Localizing Acoustic Energy in Sound Field Synthesis by Weighted Exterior Radiation Suppression

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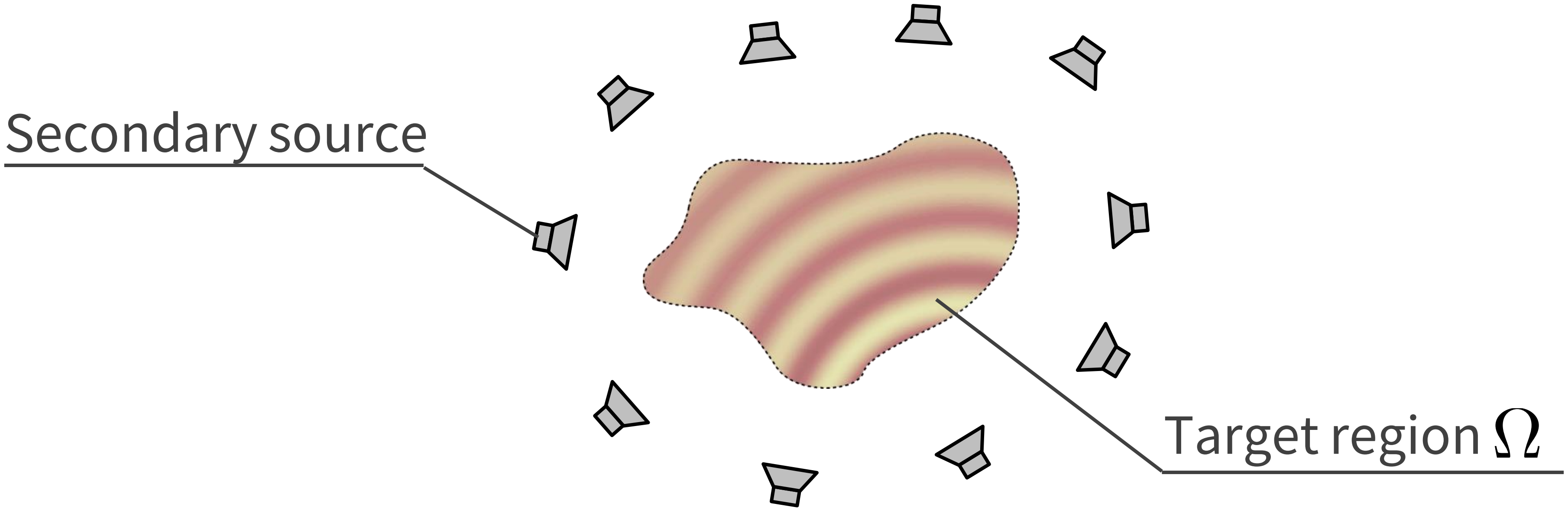
Sound Field Synthesis

Synthesizing desired spatial sound inside target region using multiple loudspeakers (secondary sources)



Applicable to spatial audio for VR/AR, personal audio systems, and spatial active noise control

Sound Field Synthesis

Synthesizing desired spatial sound inside target region using multiple loudspeakers (secondary sources)

- Sound field synthesized by L secondary sources at angular frequency ω

Transfer function of l th secondary source

$$u_{\mathrm{syn}}(\boldsymbol{r}, \omega) = \sum_{l=1}^{L} d_l(\omega) g_l(\boldsymbol{r}, \omega)$$

Driving signal of l th secondary source

- Optimization problem to obtain driving signal $\boldsymbol{d} = [d_1, \ldots, d_L]^{\mathsf{T}}$

Regularization term for $\boldsymbol{d}$

$$\underset{\boldsymbol{d} \in \mathbb{C}^L}{\mathrm{minimize}}\, \mathcal{J}(\boldsymbol{d}) + \alpha \mathcal{R}(\boldsymbol{d})$$

Synthesis error inside Ω

Sound Field Synthesis

Synthesizing desired spatial sound inside target region using multiple loudspeakers (secondary sources)

- Examples of optimization problem for sound field synthesis
  - Pressure matching [Kirkeby+ 1993]

Desired pressure

$$\underset{\boldsymbol{d}\in\mathbb{C}^L}{\text{minimize}} \|\boldsymbol{G}\boldsymbol{d} - \boldsymbol{u}_{\text{des}}\|^2 + \alpha\|\boldsymbol{d}\|^2$$

