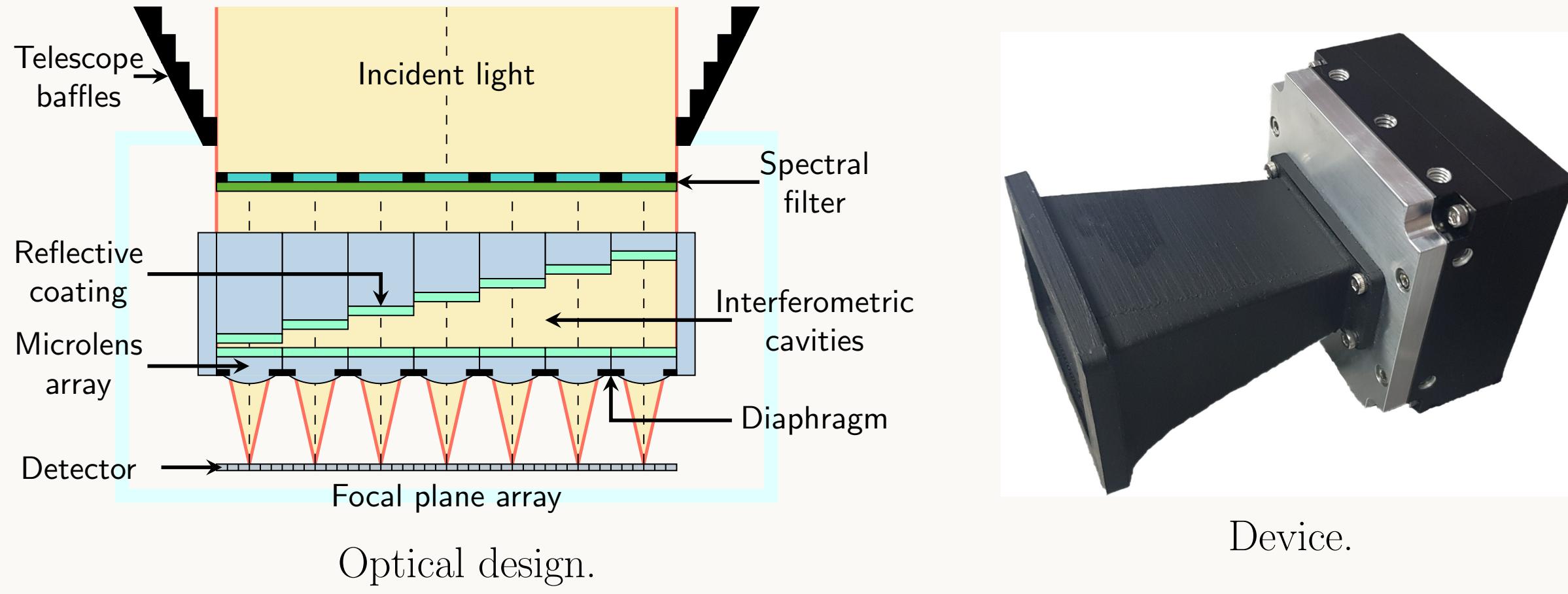


# Spectro-spatial hyperspectral image reconstruction from interferometric acquisitions

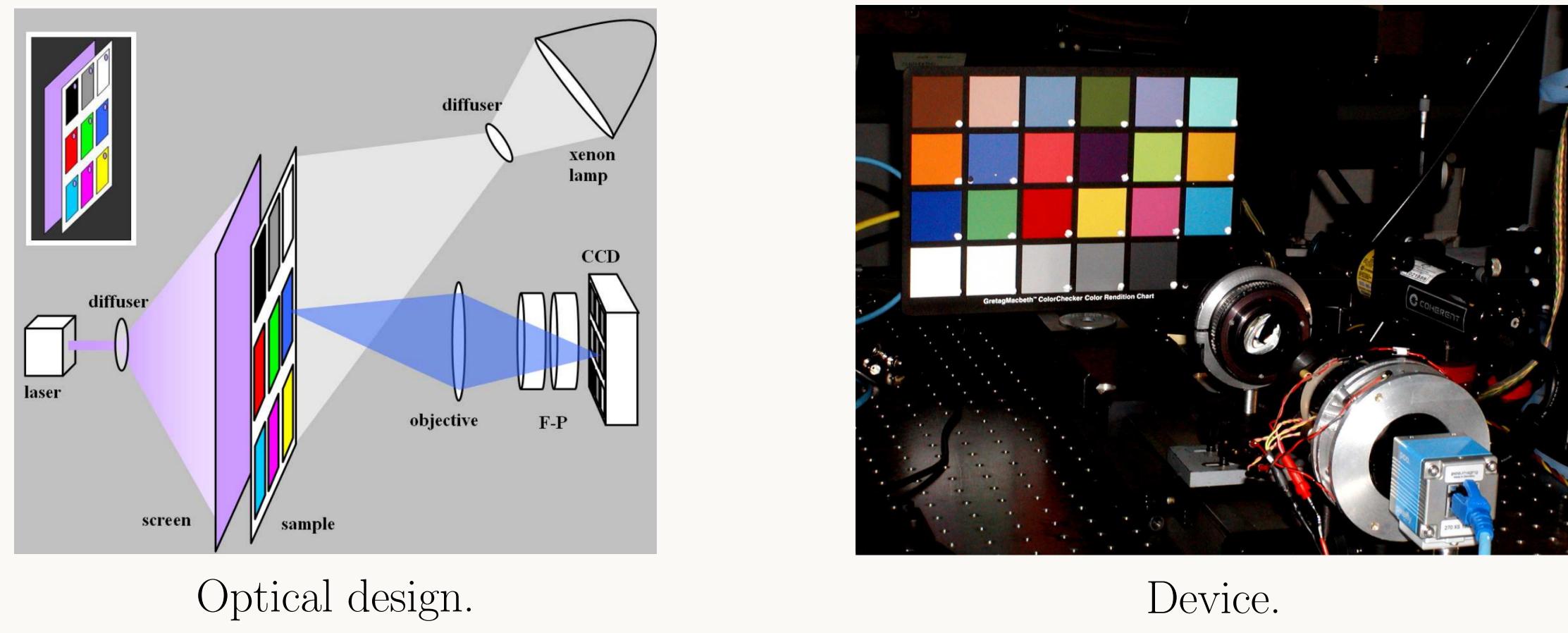
