DYNAMIC BANDWIDTH VARIATIONAL MODE DECOMPOSITION

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Introduction

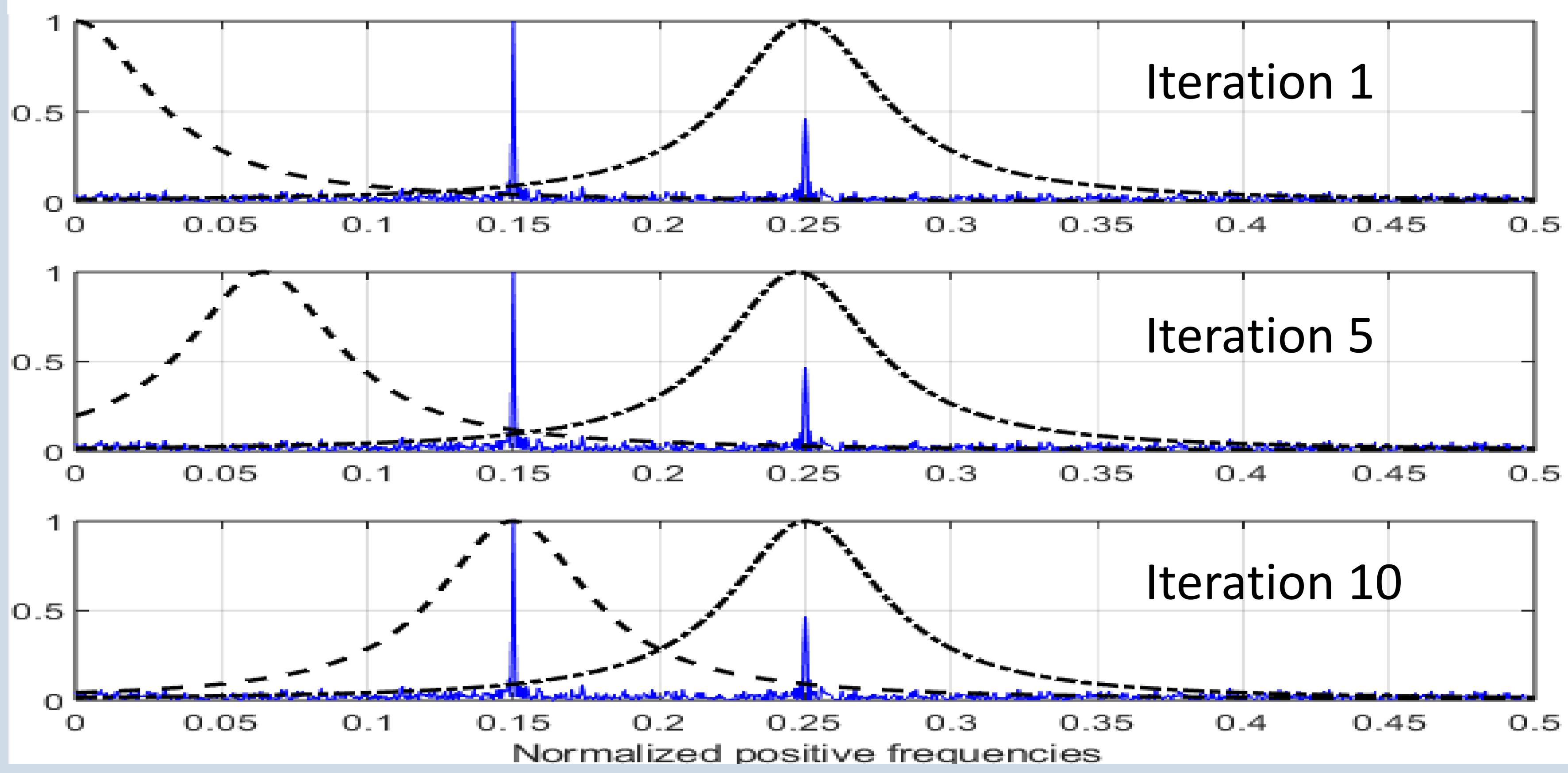
Variational Mode Decomposition (VMD) [1] is a well-known optimization-based signal decomposition method using constant-bandwidth Wiener filters to extract narrowband components from the input signal. However, limitations include constant bandwidth and the need for a predefined constituent count. We propose the **Dynamic Bandwidth VMD (DB-VMD)** to address the constant bandwidth limitation by enhancing the optimization problem with an additional constraint. Experiments on synthetic signals demonstrate DB-VMD's noise robustness and adaptability compared to VMD.