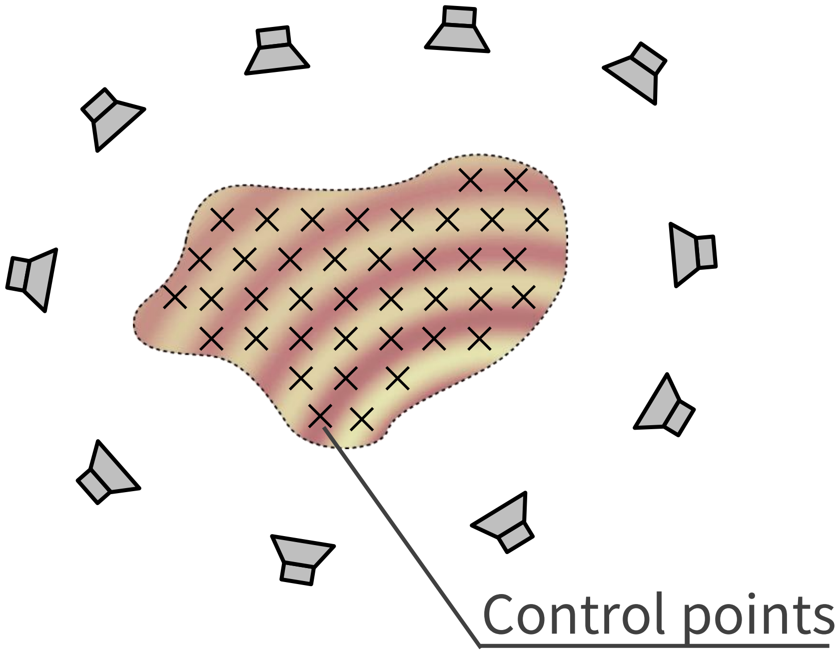
Transfer function matrix

  - Amplitude matching [Abe+ 2023]

Desired magnitude

$$\underset{\boldsymbol{d}\in\mathbb{C}^L}{\text{minimize}} \||\boldsymbol{G}\boldsymbol{d}| - \boldsymbol{a}_{\text{des}}\|^2 + \alpha\|\boldsymbol{d}\|^2$$

