Detector Design for Distributed Multichannel Radar Sensors in Colored Interference Environments

Moein Ahmadi, Mohammad Alaee-Kerahroodi, Linlong Wu, Bhavani Shankar M. R., and Björn Ottersten

Interdisciplinary Centre for Security, Reliability and Trust (SnT), University of Luxembourg

Introduction

In this paper, we present a generic signal model applicable to various distributed radar setups, encompassing both phased array (PA) and MIMO radar configurations. We consider a range of waveform modulation methods, including TDM, BPM, DDM, and fast time CDM. We devise a GLRT based detector for scenarios where the interference consists of colored noise plus a signal in a low-rank subspace and prove that the designed detector is CFAR. We demonstrate that when the CPI time is similar for the systems, the PA radar system exhibits better detection performance than MIMO, irrespective of the waveform modulation approach adopted. However, if the CPI time of the PA system is divided to the number of transmit waveforms utilized in the MIMO radar case (to account for the time needed for a PA radar to scan all angles), then in the presence of non-uniform interference, MIMO techniques, except TDM, surpass the performance of PA. Conversely, in cases of uniform interference, the performance of both MIMO techniques and PA are equivalent.

Signal Model

The received signal from pulses in one Coherent Processing Interval (CPI)

$$\mathbf{x}_q = \sum_{i=1}^{I_q} \alpha_{q,i}\mathbf{s}_q(\theta_{q,i}, \phi_{q,i}, f_{d,q,i})$$

The signal steering vector

$$\mathbf{s}_q(\theta,\phi,f_d) = \mathbf{u}_q(\theta,\phi,f_d) \otimes \mathbf{b}_q(\theta,\phi)$$

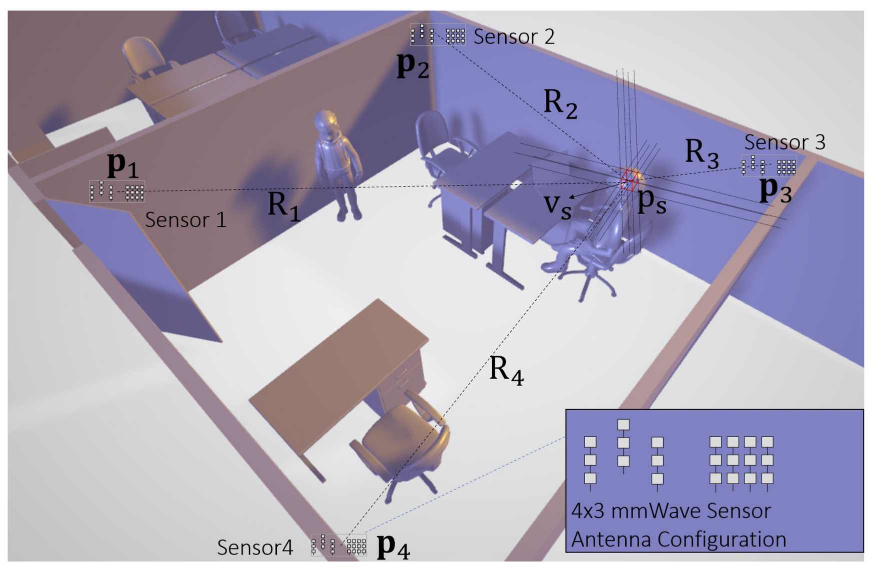
The transmit-Doppler steering vector

For CDM radars (we call it herein single pulse waveform multiplexing technique)

$$\mathbf{u}_q^S(\theta,\phi,f_d) = \mathbf{a}_q(\theta,\phi) \otimes \mathbf{d}_q(f_d) \in \mathbb{C}^{M_q L_q}$$

For TDM, BPM, and DDM-MIMO techniques (we call it herein multiple pulse waveform multiplexing techniques) and for the PA radars

$$\mathbf{u}_q^M(\theta,\phi,f_d) = (\mathbf{W}_q\mathbf{a}_q(\theta,\phi)) \odot \mathbf{d}_q(f_d) \in \mathbb{C}^{L_q}$$



At the fusion center, the received signals can be stacked:

$$\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, ..., \mathbf{x}_Q^T]^T$$

Multichannel Radar Techniques

$$\mathbf{W}_q^{TDM} = \underbrace{[\mathbf{I}_{M_q}, \mathbf{I}_{M_q}, \cdots, \mathbf{I}_{M_q}]^T}_{K|L_q=KM_q} \qquad \mathbf{W}_q^{BPM} = \underbrace{[\mathbf{H}_{M_q}, \mathbf{H}_{M_q}, \cdots, \mathbf{H}_{M_q}]^T}_{K|L_q=KM_q}$$

$$\mathbf{W}_{q,l,m}^{DDM} = e^{-j2\pi ml/M_q} \qquad \mathbf{W}_{q,l,m}^{DDM_2} = e^{-j2\pi ml/L_q}$$

