IMPERIAL

JOINT MULTI-BAND DOA ESTIMATION USING LOW-RANK MATRIX RECOVERY*

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Signal Model

Consider S wideband signals from $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_S]^T$ impinging on a uniform linear array (ULA) of M identical and isotropic sensors located at $[0, d, \cdots, (M-1)d]^T$. The array output at the nth subband f_n is

$$\mathbf{Y}(f_n) = \mathbf{A}(f_n, \boldsymbol{\theta})\mathbf{C}(f_n) + \mathbf{W}(f_n). \tag{1}$$

where the steering matrix $\mathbf{A}(f_n, \boldsymbol{\theta})$ and the vector $\mathbf{a}(f_n, \theta_s)$ at f_n are

$$\mathbf{A}(f_n, \boldsymbol{\theta}) = [\mathbf{a}(f_n, \theta_1) \quad \mathbf{a}(f_n, \theta_2) \cdots \mathbf{a}(f_n, \theta_S)], \tag{2a}$$

$$\mathbf{a}(f_n, \theta_s) = [1 \quad e^{-j2\pi f_n \tau_1(\theta_s)} \cdots e^{-j2\pi f_n \tau_{M-1}(\theta_s)}]^T, \tag{2b}$$

where $\tau_m(\theta_s) = (md/v)\sin\theta_s$ denotes the time delay of the sth signal.

Literature and Challenges

- **Conventional subspace methods**: MUSIC (Schmidt, 1986), ESPRIT (Roy and Kailath, 1989), ISM (Wax, Shan, and Kailath, 1984), CSM (Wang and Kaveh, 1985), WAVES (Claudio and Parisi, 2001), and TOPS (Yoon, Kaplan, and McClellan, 2006);
- **Covariance-based methods**: quasi-stationary (Ma, Hsieh, and Chi, 2010) and sparse array (Shen et al., 2015);
- **Recent advanced methods**: atomic norm minimization (ANM) (Wang et al., 2021; Wu, Wakin, and Gerstoft, 2023).

Challenges

1. **Gridless and covariance-free** wideband DOA estimation problem;
2. **Jointly process multiple frequencies** for wideband signals;
3. **Nonlinearity of steering matrices** from multiple frequencies.

Joint Multi-Band Representation

To address this issue, we use the **greatest common divisor (GCD)** of involved frequencies to construct a **unified frequency grid (UFD)** $\mathbf{f}_{gd}$.