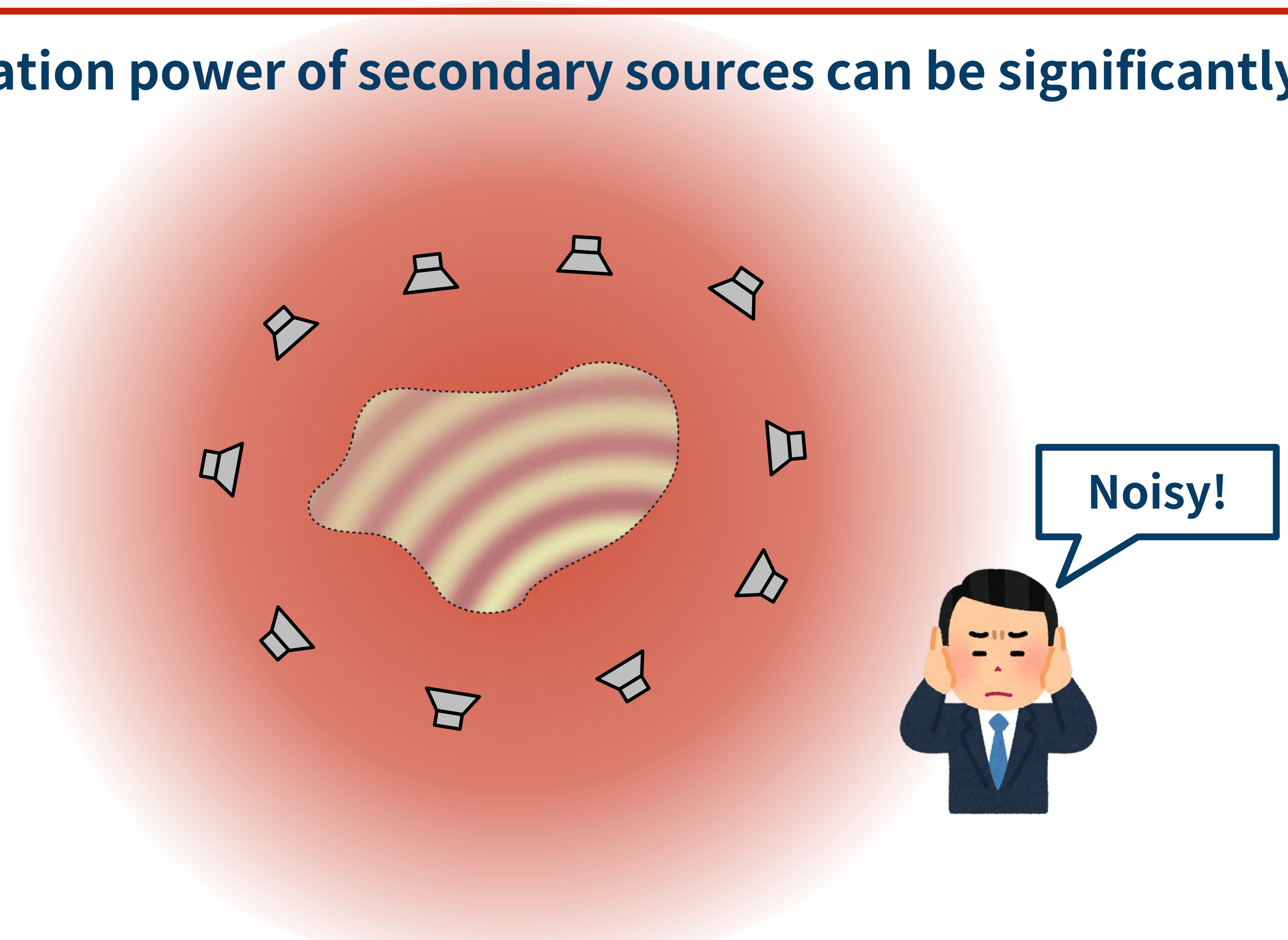
Element-wise absolute value



Exterior Radiation in Sound Field Synthesis

Most sound field synthesis methods do not take exterior radiation of secondary sources into consideration

➡ **Exterior radiation power of secondary sources can be significantly large**



Exterior Radiation in Sound Field Synthesis

- Current methods for sound field synthesis with exterior radiation suppression

Penalty-based [Poletti+ 2012, Ueno+ 2018]

Exterior radiation power

$$\underset{\boldsymbol{d}\in\mathbb{C}^L}{\text{minimize}}\,\mathcal{J}(\boldsymbol{d}) + \underline{\gamma\mathcal{E}(\boldsymbol{d})} + \alpha\mathcal{R}(\boldsymbol{d})$$

Constraint-based [Arikawa+ 2022, Kojima+ 2023]

$$\underset{\boldsymbol{d}\in\mathbb{C}^L}{\text{minimize}}\,\mathcal{J}(\boldsymbol{d}) + \alpha\mathcal{R}(\boldsymbol{d})$$

Constant

$$\text{subject to}\ \ \underline{\mathcal{E}(\boldsymbol{d}) = C}\ \text{or}\ \underline{\mathcal{E}(\boldsymbol{d}) \leq C}$$

➡ **Appropriate formulation of $\underline{\mathcal{E}(\boldsymbol{d})}$ is essential for exterior radiation suppression**