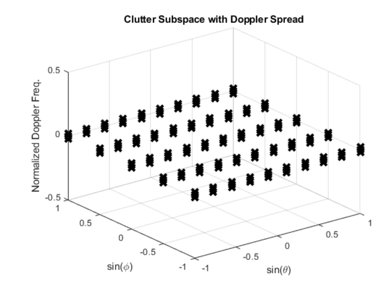
$$\mathbf{W}_{q,l,m}^{PA,(\theta_0,\phi_0)} = e^{j\mathbf{k}^T(\theta_0,\phi_0)\mathbf{p}_{q,m}^{tx}}$$

PA precoding matrix elements are independent from pulse index

Detector

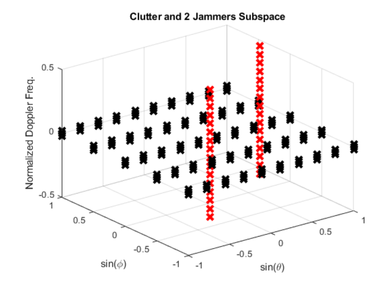
- **Signal Model and Detection Problem**

$$\begin{cases} \mathcal{H}_0: & \mathbf{x} = \mathbf{A}\mathbf{g} + \mathbf{n} \\ \mathcal{H}_1: & \mathbf{x} = \mathbf{S}\boldsymbol{\alpha} + \mathbf{A}\mathbf{g} + \mathbf{n} \end{cases} \qquad \mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{M})$$



Generalized likelihood Ratio Test:

$$t_{\text{GLRT}} = \ln \frac{\max_{\mathbf{g},\boldsymbol{\alpha}} p(\mathbf{x}|\mathcal{H}_1)}{\max_{\mathbf{g}} p(\mathbf{x}|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$



ML estimations of unknown parameters:

$$\hat{\mathbf{g}}_1 = (\mathbf{A}^H\mathbf{M}^{-1}\mathbf{A})^{-1}\mathbf{A}^H\mathbf{M}^{-1}(\mathbf{x} - \mathbf{S}\boldsymbol{\alpha})$$

$$\hat{\mathbf{g}}_0 = (\mathbf{A}^H\mathbf{M}^{-1}\mathbf{A})^{-1}\mathbf{A}^H\mathbf{M}^{-1}\mathbf{x}$$

$$\hat{\boldsymbol{\alpha}} = (\mathbf{S}^H\mathbf{P}_{\mathbf{M},\mathbf{A}}^{\perp}\mathbf{S})^{-1}\mathbf{S}^H\mathbf{P}_{\mathbf{M},\mathbf{A}}^{\perp}\mathbf{x}$$

$$\mathbf{P}_{\mathbf{M},\mathbf{A}}^{\perp} = \mathbf{M}^{-1} - \mathbf{M}^{-1}\mathbf{A}(\mathbf{A}^H\mathbf{M}^{-1}\mathbf{A})^{-1}\mathbf{A}^H\mathbf{M}^{-1}$$

Proposed detector:

$$t_{\text{GLRT}} = \mathbf{x}^H\mathbf{P}_{\mathbf{M},\mathbf{A}}^{\perp}\mathbf{S}(\mathbf{S}^H\mathbf{P}_{\mathbf{M},\mathbf{A}}^{\perp}\mathbf{S})^{-1}\mathbf{S}^H\mathbf{P}_{\mathbf{M},\mathbf{A}}^{\perp}\mathbf{x} \underset{\text{H}_0}{\overset{\text{H}_1}{\gtrless}} \eta$$

Detector statistics:

$$t_{\text{GLRT},\mathcal{H}_0} \sim \chi_{2Q}^2(0) \qquad P_{fa} = \Pr(t_{\text{GLRT}} > \eta|\mathcal{H}_0) = e^{-\eta}\sum_{q=0}^{Q-1}\frac{\eta^q}{q!}$$

CFAR Detector ✓

$$t_{\text{GLRT},\mathcal{H}_1} \sim \chi_{2Q}^2(\boldsymbol{\alpha}^H\mathbf{S}^H\mathbf{P}_{\mathbf{M},\mathbf{A}}^{\perp}\mathbf{S}\boldsymbol{\alpha}) \qquad P_d = \Pr(t_{\text{GLRT}} > \eta|\mathcal{H}_1) = \mathcal{M}_Q\left(\sqrt{2\boldsymbol{\alpha}^H\mathbf{S}^H\mathbf{P}_{\mathbf{M},\mathbf{A}}^{\perp}\mathbf{S}\boldsymbol{\alpha}}, \sqrt{2\eta}\right)$$

