NEURAL NETWORK-BASED SYMBOLIC REGRESSION FOR EMPIRICAL MODELING OF THE BEHAVIOR OF A PLANETARY GEARBOX

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Motivation

- Planetary gearbox condition monitoring and quality surveillance.
- Get a clear interpretation of the vibration spectrum contents.
- Identify an empirical model with an automatic fashion for a planetary gearbox vibration signal.
- Past practice: visually and manually inspecting the contents of the spectrum.

Introduction

- A single-stage planetary gearbox consists of a sun gear at the center that meshes with P planet gears placed at different angular positions around it.

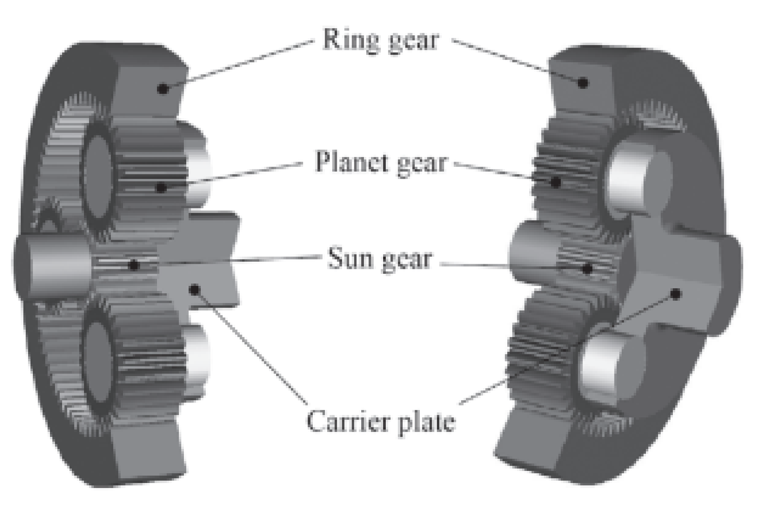


Fig. 1. A planetary gear transmission with several planets.

- Physics knowledge : Gear meshing generates vibration signals that interact with the rotational motion of rotary system elements.
- Modeling: Identify a nonlinear mixing function that accurately interprets the spectral content of $s(t)$ as follows:

$$s(t) = \sum_{p=1}^{P} F_p(s_{1,p}, s_{2,p}, s_{3,p}, s_{4,p})$$

where:

  - F_p are unknown mixture functions,
  - $s_{i,p}$ are the elementary vibration sources depends on the planet p angular position.
  - The source s_i are a T_i-periodic signal.

- Blind Source separation : Under stationary conditions, separate the individual contributions of vibration sources. Certain sources are linked with the system's gear rotation frequencies.
- Literature Overview: Previous works have investigated different scenarios of non-linear BSS (e.g., quadratic mixtures, polynomial mixtures) and post-linear BSS.
- Available data: Gearbox's shaft speed, Vibration measurements, gearbox model kinematic.

Multivariate regression

- We consider a multivariate regression problem with a training set $\{(\mathbf{D}_t, \mathbf{s}_t)\}_{1 \le t \le T/N}$ with $\mathbf{s}_t = [s((t-1)N+1), \cdots, s(tN)]^T$ (N being a processing window and T is the total sample size) and the dictionary $\mathbf{D}_t$ is given by $\mathbf{D}_t = [\mathbf{D}_{t,1}, \mathbf{D}_{t,2}, \mathbf{D}_{t,3}, \mathbf{D}_{t,4}]$, where $\mathbf{D}_{t,i}$ is formed by the columns of $\mathbf{D}_i$ of indices $k = (t-1)N+1, \cdots, tN$, defined as:

$$\mathbf{D}_i(:,k) = [\sin(2\pi f_i k), \cos(2\pi f_i k), \cdots, \sin(2\pi H_{max} f_i k), \cos(2\pi H_{max} f_i k)]^T \quad (1)$$