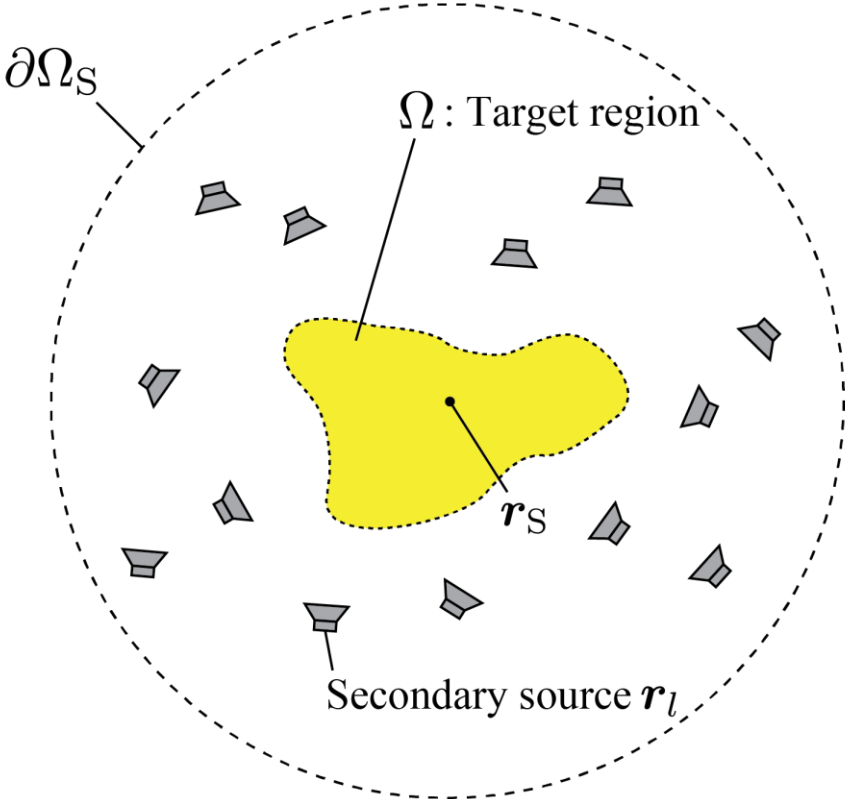
Exterior Radiation in Sound Field Synthesis

- Formulation of exterior radiation power from spherical surface $\partial\Omega_{\mathrm{S}}$

$$\mathcal{E}_{\mathrm{uni}}(\boldsymbol{d}) = \int_{\partial\Omega_{\mathrm{S}}} I_r(\boldsymbol{r})\mathrm{d}\boldsymbol{r}$$

Acoustic intensity in outward normal direction

$$I_r(\boldsymbol{r}) = \frac{1}{2}\mathrm{Re}\left[u_{\mathrm{syn}}(\boldsymbol{r})\frac{\mathrm{j}}{\rho c k}\frac{\partial}{\partial r}u_{\mathrm{syn}}(\boldsymbol{r})^*\right]$$

