Space-Time Adaptive Processing for radars in Connected and Automated Vehicular Platoons

Zahra Esmaeilbeig[1], Kumar Vijay Mishra[2] and Mojtaba Soltanalian[1]
[1]ECE Department, University of Illinois Chicago
[2]United States DEVCOM Army Research Laboratory

April 11, 2024

Motivation

- Performance of single automotive radar is subject to either small aperture or limited field of view(FoA). It is of great interests to improve the automotive radar performance through radar networking.
- In a Connected and Automated Vehicular (CAV) system:
  $\star$ vehicle-to-vehicle (V2V) communications
  $\star$ vehicle-to-infrastructure (V2I) communications
  Enables collaborative space-time adaptive processing (STAP).
- We formulate a collaborative target detection problem in automotive radar.
- The transmitted signals need to be orthogonal to be distinguishable at the receiver $\rightarrow$ In a CPI, the antennas need to take turns to transmit $\rightarrow$ We propose to incorporate TDM by designing a transmitter scheduling matrix for the platoon of vehicles.

Motivation

A simplified illustration of a CAV platoon consisting of three vehicles, sensing a target in the FoV of all three vehicles. The radar on vehicle 1, denoted by RX/TX1, leads the platoon and is assisted by two other radars, denoted by TX2 and TX3.

Collaborative STAP

- We consider a network of K cooperative vehicles, the received signal at the designated range bin at the m-th Rx on vehicle i from the n-th Tx on vehicle k is

$$
\begin{aligned}
s_{ki}(l,n,m) =&\alpha_k e^{-\mathrm{j}\frac{2\pi f_c}{c}\nu_k((l-1)N+(n-1))T_c} \\
&e^{-\mathrm{j}\frac{2\pi f_c}{c}\nu_i((l-1)M+(m-1))T_c} \\
&e^{\mathrm{j}2\pi f_c\left(\mathbf{p}_{T,kn}^{\top}\mathbf{p}_{tk}\right)} e^{\mathrm{j}2\pi f_c\left(\mathbf{p}_{R,im}^{\top}\mathbf{p}_{ti}\right)},
\end{aligned}
\tag{1}
$$

Collaborative STAP

- We consider a network of K cooperative vehicles, the received signal at the designated range bin at the m-th Rx on vehicle i from the n-th Tx on vehicle k is

$$\begin{aligned} s_{ki}(l,n,m) =& \alpha_k e^{-\mathrm{j}\frac{2\pi f_c}{c}\nu_k((l-1)N+(n-1))T_c} \\ & e^{-\mathrm{j}\frac{2\pi f_c}{c}\nu_i((l-1)M+(m-1))T_c} \\ & e^{\mathrm{j}2\pi f_c\left(\mathbf{p}_{T,kn}^{\top}\mathbf{p}_{tk}\right)} e^{\mathrm{j}2\pi f_c\left(\mathbf{p}_{R,im}^{\top}\mathbf{p}_{ti}\right)}, \end{aligned} \quad (1)$$

By stacking the echoes from all N Tx on vehicle k, we obtain

$$\begin{aligned} \mathbf{s}_{ki} &= \begin{bmatrix} \mathbf{s}_{ki}(1) \\ \vdots \\ \mathbf{s}_{ki}(N) \end{bmatrix} \\ &= \left(\mathbf{a}_{T,k}(\theta) \odot \mathbf{a}_{D,N}(\nu_k)\right) \otimes \Big(\left(\mathbf{a}_{d,N}(\nu_k) \odot \mathbf{a}_{d,M}(\nu_i)\right) \\ & \otimes \left(\mathbf{a}_{R,i}(\theta) \odot \mathbf{a}_{D,M}(\nu_i)\right) \Big) \in \mathbb{C}^{NLM\times 1}. \end{aligned} \quad (2)$$

Collaborative STAP

In order to decide whether a target is present in a particular known range-cell, we perform binary hypothesis testing between $\mathcal{H}_0$ (target-free hypothesis) and $\mathcal{H}_1$ (target-present hypothesis), i.e.,

$$\begin{aligned} \mathcal{H}_0 : &\quad \mathbf{y}_k = \mathbf{n}_k \\ \mathcal{H}_1 : &\quad \mathbf{y}_k = \alpha_k \mathbf{s}_k + \mathbf{n}_k, \end{aligned} \tag{3}$$

where α_k is the complex target reflectivity factor and $\mathbf{n}_k$ is the noise and interference with covariance $\mathbf{R}_k$. The log-likelihood ratio test statistic is given by,

$$\zeta = \sum_{k=1}^{K} \frac{|\mathbf{s}_k^{\mathrm{H}} \mathbf{R}_k^{-1} \mathbf{y}_k|^2}{\frac{|\alpha_k|^2}{2} + \mathbf{s}_k^{\mathrm{H}} \mathbf{R}_k^{-1} \mathbf{s}_k} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma. \tag{4}$$

TDM Design

- The transmitted signals need to be orthogonal to be distinguishable at the receiver. For this matter in a CPI the antennas need to take turns to transmit.