Background

Variational Mode Decomposition (VMD) breaks down an input signal into K narrowband oscillatory modes. VMD formulates an optimization problem that aims to minimize the modes' collective bandwidth subject to the reconstruction constraint. The quadratic penalty and the Lagrange multipliers are used and a parameter $\boldsymbol{\alpha}$ is introduced to tune the importance of minimum collective bandwidth. The Lagrangian is obtained

$$\mathcal{L} = \boldsymbol{\alpha} \sum_k BW_k + \left\| x - \sum_k u_k \right\|_2^2 + < \lambda, x - \sum_k u_k >$$

and the ADMM [2] is used to derive the algorithm of VMD. That is equivalent to introducing **constant-bandwidth** bandpass filters to the input's spectrum and updating their central frequencies.

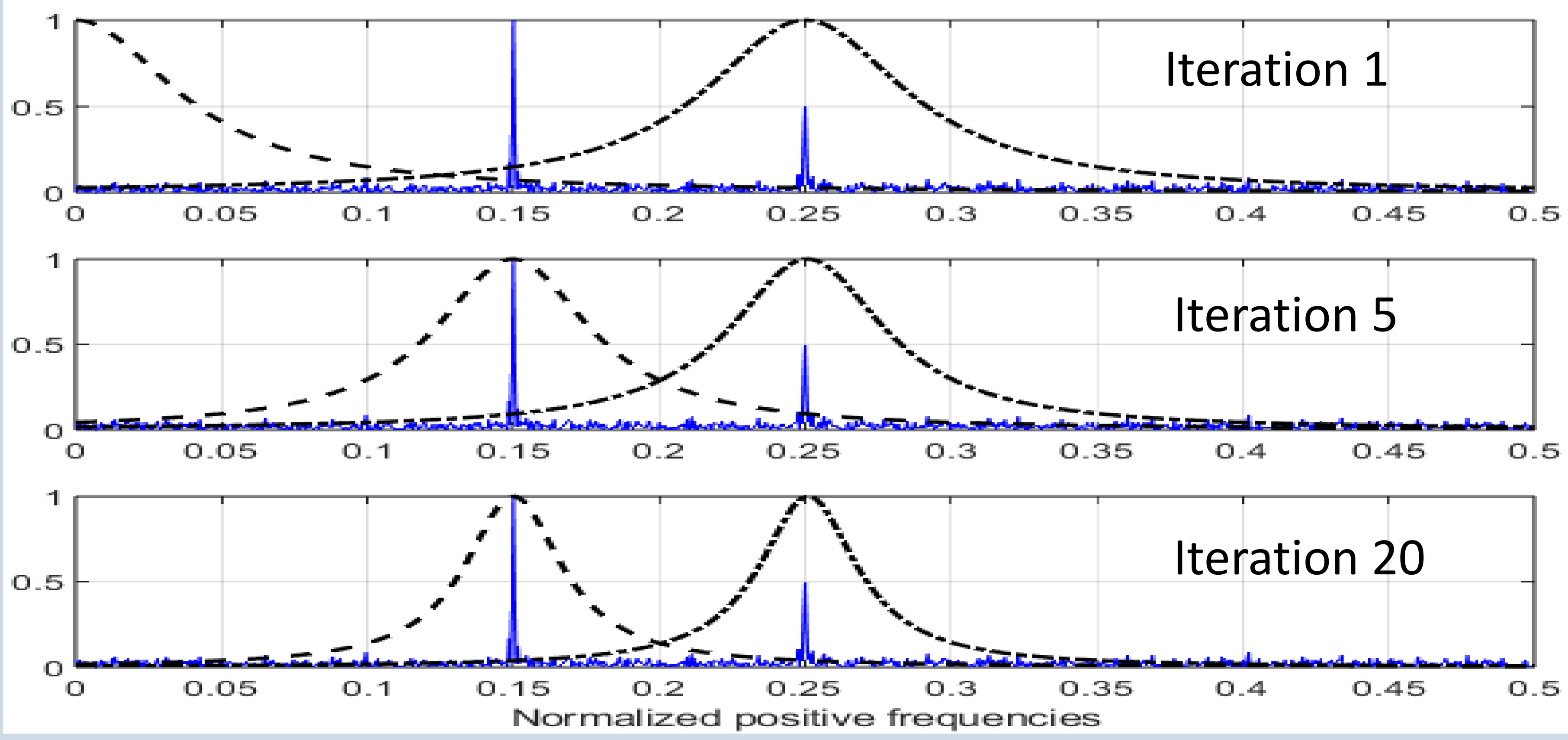


Proposed method

We propose the **Dynamic Bandwidth Variational Mode Decomposition (DB-VMD)** to generalize VMD by imposing an additional constraint to the optimization problem which bounds the collective bandwidth between two scalar parameters. The Lagrange multipliers are used to enforce the new constraint. The Lagrangian is obtained and the parameter $\boldsymbol{\alpha}$ is replaced by a function of the new Lagrange multipliers.