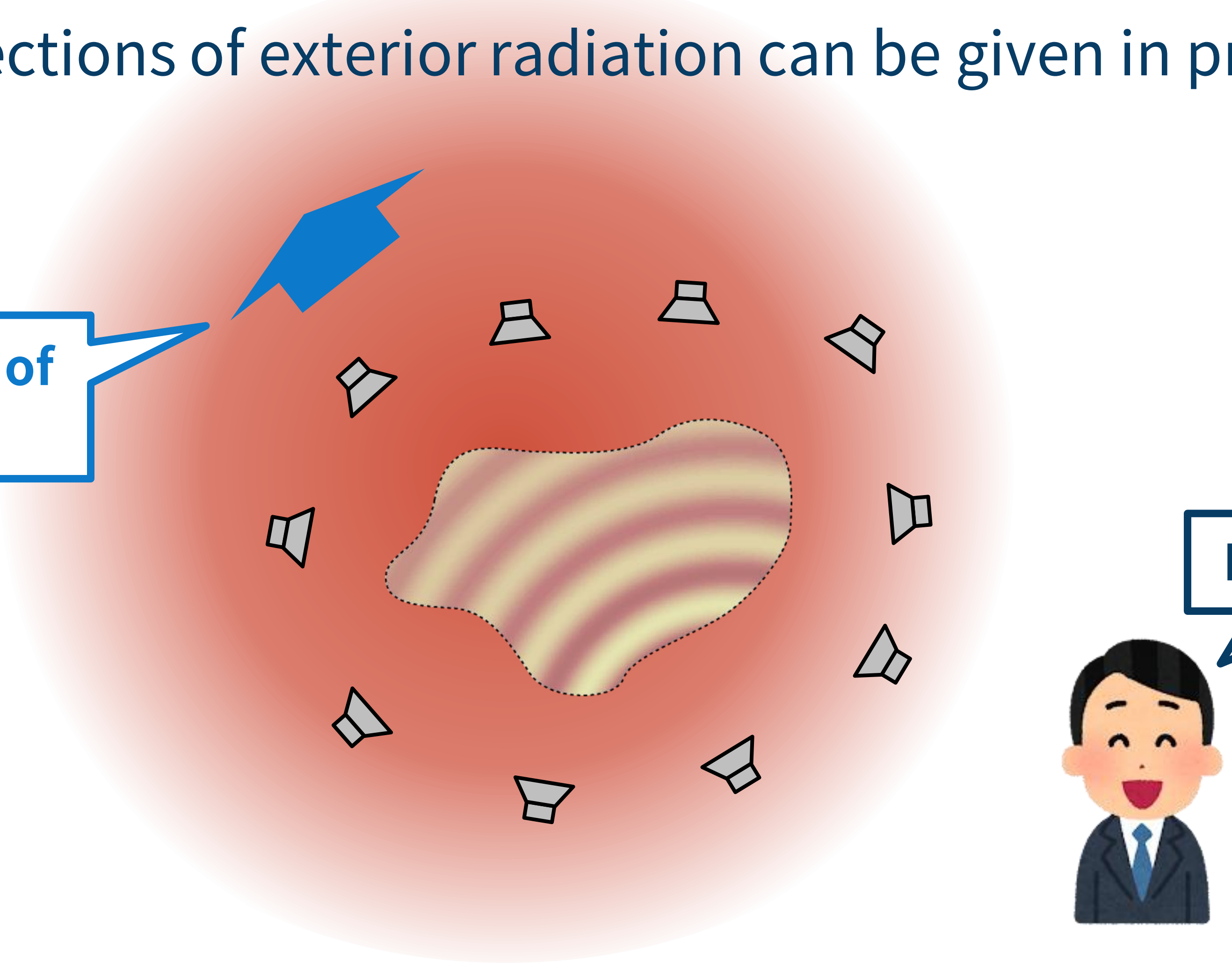


Since suppressing exterior radiation in all outward directions is difficult, synthesis accuracy in the interior region can deteriorate

Idea

Prioritizing suppression directions of exterior radiation to maintain high synthesis accuracy in the interior region

➡ Allowable directions of exterior radiation can be given in practical situations



Allowable direction of exterior radiation

Not so noisy!

Directionally Weighted Exterior Radiation Suppression

- Formulation of directionally weighted exterior radiation power

$$\mathcal{E}_{\mathrm{dir}} = \int_{\partial\Omega_{\mathrm{S}}} I_r(\boldsymbol{r}) w(\theta, \phi) \mathrm{d}\boldsymbol{r}$$

Directional weighting function

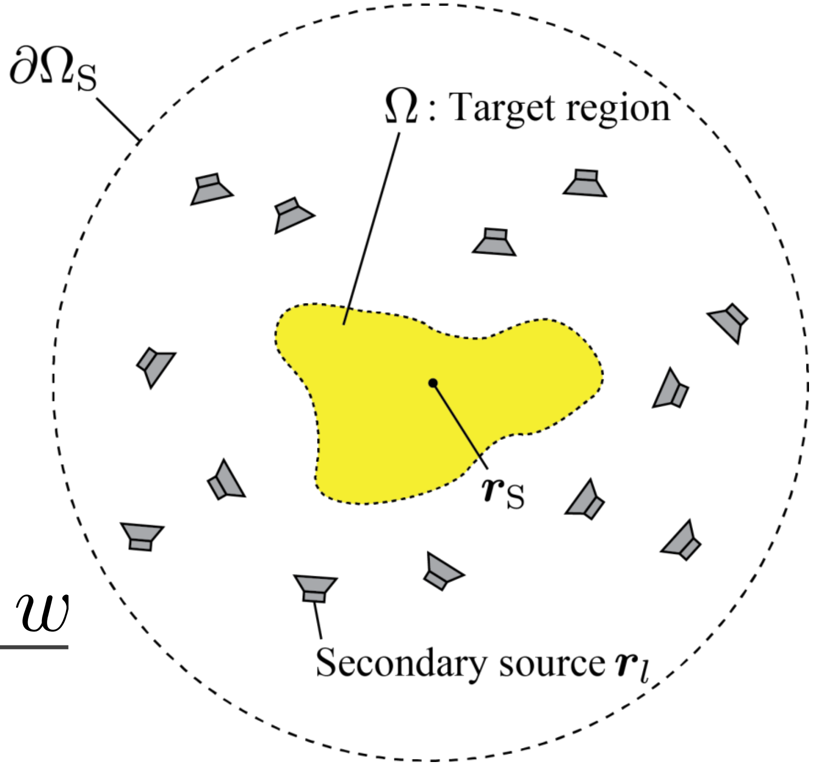
Represented by quadradic form of $\boldsymbol{d}$

$$\mathcal{E}_{\mathrm{dir}}(\boldsymbol{d}) = \boldsymbol{d}^{\mathsf{H}} \boldsymbol{A} \boldsymbol{d}$$