Numerical Results

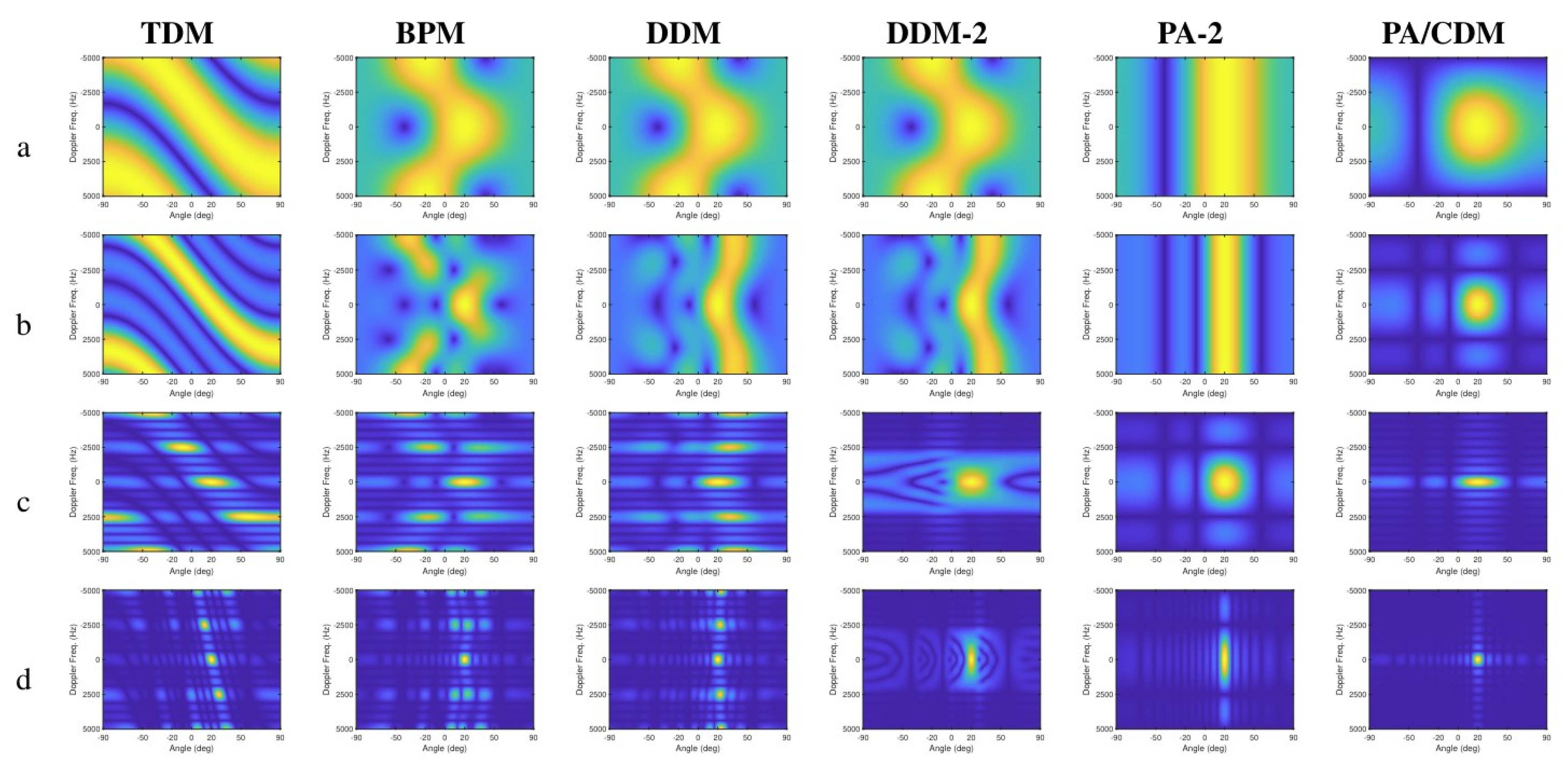


Fig. 1: Doppler angle beampattern for TDM, BPM, DDM, PA and CDM techniques ,a) $M = 2$, $N = 1$, $L = 2$, b) $M = 4$, $N = 1$, $L = 4$, c) $M = 4$, $N = 1$, $L = 16$, d) $M = 4$, $N = 4$, $L = 16$. PA-2 refers to the case when the same scan time for PA and MIMO has been considered, and consequently the CPI time for PA is divided to the number of transmit waveforms.