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## I. Interferometric imaging systems

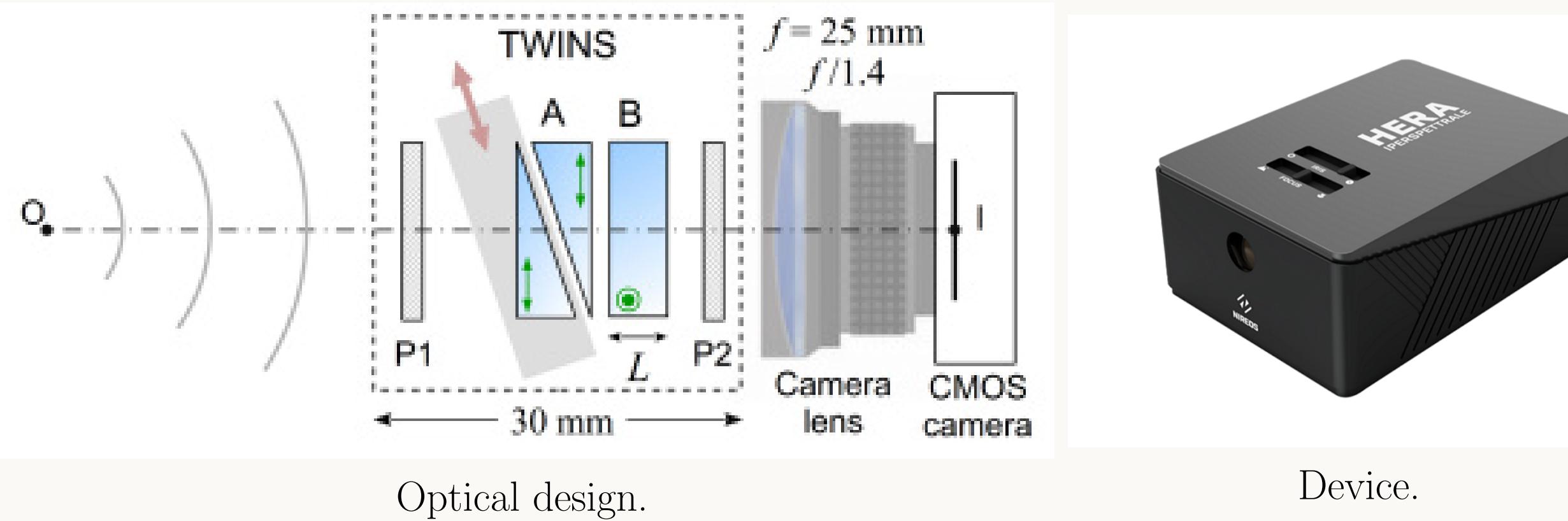
### Multi-aperture Fabry-Perot interferometers [1-2]



