Hardware Impairments-Aware Design of Noncoherent Grassmannian Constellations

D. Cuevas[1], J. Álvarez-Vizoso[1], M. Gutiérrez[1], I. Santamaría[1],
V. Tuček[2] and G. Peters[2]

[1]Universidad de Cantabria, Santander, Spain
[2]Huawei Technologies, Kista, Sweden

Grassmannian Constellations

- **Noncoherent Communications** → Neither CSIT nor CSIR
- **System Model**: M transmitter antennas, N receiver antennas, Rayleigh block-fading channel $\mathbf{H}$ with coherence time $T \geq 2M$ symbol periods, additive Gaussian noise $\mathbf{W}$, signal-to-noise ratio ρ

$$\mathbf{Y} = \mathbf{X}\mathbf{H} + \sqrt{\frac{M}{T\rho}}\,\mathbf{W}$$

- **MIMO Grassmannian Constellation**: TX codewords $\mathbf{X}_1, \ldots, \mathbf{X}_K \in \mathbb{C}^{T \times M}$ are Stiefel matrices, $\mathbf{A}^H\mathbf{A} = \mathbf{I}_M$, representing K subspaces in $\mathbb{G}(M, \mathbb{C}^T)$
- **Motivation**: Grassmannian constellations performance degradation due to hardware-impairments (HWIs) caused by non-idealities of RF transceivers
- **Goal**: Design robust Grassmannian constellations that account for HWIs [1]

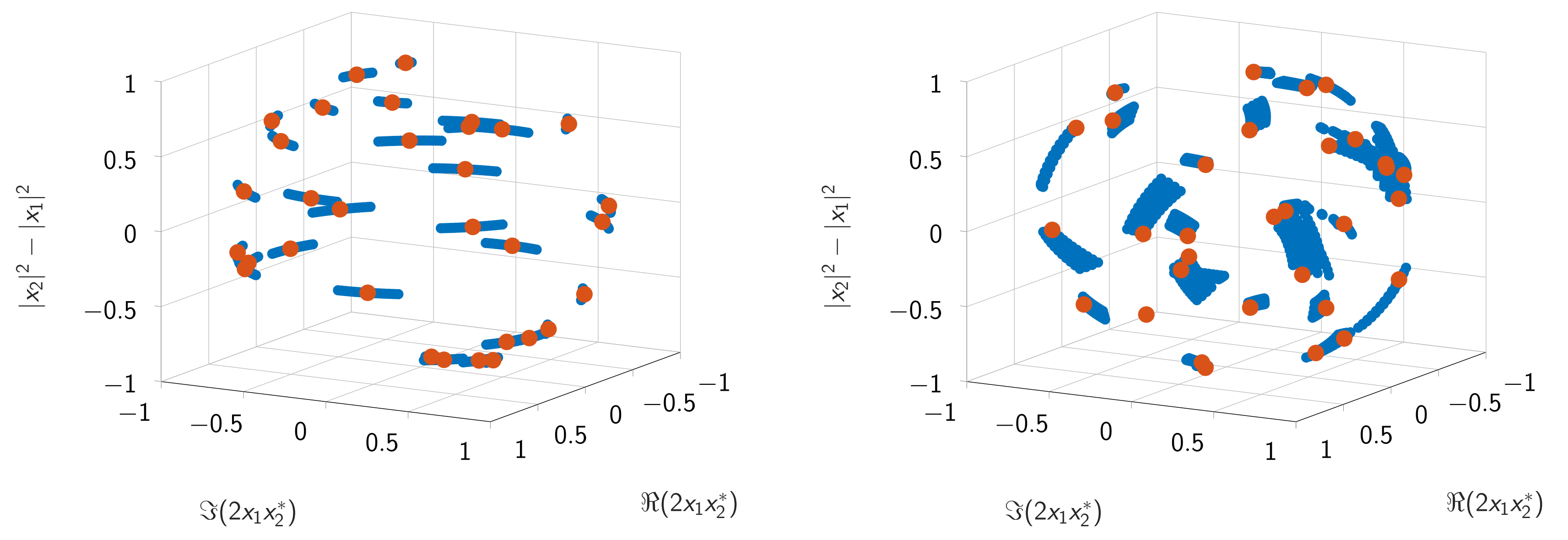
Robust Constellation Design

- We consider two types of HWIs that particularly affect noncoherent Grassmannian constellations:
  - Carrier frequency offset (CFO): slight shift between Tx and Rx carrier frequencies