Spherical harmonic exp coef of w

$$\boldsymbol{A} = -\frac{R_{\mathrm{S}}^2}{2\rho c} \mathrm{Im} \left[\sum_{n,m} \tilde{w}_n^m(\boldsymbol{r}_{\mathrm{S}}) \boldsymbol{B}_n^m \right]$$

Matrix defined by exp coefs of transfer funcs



Simply applicable to various sound field synthesis techniques

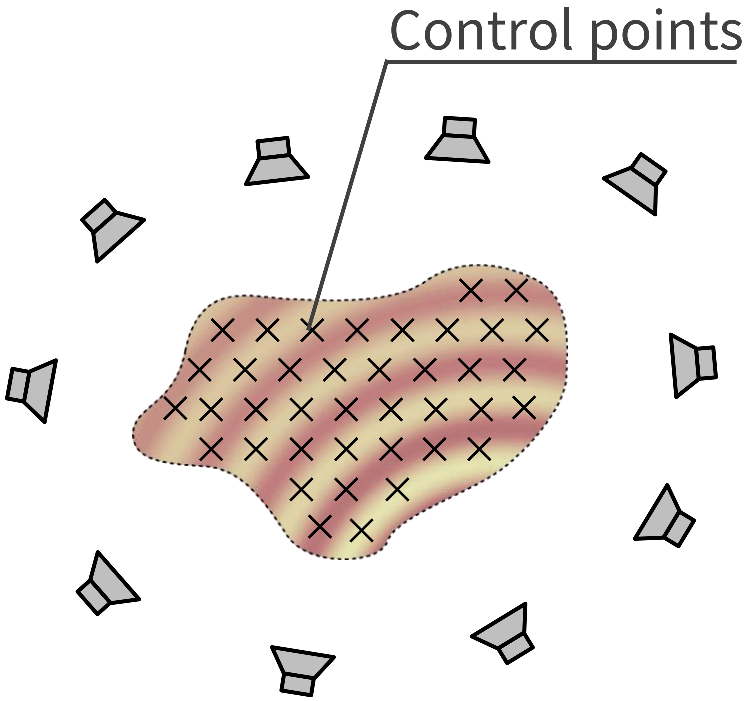
Application to Amplitude Matching [Abe+ 2023]

- Synthesizing desired magnitude distribution in target region while suppressing exterior radiation power

Desired magnitude

$$\underset{\boldsymbol{d}\in\mathbb{C}^L}{\mathrm{minimize}}\, \left\| |\boldsymbol{G}\boldsymbol{d}| - \boldsymbol{a}_{\mathrm{des}} \right\|^2 + \gamma \boldsymbol{d}^{\mathsf{H}} \boldsymbol{A} \boldsymbol{d} + \alpha \|\boldsymbol{d}\|^2$$