### Scanning Fabry-Perot interferometers [3]

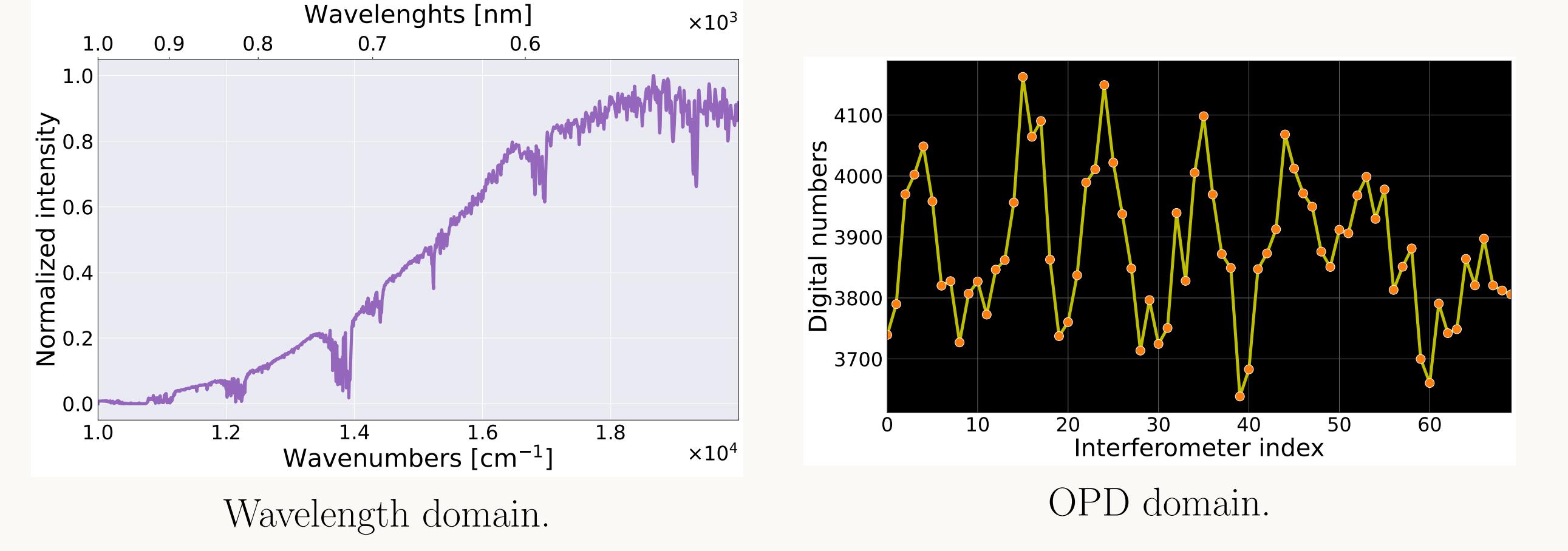


### Birifrangent interferometers [4]

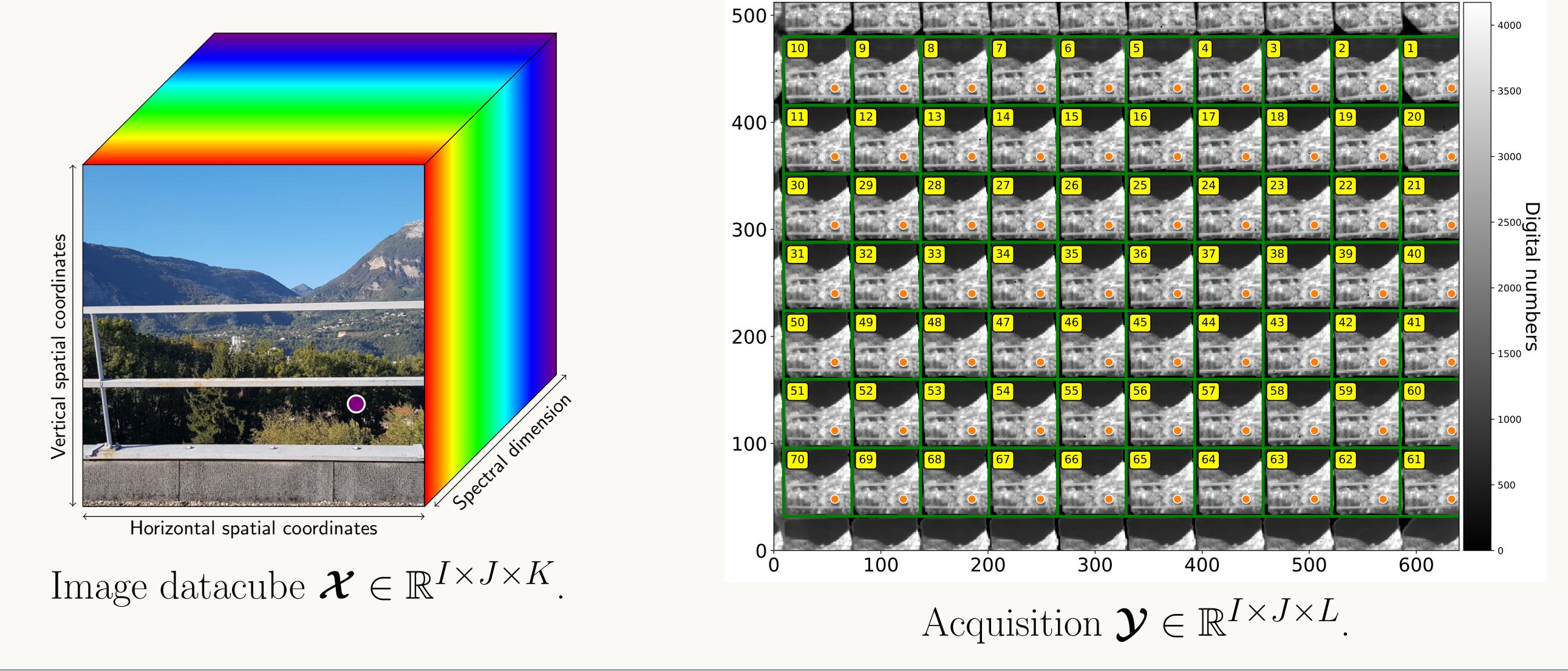


## II. Spectrometers example acquisitions

### Single pixel interferometric spectrometer



### Interferometric imaging spectrometer [5]



## III. Problem statement

**Objective:** Estimate  $\hat{\mathcal{X}} : \hat{\mathcal{X}} = \arg \min_{\mathcal{X}} \left( \frac{1}{2} \|\mathcal{X} \bullet_3 \mathbf{A} - \mathcal{Y}\|_2^2 + h(\mathbb{L}\mathcal{X}) \right)$ .

- $f(\mathcal{X}) = \frac{1}{2} \|\mathcal{X} \bullet_3 \mathbf{A} - \mathcal{Y}\|_2^2$ : data fidelity term.
- $g(\mathcal{X}) = h(\mathbb{L}\mathcal{X})$ : prior (with linear operator  $\mathbb{L}$ ).
- $\mathbf{A} \in \mathbb{R}^{K \times L}$ : device transmittance function.