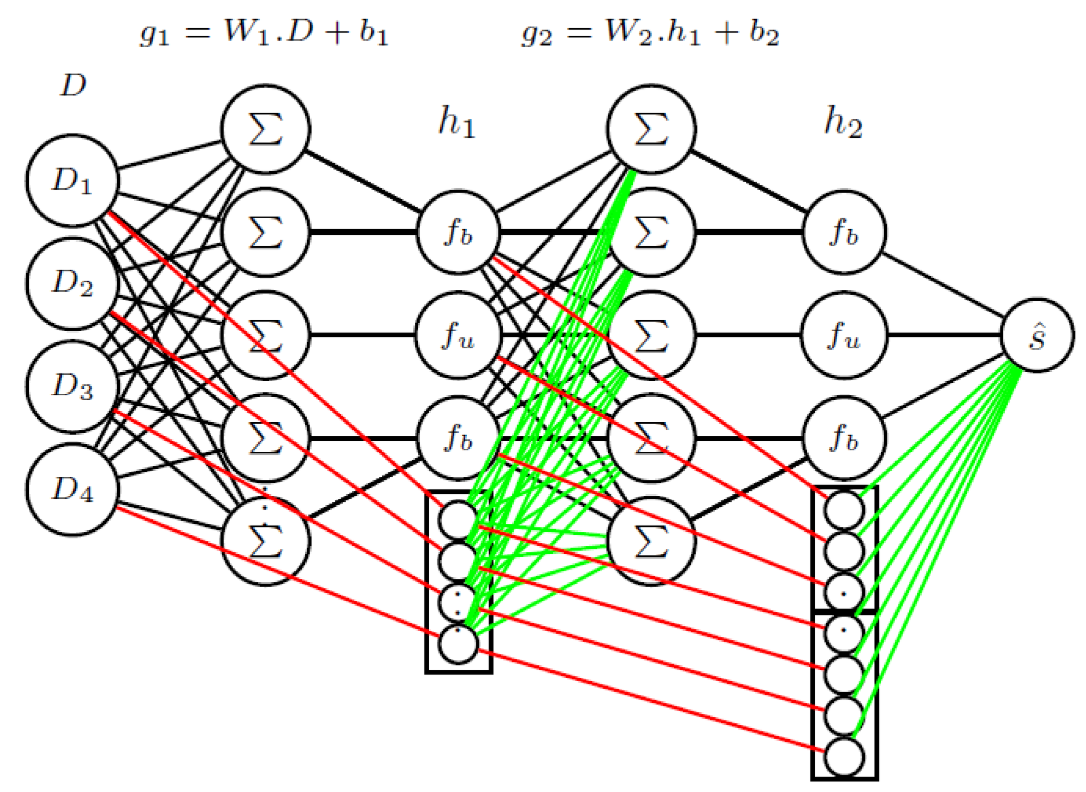
- The signal $\mathbf{s}$ originates from an unknown analytical function F of sources $\mathbf{s}_i$, corrupted by an additive zero-mean noise, ϵ, i.e

$$\mathbf{s} = F(\mathbf{s}_1, \cdots, \mathbf{s}_4) + \epsilon \quad (2)$$

- Task: Recover elementary sources $\mathbf{s}_i$ and learn a function ψ that approximates the true functional relation F.

Neural Network

- We suggest using the equation learner (EQL) architecture, which is based on a dense layer that incorporates some elements designed for Symbolic regression, encompassing both binary f_b and unary operators f_u.



- The red and green colors represent the skip connections.
- Objective function:

$$J(\theta) = \frac{1}{T} ||\psi_\theta(\mathbf{D}) - \mathbf{s}||^2 + \lambda R_q(\theta) \quad (3)$$

- R_q is the sparsity regularization applied to the network weights $\theta = \{\mathbf{W}, \mathbf{b}\}$ defined as:

$$R_q = \sum_{i=1}^{L+1} \sum_{j,k} |W^i_{j,k}|^q + \sum_{i=1}^{L+1} \sum_{j} |b^i_j|^q \quad (4)$$

- We use a smoothed version of the $\mathcal{L}^*_{0.5}$ that enforces sparsity.

Experiments

- For simulated data generation, we investigate the healthy case corresponding to $P = 1$.
- Normalized fundamental frequencies for the four sources in are set to $\{f_1 = 0.045, f_2 = 0.016, f_3 = 0.023, f_4 = 0.0013\}$.
- Signal length is set equal to $T = 10,000$ samples.
- Harmonics number of each source is chosen arbitrarily between two and three harmonics,
- Overestimated harmonics number is $H_{max} = 10$.
- Optimizer: RMSprop, batch size 128, learning rate=0.001
- The EQL network shows a good fit of both the complexity of the model measured by the $\mathcal{L}_1$ norm of the network weight and the reconstruction error in Fig.3.