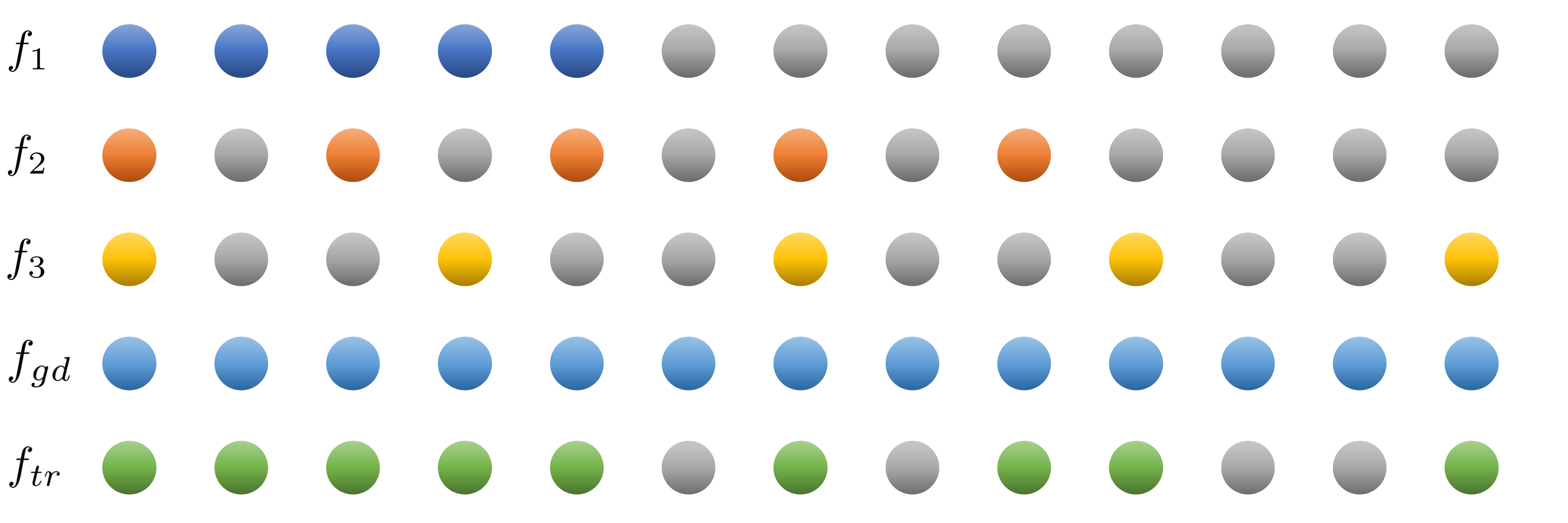


Fig. 1. A diagram of the unified frequency grid.

- **Definition**: The UFD is defined as a set of linear and uniformly spaced frequencies $\mathbf{f}_{gd}$, $f_n \subset \mathbf{f}_{gd}$ and $f_n = p_n \delta_f$, where $f_n, \delta_f \in \mathbb{Q}^+$, $p_n \in \mathbb{Z}^+$.

Let $\tilde{M} = 2p_N(M-1)$, the upsampled array output at f_n is

$$\mathbf{u}^{l,n} = \sum_{s=1}^{S} \tilde{\mathbf{a}}(f_n, \theta_s) c_s^l(f_n), \tag{3a}$$

$$\tilde{\mathbf{a}}(f_n, \theta_s) = [1 \quad e^{-j2\pi\delta_f \tau_1(\theta_s)} \cdots e^{-j2\pi\delta_f \tau_{\tilde{M}-1}(\theta_s)}]^T, \tag{3b}$$

where $\tilde{\mathbf{a}}(f_n, \theta_s)$ is an upsampling of $\mathbf{a}(f_n, \theta_s)$.

Low-Rank Hankel Matrix Recovery

- The UFD allows for formulating the joint multi-band DOA estimation problem as a low-rank Hankel matrix recovery problem.

Given $\mathbf{u}^{l,n}$, the truncated Hankel matrix is

$$\mathbf{H}_{I_r, I_r}^{u,l,n} = \sum_{s=1}^{S} \tilde{\mathbf{a}}_{I_r}(f_n, \theta_s) \tilde{\mathbf{a}}_{I_r}(f_n, \theta_s)^T c_s^l(f_n), \tag{4}$$

where I_r denotes the truncated index sets. $\mathbf{H}_{I_r, I_r}^{u,l,n}$ contains Hankel submatrices $\mathbf{H}_n^{u,l,n}$ at all subbands, with the minimum size.

Let $\mathbf{X} \in \mathbb{C}^{\tilde{M} \times LN}$, $\mathbf{X}_{:,(n-1)L+l} := \mathbf{u}_I^{l,n}$. $\mathcal{A}(\mathbf{X}) \in \mathbb{C}^{M \times L \times N}$ approximates the array output. $\mathcal{H}(\mathbf{X}) \in \mathbb{C}^{\bar{M} \times \bar{M}LN}$ forms the truncated Hankel matrices. The nonconvex optimization problem of low-rank matrix recovery is