$$\mathbf{F}(\Delta\omega) = \mathrm{diag}\left(1, e^{j\Delta\omega}, e^{j2\Delta\omega}, \ldots, e^{j(T-1)\Delta\omega}\right) \cdot \mathbf{X} \tag{1}$$

  - I/Q imbalance (IQI): phase difference of not exactly 90° and amplitude difference between I and Q branches

$$\mathbf{Z}(G,\theta) = \left(\frac{1 + Ge^{j\theta}}{2}\right)\mathbf{X} + \left(\frac{1 - Ge^{-j\theta}}{2}\right)\mathbf{X}^* \tag{2}$$



(a) CFO ($|\Delta\omega|_{max} = 10°$) (b) IQI ($G_{max} = 2$ dB, $\theta_{max} = 7.5°$)

- **HWIs robust algorithm**:
  - **Input**: Initial non-HWIs-aware constellation $\mathcal{C}_{ini} = \{\mathbf{X}_k\}_{k=1}^K$ designed to maximize the minimum diversity product [2]

$$\boxed{\underset{[\mathbf{X}_1],\ldots,[\mathbf{X}_K]}{\mathrm{argmax}} \quad \min_{k \neq j} \det\left(\mathbf{I}_M - \mathbf{X}_k^H\mathbf{X}_j\mathbf{X}_j^H\mathbf{X}_k\right)} \tag{3}$$

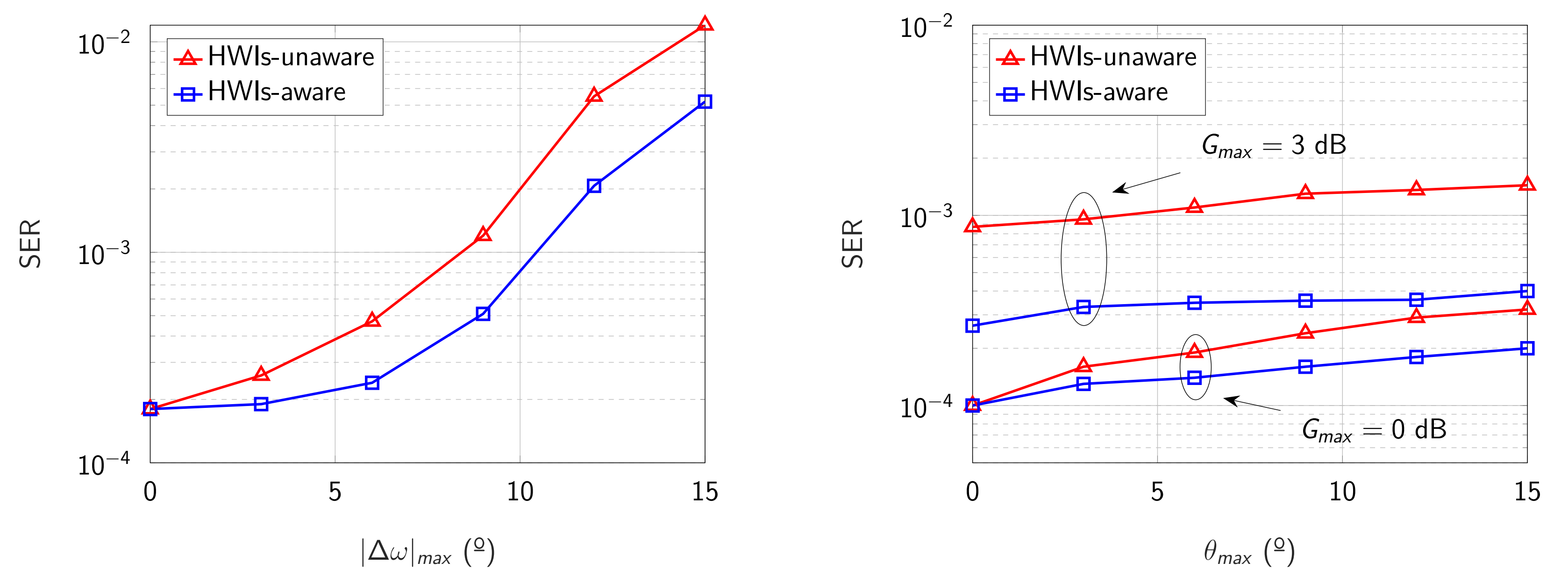
1. Robustness against CFO: for every codeword X_k
   1.1 Draw i.i.d. $\Delta\omega_p \sim U(-\Delta\omega_{max}, \Delta\omega_{max})$, $p = 1, \ldots, N_{CFO}$
   1.2 Generate a set of perturbed subspaces $\mathbf{F}_k(\Delta\omega_p)$, $p = 1, \ldots, N_{CFO}$ as in (1)
   1.3 Compute CFO-perturbed mean projection matrix