## IV. Image reconstruction methodology

### ADMM

$$\begin{aligned} \mathcal{Z}^{[q+1]} &= \text{prox}_{\tau f} \left( \mathcal{X}^{[q]} - \mathcal{U}^{[q]} \right) \\ \mathcal{X}^{[q+1]} &= \text{prox}_{\tau g} \left( \mathcal{Z}^{[q+1]} + \mathcal{U}^{[q]} \right) \\ \mathcal{U}^{[q+1]} &= \mathcal{U}^{[q]} + \left( \mathcal{Z}^{[q+1]} - \mathcal{X}^{[q+1]} \right) \end{aligned}$$

- $\mathcal{X}^{[q]}$  is the estimation  $\hat{\mathcal{X}}$  at the  $q$ -th iteration (with  $\tau, \eta \in \mathbb{R}^+$ ).
- $\text{prox}_{\gamma f}(\mathcal{X}) = \arg \min_{\mathcal{V}} \left( f(\mathcal{V}) + \frac{1}{2\gamma} \|\mathcal{V} - \mathcal{X}\|_2^2 \right)$  is the proximal operator of  $f$ .
- $h^*(\mathcal{X}) = \sup_{\mathcal{V}} (\langle \mathcal{V}, \mathcal{X} \rangle - h(\mathcal{V}))$  is the Fenchel conjugate of  $h$ .

#### Approach 1: Spectro-spatial priors

- $g(\cdot)$  operates in both domains:
- $\mathbb{L}(\mathcal{X})$ : TV spatially, DCT spectrally.
  - $h(\mathcal{X}) = \|\cdot\|_{221}$  ( $\ell_2$  on gradients/bands,  $\ell_1$  on pixels).

### Loris-Verhoeven (LV)

$$\begin{aligned} \mathcal{Z}^{[q+1]} &= \mathcal{X}^{[q]} - \tau \left( \nabla_f \mathcal{X}^{[q]} + \mathbb{L}^T \mathcal{U}^{[q]} \right) \\ \mathcal{U}^{[q+1]} &= \text{prox}_{\eta h^*} \left( \mathcal{U}^{[q]} + \eta \mathbb{L} \mathcal{Z}^{[q+1]} \right) \\ \mathcal{X}^{[q+1]} &= \mathcal{X}^{[q]} - \tau \left( \nabla_f \mathcal{X}^{[q]} + \mathbb{L}^T \mathcal{U}^{[q+1]} \right) \end{aligned}$$

- Approach 2: Plug-and-Play [6]**
- $\text{prox}_{\tau g}$  is substituted with any denoiser, such as BM3D.
  - $g(\cdot)$  is not necessarily known.