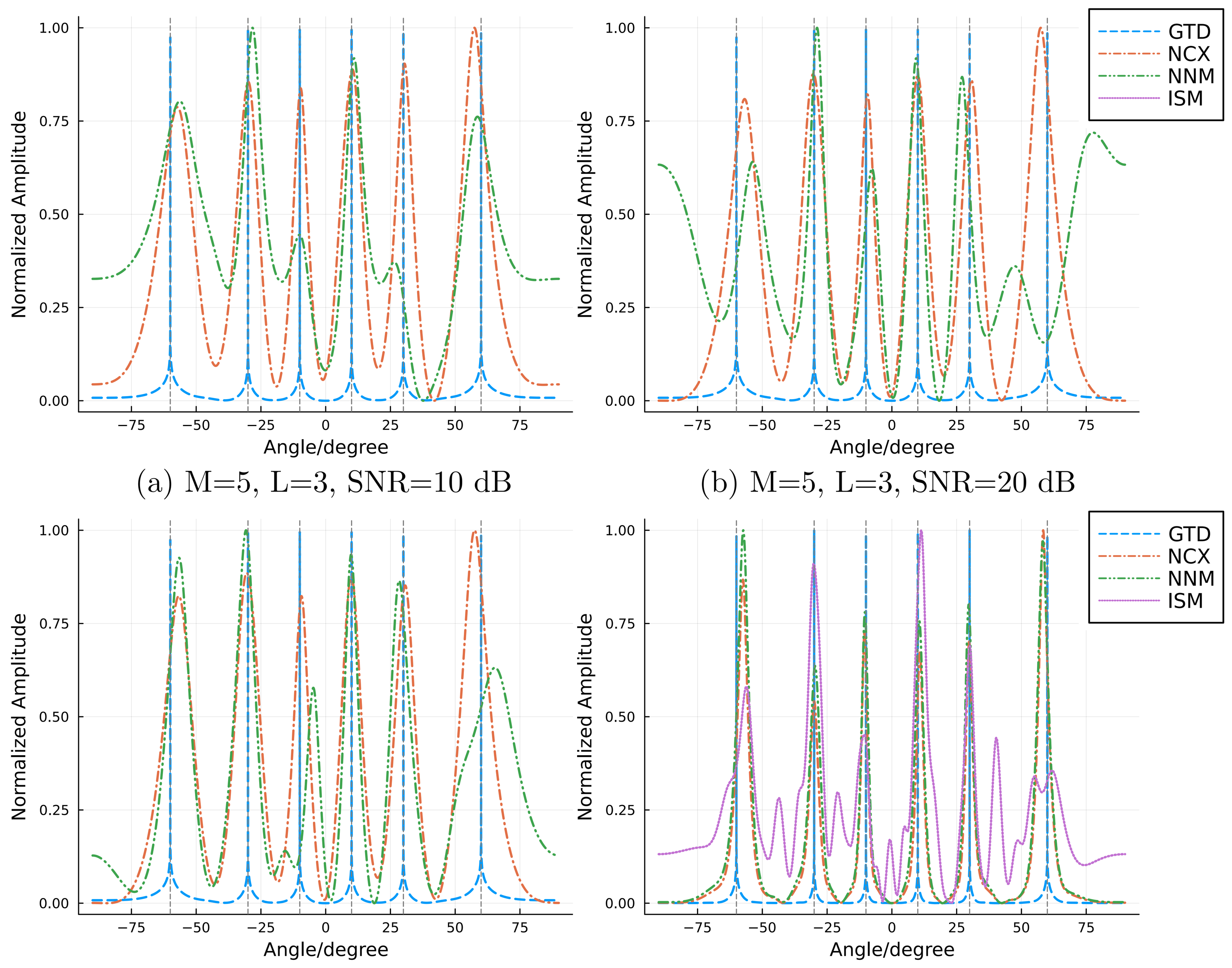
$$\min_{\mathbf{X}} \ \frac{1}{2} \| \mathbf{Y} - \mathcal{A}(\mathbf{X}) \|_F^2 + \frac{\beta}{2} \| \mathbf{X} \|_F^2 + \delta\big(\text{rank}(\mathcal{H}(\mathbf{X})) \le S\big), \tag{5}$$

where β is the regularization parameter, $\delta(\text{rank}(\cdot) \le S)$ denotes an indicator function, which makes the above problem nonconvex.

- To tackle the nonconvex optimization problem directly, an iterative algorithm is developed using **proximal gradient descent (PGD)**.

Numerical Results

- Experimental set-up: a ULA of M = 5, S = 6, N = 3, L = 3.



(a) M=5, L=3, SNR=10 dB
(b) M=5, L=3, SNR=20 dB
(c) M=5, L=3, SNR=30 dB
(d) M=12, L=3, SNR=10 dB

Fig. 2. Simulation results of a comparative study.

Conclusions

1. Recover **more source angles** than the number of sensors in a ULA;
2. Gridless and covariance-free DOA estimation with **a few snapshots**;
3. Average RMSE performance is better than that of the ISM and NNM.

Acknowledgements

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