$$\mathbf{P}_{\mathbf{F}_k} = \frac{1}{N_{CFO}} \sum_{p=1}^{N_{CFO}} \mathbf{F}_k(\Delta\omega_p)\mathbf{F}_k(\Delta\omega_p)^H$$

   1.4 Substitute $\mathbf{X}_k \mapsto \mathbf{U}(:, 1:M)$, where $\mathbf{U\Sigma V}^H = \mathbf{P}_{\mathbf{F}_k}$
2. Robustness against IQI: for every codeword X_k
   2.1 Draw $G_n \sim U(0, G_{max})$ and $\theta_n \sim U(0, \theta_{max})$
   2.2 Substitute $\mathbf{X}_k \mapsto QR\left(\mathbf{Z}_k(G,\theta)\right)$, where $\mathbf{Z}_k(G,\theta)$ is the IQI-perturbed codeword from $\mathbf{X}_k$ given in (2)
3. Gradient ascent algorithm: CFO and/or IQI perturbed constellation is optimized according to the criterion in (3)

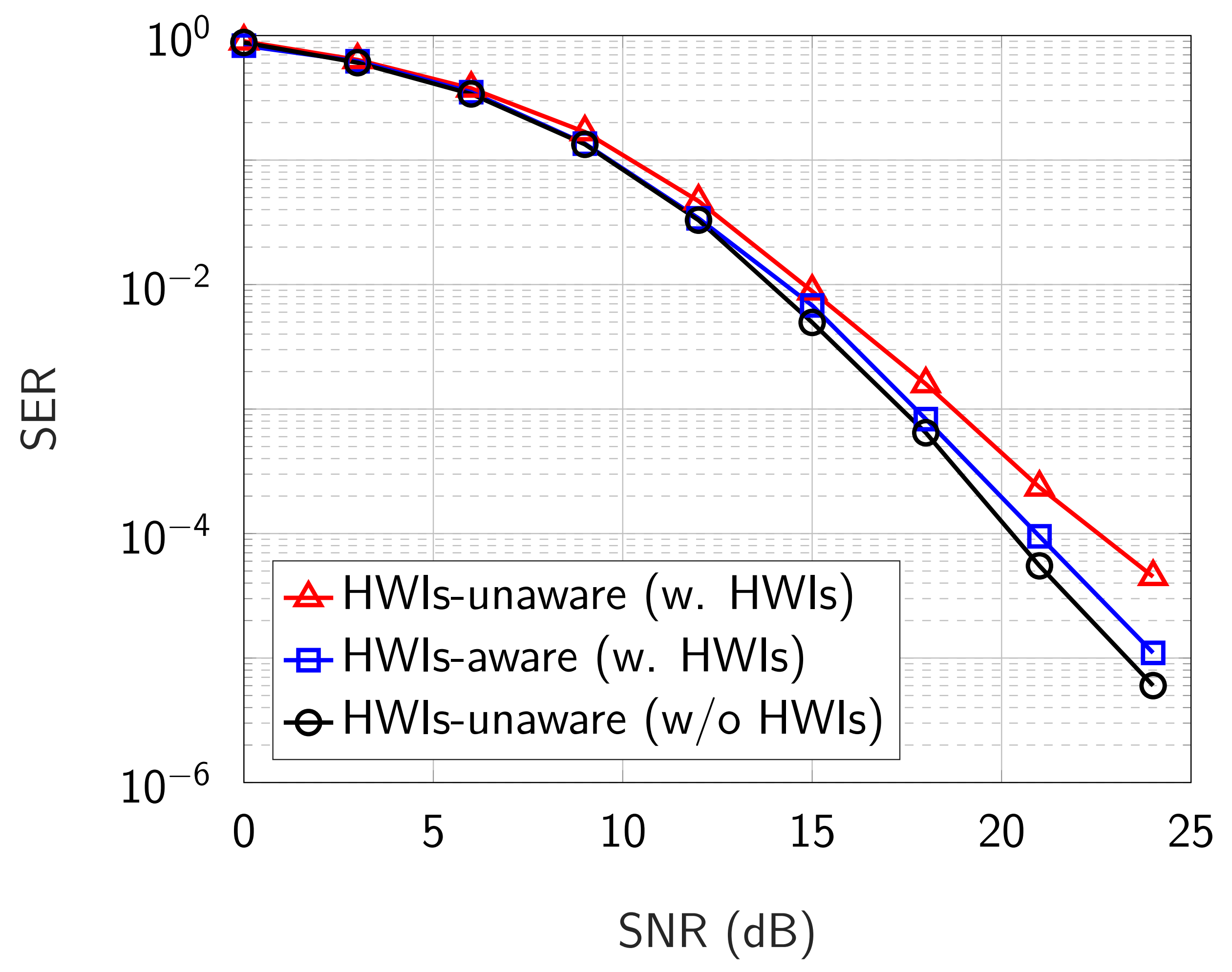
Simulation Results

- **SER vs. CFO & IQI values**: SNR = 20 dB, $K = 64$ codewords, $T = 4$ symbol periods, $M = 2$ antennas, $N = 2$ antennas