$$\mathcal{L} = (\mathbf{1} + \boldsymbol{\rho_\beta} - \boldsymbol{\rho_\alpha}) \sum_k BW_k + \left\| x - \sum_k u_k \right\|_2^2 + < \lambda, x - \sum_k u_k >$$

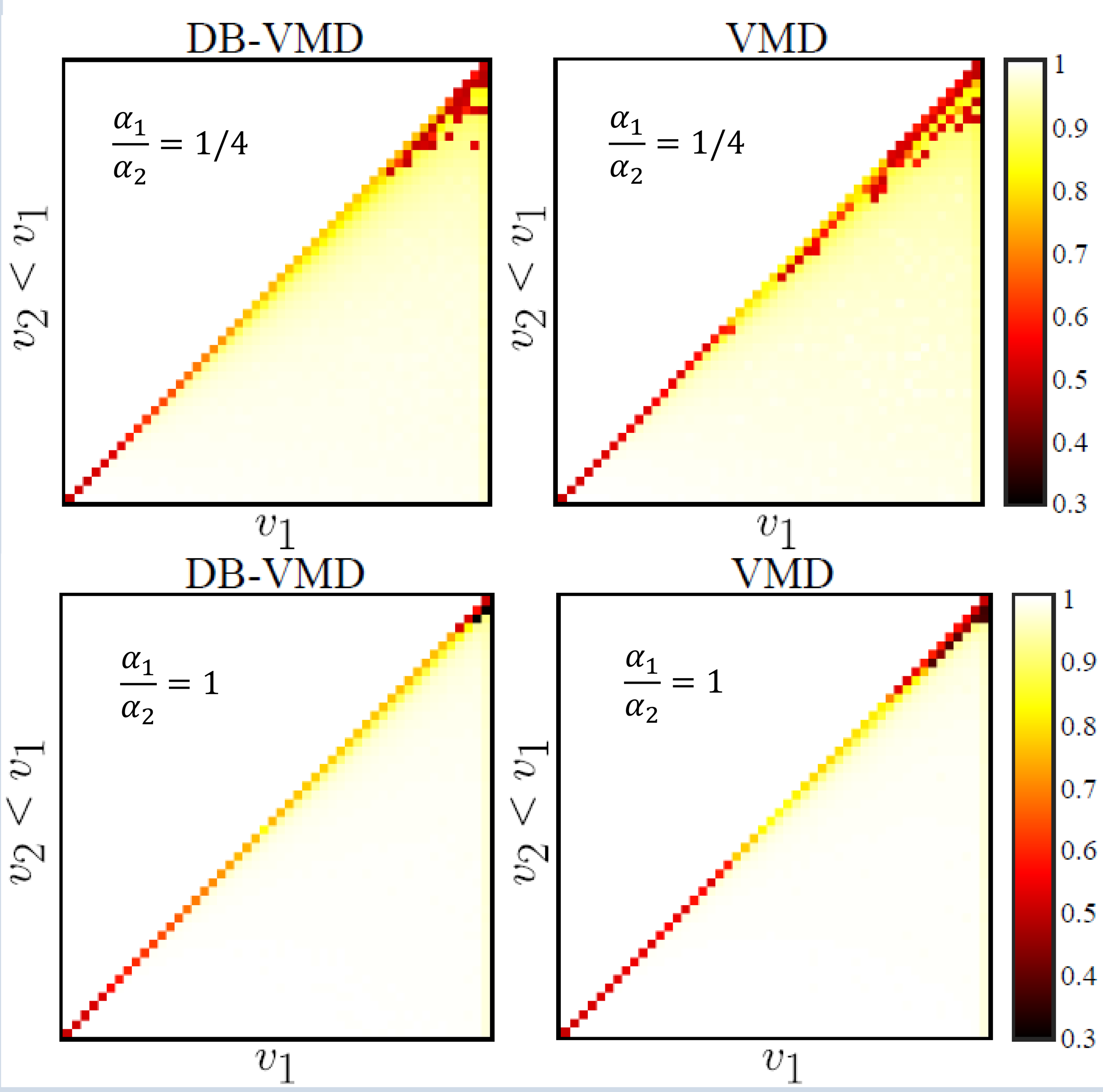
ADMM is also employed here to derive the DB-VMD algorithm. This process is equivalent to introducing **dynamic-bandwidth** bandpass filters to the input's spectrum and updating their central frequencies.