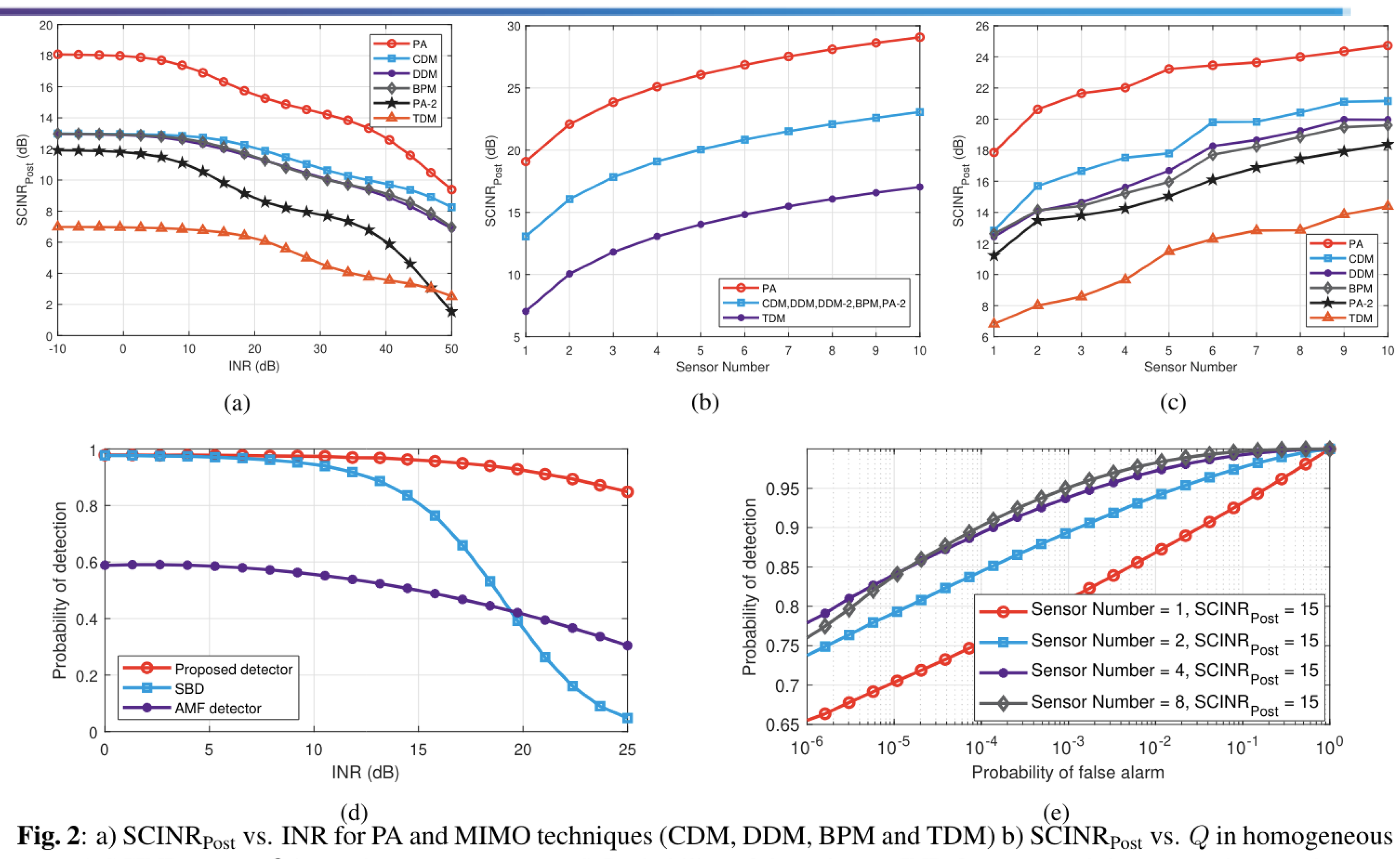


Fig. 2: a) $\text{SCINR}_{\text{Post}}$ vs. INR for PA and MIMO techniques (CDM, DDM, BPM and TDM) b) $\text{SCINR}_{\text{Post}}$ vs. Q in homogeneous env., c) $\text{SCINR}_{\text{Post}}$ vs. Q in nonhomogeneous env. d) Probability of detection vs INR for proposed detector, subspace detector [16] and AMF detector [19]. e) Receiver Operating Characteristic (ROC) for proposed detector and $Q = 1, 2, 4, 8$ distributed sensors with same $\text{SCINR}_{\text{Post}}$ and fluctuant RCS.

Conclusion

We introduced a generic signal model suitable for a variety of distributed radar systems, including both PA and MIMO configurations. Utilizing a GLRT based detector, our detection performance analysis revealed distinct operational characteristics under different interference scenarios. Specifically, PA outperformed MIMO when the CPI time was kept constant across setups. However, when CPI time of PA was divided to number of transmit waveforms in MIMO case, and in the presence of nonhomogeneous interference, MIMO techniques generally surpassed the PA radar except in TDM configurations. In homogeneous interference situations, both system types exhibited equivalent performance.

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