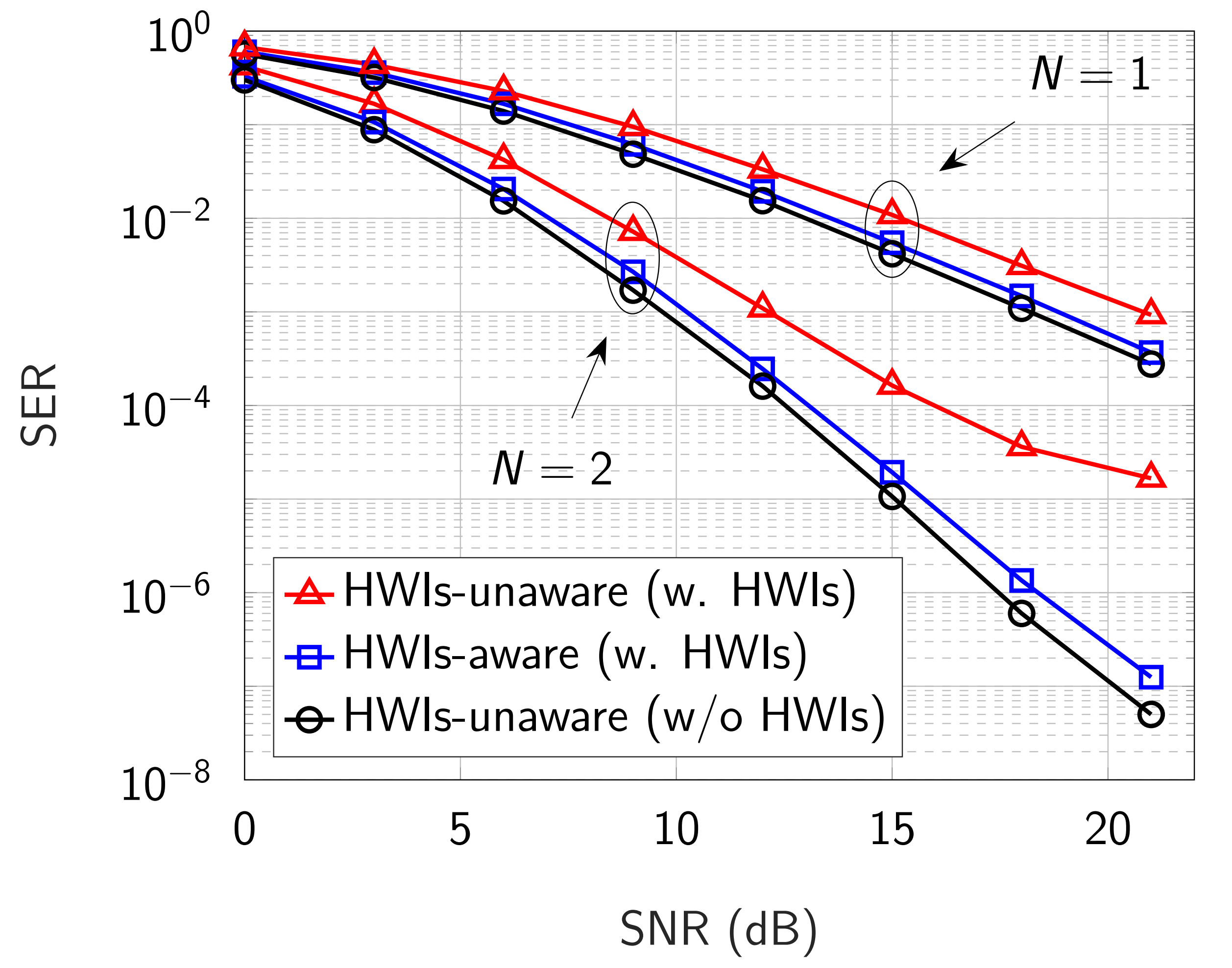


(a) $G_{max} = 0$ dB, $\theta_{max} = 0°$ (b) $|\Delta\omega|_{max} = 0°$

- **SER vs. SNR**: $|\Delta\omega|_{max} = 10°$, $G_{max} = 3$ dB, $\theta_{max} = 15°$, $K = 64$ codewords, $T = 4$ symbol periods, $M = 2$ antennas, $N = 2$ antennas



- **SER vs. SNR**: $|\Delta\omega|_{max} = 10°$, $G_{max} = 3$ dB, $\theta_{max} = 15°$, $K = 64$ codewords, $T = 8$ symbol periods, $M = 2$ antennas, $N \in \{1, 2\}$ antennas



References

[1] D. Cuevas, J. Álvarez-Vizoso, M. Gutiérrez, I. Santamaría, V. Tuček and G. Peters,“Hardware impairments-aware design of noncoherent grassmannian constellations,” in *2024 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2024)*, (Seoul, Korea), Apr. 2024.

[2] J. Álvarez-Vizoso, D. Cuevas, C. Beltrán, I. Santamaría, V. Tuček and G. Peters, “Coherence-based subspace packings for MIMO noncoherent communications,” in *2022 30th European Signal Processing Conference (EUSIPCO 2022)*, (Belgrade, Serbia), Sept. 2022.

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