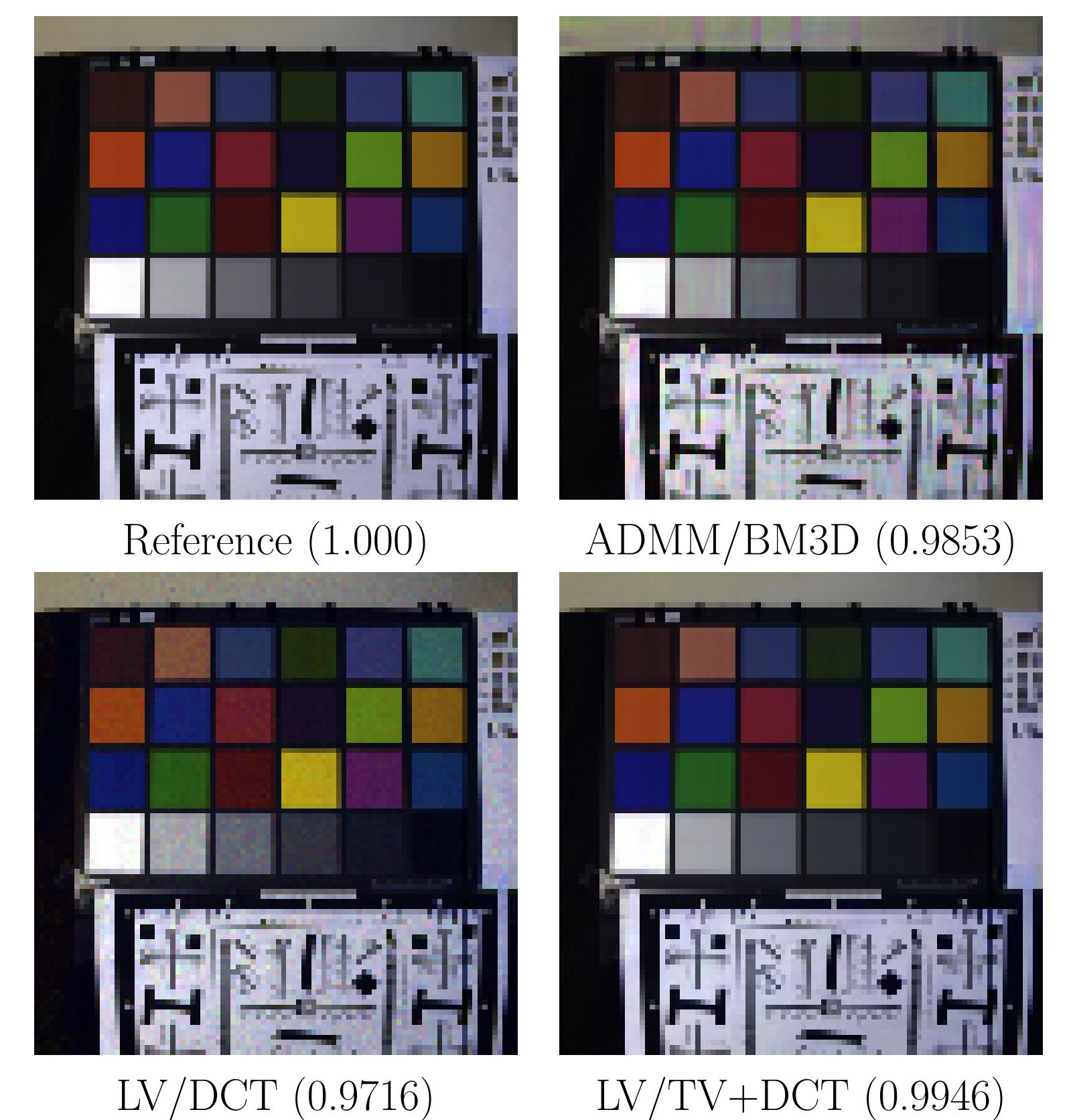
## V. Reconstruction experiments

### Setup

- Image to reconstruct:  
  - 96 × 96 px.
  - 366 channels.
  - Wavelengths: [400, 700] nm.
- Simulated model  $\mathbf{A} = \{a_{lk}\}$ :

$$a_{lk} = \frac{(1 - \mathcal{R})^2}{1 + \mathcal{R}^2 - 2\mathcal{R} \sin^2(\frac{\pi \delta_l}{\lambda_k})}.$$

- Reflectivity  $\mathcal{R} = 0.255$ .
- OPD range: [0, 8000] nm.
- OPD step size: 25 nm.
- SSIM values in parentheses.



## VI. References

- [1] Oiknine et al., "Multi-aperture snapshot compressive hyperspectral camera," Opt. Lett., vol. 43, no. 20, pp. 5042–5045, 2018.
- [2] Gousset et al., "NanoCarb hyperspectral sensor: On performance optimization and analysis for greenhouse gas monitoring from a constellation of small satellites," CEAS Space Journal, vol. 11, no. 4, pp. 507–524, 2019.
- [3] Pisani et al., "Compact imaging spectrometer combining Fourier transform spectroscopy with a Fabry-Perot interferometer," Opt. Express, vol. 17, no. 10, pp. 8319–8331, 2009.
- [4] Perri et al., "Hyperspectral imaging with a TWINS birefringent interferometer," Opt. Express, vol. 27, no. 11, p. 15956, 2019.
- [5] Picone et al., "Interferometer response characterization algorithm for multi-aperture Fabry-Perot imaging spectrometers," Opt. Express, vol. 31, no. 14, pp. 23066–23085, 2023.
- [6] Venkatakrishnan et al., "Plug-and-Play priors for model based reconstruction," in IEEE GlobalSIP, 2013.