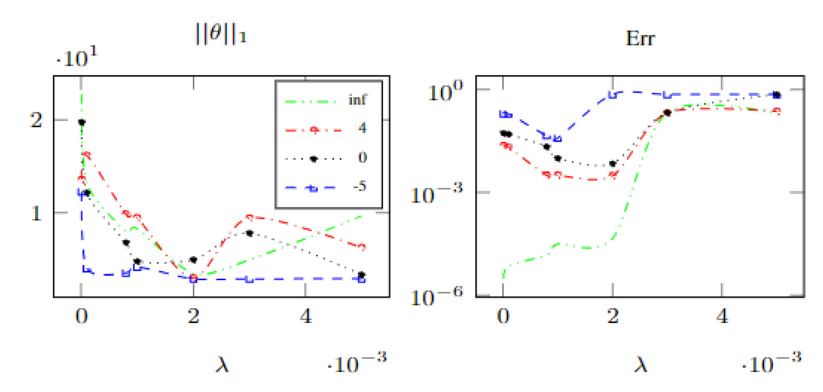


Fig. 3. Signal reconstruction error and the $\mathcal{L}_1$ norm of the network weights versus λ and over multiple levels of SNR.

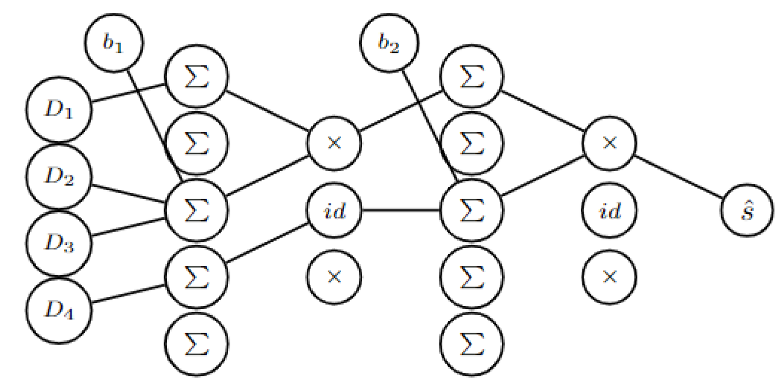


Fig. 4. The ultimate neural network after regularization and thresholding where only the desired inputs are activated.

- The model identified in Fig.4 by the EQL is as follows:

$$\hat{s}(t) = a\hat{s}_1(t)(a + b\hat{s}_2(t) + c\hat{s}_3(t))(d + e\hat{s}_4(t)),$$

where α, a, b, c, d, e are constants.

- Our method determines accurately the exact sources, for different levels of noise, as shows in Table 1.

SNR	$c(s_1, \hat{s}_1)$	$c(s_2, \hat{s}_2)$	$c(s_3, \hat{s}_3)$	$c(s_4, \hat{s}_4)$	$c(s, \hat{s})$
inf dB	0.996	0.998	1.0	0.998	0.99
0 dB	0.989	0.998	0.998	0.998	0.998
-5 dB	0.993	0.996	0.998	0.999	0.991

Table 1. Correlation between real sources s_i and estimated ones for different levels of SNR.The correlation between the input signal s and its estimated $\hat{s}$ is also added to illustrate the reconstruction performance.

Summary

- An end-to end framework is introduced for explaining the physical phenomena and representing it in a concise mathematical form.
- Modeling/separation of the elementary sources from an observed nonlinear mixture.
- Extend the proposed metholodlogy to deal with non-stationary signals.

KEY REFERENCES

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[2] Samuel Kim, Peter Y Lu, Srijon Mukherjee, Michael Gilbert, Li Jing, Vladimir Ceperi c, and Marin Soljaci c, "Integration of neural network-based symbolic regres- sion in deep learning for scientific discovery," IEEE transactions on neural networks and learning systems, vol. 32, no. 9, pp. 4166–4177, 2020

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