_F as compared to the conventional system.

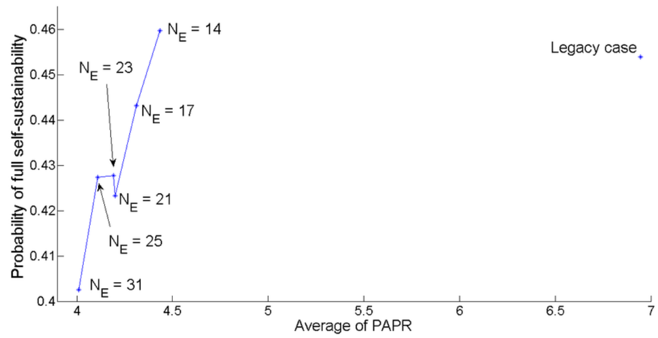

Fig. 4. Probability of full self-sustainability as a function of $\mathrm{PAPR_{CP}}$.

VI. Conclusion

In this project, we proposed a new system architecture that employs a frame-theoretic method (EPS-CP) and demonstrated it significantly improves PAPR of the CP in self-sustainable OFDM without much loss in other system parameters. In particular, we showed that for the same transmit power constraint, self-sustainability behavior of the new system is almost the same as in prior work, but $\mathrm{PAPR_{CP}}$ is greatly reduced because of the probabilistic erasure selection. A tradeoff between PAPR of the CP and self-sustainability is possible by varying the number of erasures. This robustness of energy delivery to energy smoothness requirements is contrary to information-theoretic limits which are severely impacted by energy smoothness requirements [9]; somehow the practical scheme is more robust than the fundamental limits.

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