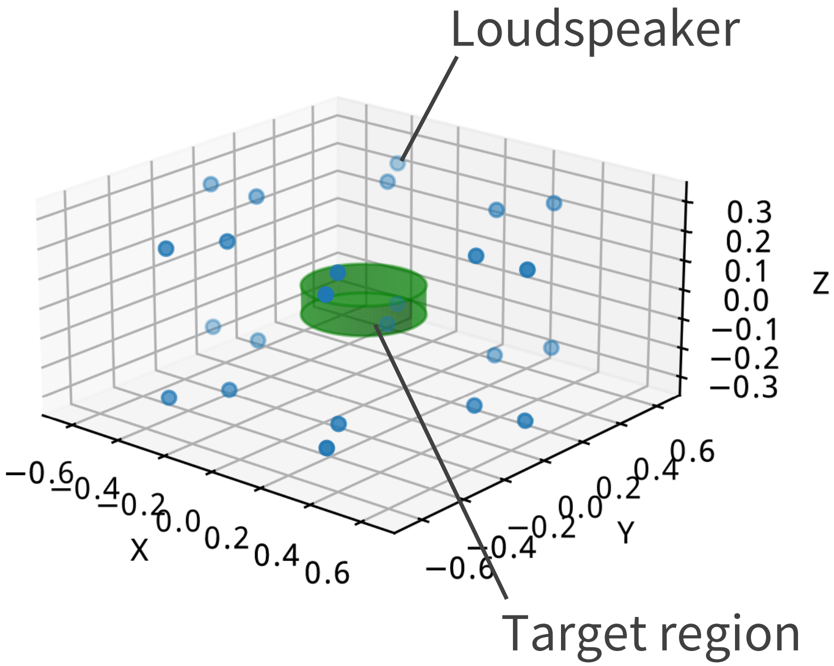
Synthesized magnitude



- Non-convex and non-differentiable cost function, but this optimization problem can be solved by ADMM
- Algorithm is derived in a similar manner to amplitude matching without exterior radiation suppression
- Differential-norm penalty proposed in [Abe+ 2023] is also applied to induce smoothness between frequency bins

Experiments

- Experimental setting
  - Cylindrical target region Ω of radius 0.2 m and height 0.1 m
  - 24 loudspeakers (point sources) on circles of radii 0.45 m and 0.65 m at z = $\pm$0.2 m
  - 147 control points uniformly distributed over Ω at intervals of 0.05 m
  - Comparison:
    - Amplitude matching without exterior radiation suppression (**AM**)
    - AM with uniformly weighted exterior radiation suppression (**AM-Rad**)
    - AM with directionally weighed exterior radiation suppression (**AM-Rad-Dir**; Proposed method)
  - Desired magnitude: uniform distribution
  - Directional weighting function for **AM-Rad-Dir**: $w(\theta, \phi) = 1 + \cos\phi \sin\theta$



Experiments

- Evaluation measures
  - Mean square error of magnitude synthesized at control points

$$\mathrm{MSE}(\omega) = \frac{1}{M} \left\| \boldsymbol{a}_{\mathrm{syn}}(\omega) - \boldsymbol{a}_{\mathrm{des}}(\omega) \right\|^2$$

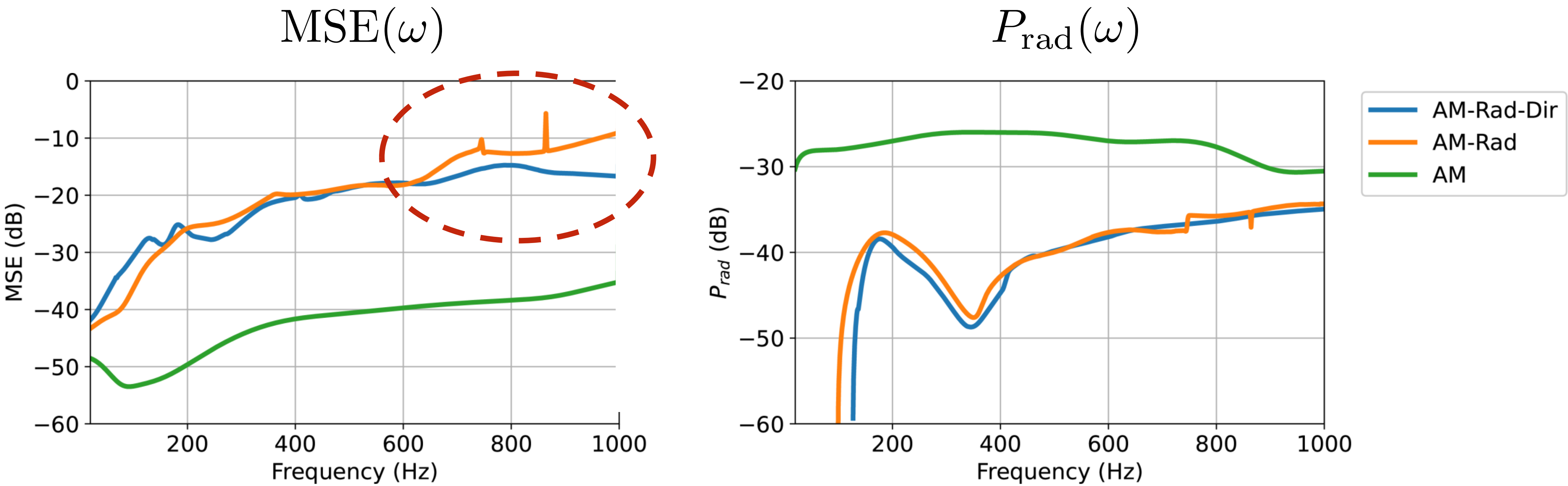
  - Exterior radiation power in suppression directions

$$P_{\mathrm{rad}}(\omega) = \int_{\phi_1}^{\phi_2} \mathrm{d}\phi \int_{\theta_1}^{\theta_2} \mathrm{d}\theta I_r(\theta, \phi) r^2 \sin\theta$$