-At most one antenna within the platoon is allowed to transmit during each pulse.

TDM Design

- The transmitted signals need to be orthogonal to be distinguishable at the receiver. For this matter in a CPI the antennas need to take turns to transmit.

-At most one antenna within the platoon is allowed to transmit during each pulse.

-After performing TDM the received signal $\bar{\mathbf{s}} = [\bar{\mathbf{s}}_1, \cdots, \bar{\mathbf{s}}_K]^T$ is

$$\bar{\mathbf{s}} = \begin{bmatrix} \left(\mathrm{vec}\left(\mathbf{J}_1\right) \otimes \mathbf{1}\right) \odot \mathbf{s}_1 \\ \vdots \\ \left(\mathrm{vec}\left(\mathbf{J}_k\right) \otimes \mathbf{1}\right) \odot \mathbf{s}_K \end{bmatrix} = \left(\mathrm{vec}\left([\mathbf{J}_1 \mid \ldots \mid \mathbf{J}_K]\right) \otimes \mathbf{1}_M\right) \odot \mathbf{s}$$
$$= \left(\mathrm{vec}\left(\mathbf{J}\right) \otimes \mathbf{1}_M\right) \odot \mathbf{s} \in \mathbb{C}^{KNLM \times 1}, \quad (5)$$

where $\mathbf{J} = \left[\mathbf{J}_1 \mid \ldots \mid \mathbf{J}_K\right] \in \{0,1\}^{L \times KN}$ is a permutation matrix and called the waveform selection matrix.

TDM Design

We intend to maximize the target detection performance. We use the mean of the test statistic as the design criteria. Consequently the TDM design problem is

$$
\begin{aligned}
\mathcal{P}_1 : \ \underset{\mathbf{J}}{\text{maximize}} \quad & \mathbb{E}\left\{\zeta | \mathcal{H}_1\right\} \\
\text{subject to} \quad & \sum_{p} \mathbf{J}_{pn} = 1, \qquad p, n \in \{1, \ldots, L\}; \\
& \sum_{n} \mathbf{J}_{pn} = 1; \\
& \mathbf{J}_{pn} \in \{0, 1\}.
\end{aligned} \tag{6}
$$

TDM Design

- The probability of detection and false alarm are obtained respectively as

$$\begin{aligned} \mathsf{P}_{\mathsf{D}} &= \mathbb{P}\mathrm{r}\left\{\zeta > \gamma|\mathcal{H}_1\right\} = 1 - \mathbb{P}\mathrm{r}\left\{\zeta \leq \gamma|\mathcal{H}_1\right\} = 1 - F_{\zeta|\mathcal{H}_1}(\gamma|\mathcal{H}_1), \\ \mathsf{P}_{\mathsf{FA}} &= \mathbb{P}\mathrm{r}\left\{\zeta > \gamma|\mathcal{H}_0\right\} = 1 - \mathbb{P}\mathrm{r}\left\{\zeta \leq \gamma|\mathcal{H}_0\right\} = 1 - F_{\zeta|\mathcal{H}_0}(\gamma|\mathcal{H}_0), \end{aligned} \tag{7}$$

where $F_{\zeta|\mathcal{H}}(.)$ is the cumulative distribution function of the test statistic with hypo-exponential distribution. We have

$$\mathbb{E}\left\{\zeta|\mathcal{H}_1\right\} = \sum_{k=1}^{K} 1 + 2|\alpha_k|^2 \mathbf{s}_k^{\mathrm{H}} \mathbf{R}_k^{-1} \mathbf{s}_k. \tag{8}$$

By substituting (5) in (8) we obtain the quadratic objective

$$\begin{aligned} \mathbb{E}\left\{\zeta|\mathcal{H}_1\right\} &= \bar{\mathbf{s}}^{\mathrm{H}} \mathbf{R}^{-1} \bar{\mathbf{s}} \\ &= \left(\left(\mathrm{vec}\left(\mathbf{J}\right) \otimes \mathbf{1}_M\right) \odot \mathbf{s}\right)^{\mathrm{H}} \mathbf{R}^{-1} \left(\left(\mathrm{vec}\left(\mathbf{J}\right) \otimes \mathbf{1}_M\right) \odot \mathbf{s}\right) \\ &= \mathrm{vec}\left(\mathbf{J}\right)^{\mathrm{H}} \mathbf{S}\, \mathrm{vec}\left(\mathbf{J}\right). \end{aligned} \tag{9}$$