Experimental results

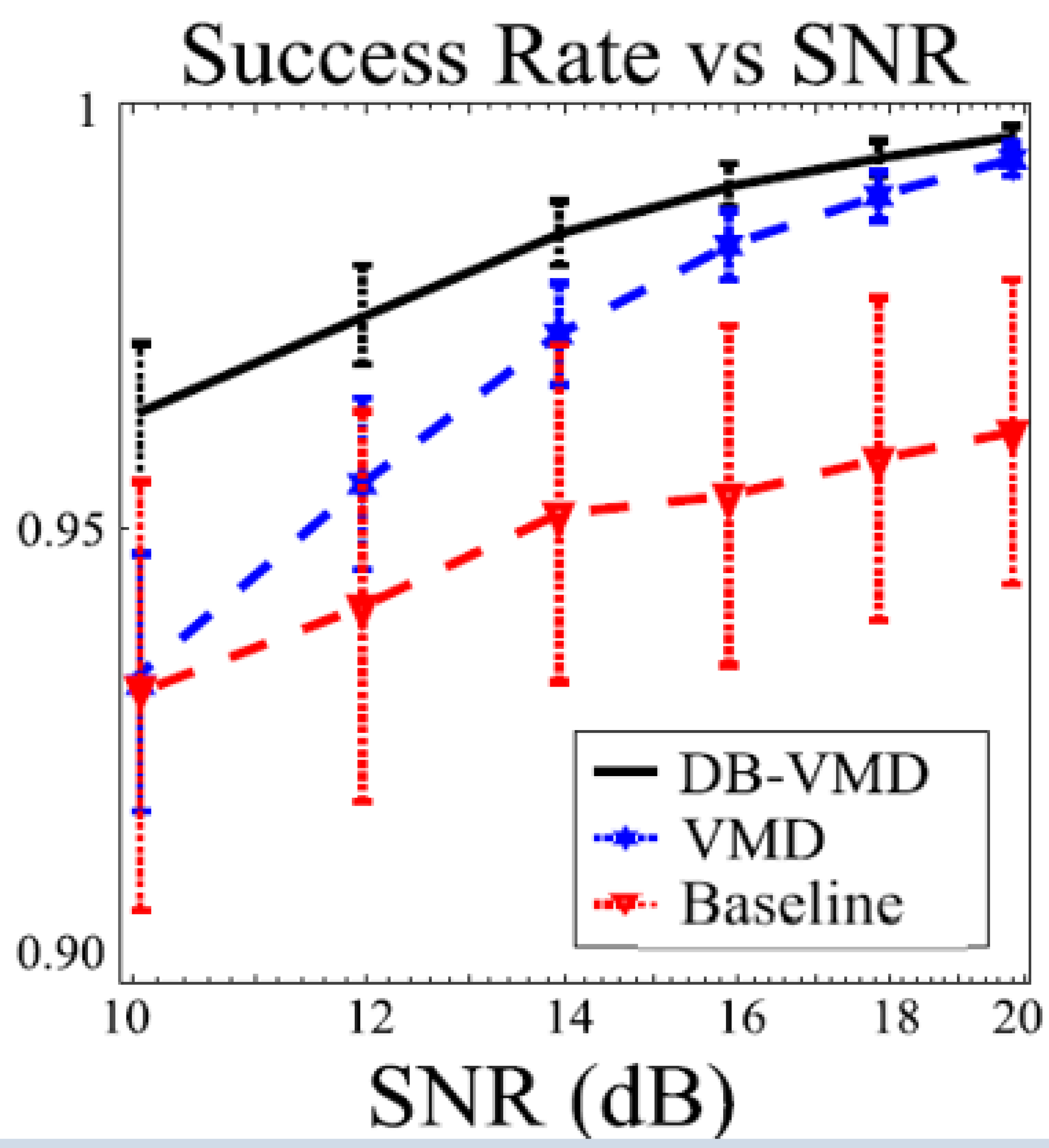
Tones Separation

$$x_{v_1,v_2}(n) = \alpha_1 \cos(2\pi v_1 n) + \alpha_2 \cos(2\pi v_2 n)$$



Noise Robustness

$$x(n) = \sum_k w_{L_k,d_k}(n) \cdot A_k \cdot \cos(\omega_k n) + \eta(n)$$



Key takeaways

Dynamic Bandwidth VMD

- generalizes VMD
- is more **adaptive** and **noise resilient** than VMD
- paves the way for applications with noise contaminated signals.

Future work

- Address the need for constituent count
- Extend to multivariate signal analysis
- Bound individual modes' bandwidth

[1] Dragomiretskiy, K., & Zosso, D. (2013). Variational mode decomposition. IEEE transactions on signal processing, 62(3), 531-544.

[2] Boyd, S., Parikh, N., Chu, E., Peleato, B., & Eckstein, J. (2011). Distributed optimization and statistical learning via the alternating direction method of multipliers. Foundations and Trends® in Machine learning, 3(1), 1-122.