    - Range of suppression direction is set by $(\theta_1, \theta_2) = (0, \pi)$ and $(\phi_1, \phi_2) = (-\pi/2, \pi/2)$

Results

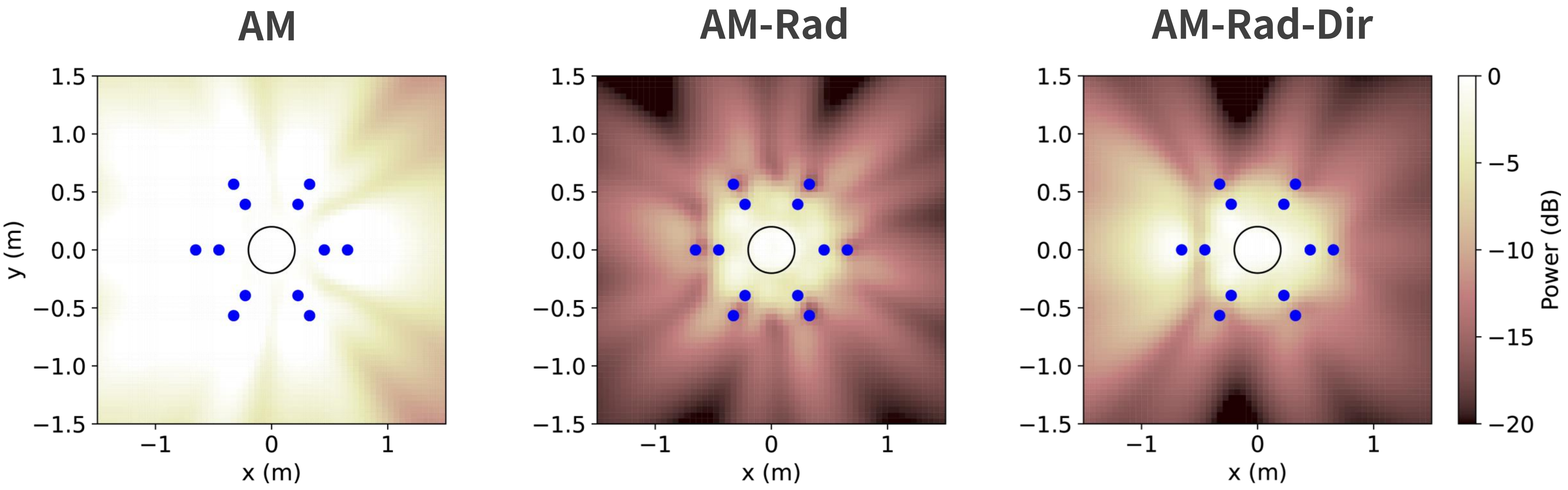
- $\mathrm{MSE}(\omega)$ and $P_{\mathrm{rad}}(\omega)$ w.r.t. frequency



Proposed AM-Rad-Dir achieved small exterior radiation power and small synthesis error above 600 Hz

Results

- Power distribution of synthesized sound field up to 1000 Hz



Exterior radiation power in the intended direction was sufficiently suppressed in AM-Rad-Dir

Conclusion

- **Sound field synthesis with directionally weighted exterior radiation suppression**
  - Suppressing exterior radiation in all outward directions is difficult, then interior synthesis accuracy can deteriorate
  - Prioritizing suppression direction using prior knowledge of allowable directions of exterior radiation
  - Directional weighting enables relaxing the constraint on the exterior radiation compared with uniform weight
  - Directionally weighted exterior radiation power is formulated as a quadratic form of loudspeaker driving signals
  - Numerical experiments indicated that amplitude matching with directionally weighted exterior radiation enables suppressing exterior radiation power while maintaining the interior synthesis accuracy high