Solution Methodology

$$\mathcal{P}_3 : \underset{\mathbf{J} \in \mathbf{\Omega}}{\text{maximize}} \quad \text{vec}\,(\mathbf{J})^{\mathrm{H}}\, \bar{\mathbf{S}}\, \text{vec}\,(\mathbf{J})\,. \tag{10}$$

where $\mathbf{\Omega}$ is the set of permutation matrices i.e.

$$\mathbf{\Omega} = \Bigg\{ \mathbf{J} \;\Bigg|\; \sum_p \mathbf{J}_{pn} = 1, \quad \sum_n \mathbf{J}_{pn} = 1, \quad \mathbf{J}_{pn} \in \{0, 1\}, \quad p, n \in \{1, \ldots, L\} \Bigg\}. \tag{11}$$

Solution Methodology

One can locally optimize $\mathcal{P}_3$ by resorting to *power method-like* iterations of the form

$$\mathcal{P}_4: \underset{\mathbf{J}^{(s+1)} \in \mathbf{\Omega}}{\text{minimize}} \quad \|\text{vec}\left(\mathbf{J}^{(s+1)}\right) - \bar{\mathbf{S}}\, \text{vec}\left(\mathbf{J}^{(s)}\right)\|_2 \tag{12}$$

We define the matrix $\mathbf{C}^{(s)} = -\text{vec}_{L,L}^{-1}\left(\bar{\mathbf{S}}\, \text{vec}\left(\mathbf{J}^{(s)}\right)\right)$. It is straightforward to see $\mathcal{P}_4$ is equivalent to

$$\mathcal{P}_5: \underset{\mathbf{J}^{(s+1)} \in \mathbf{\Omega}}{\text{minimize}} \quad \text{Tr}\left(\mathbf{J}^{(s+1)} \mathbf{C}^{(s)\text{H}}\right). \tag{13}$$

- This problem is in fact a linear assignment problem with cost matrix $\mathbf{C}^{(s)\text{H}}$ that can be solved efficiently using the Hungarian algorithm also known as Munkres assignment algorithm, with computational complexity of $\mathcal{O}(L^2)$.

TDM Design algorithm for collaborative sensing

Algorithm 1 Power method-like iterations for transmitter scheduling in CAVs.

Input The overall steering vector of the CAV $\bar{\mathbf{s}}$

Initialization $\mathbf{J}^{(0)} \in \mathbf{\Omega}$, $s = 0$

1: $\bar{\mathbf{S}} = \lambda_m \mathbf{I} - \mathbf{S}$

2: **While** $\left| \left[f(\mathbf{J}^{(s+1)}) - f(\mathbf{J}^{(s)}) \right] / f(\mathbf{J}^{(s)}) \right| \geq \epsilon$ **do**

3: $\quad \mathbf{C}^{(s)} = -\mathrm{vec}^{-1}_{L,L} \left(\bar{\mathbf{S}} \,\mathrm{vec}\left(\mathbf{J}^{(s)}\right)\right)$

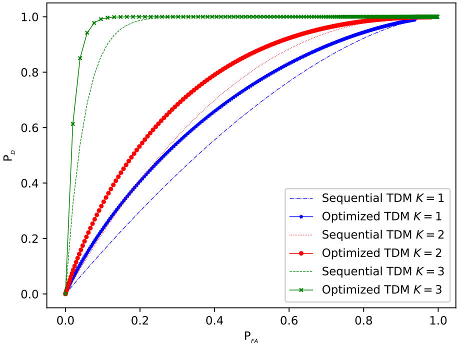
4: $\quad \mathbf{J}^{(s+1)} \leftarrow \text{Hungarian}(\mathbf{C}^{(s)\mathrm{H}})$

5: $\quad s \leftarrow s + 1$

6: $\mathbf{J}_{\text{opt}} \leftarrow \mathbf{J}^{(s)}$

7: **Output** $\mathbf{J}_{\text{opt}}$

Numerical Experiments



RoC of detection for a CAV of FMCW radars. The optimized TDM is compared with uniform transmission where the antennas are activated uniformly in a sequence.

Thank you!

Email: Zesmae2@uic.edu