LINEAR COMPLEXITY GIBBS SAMPLING FOR GENERALIZED LABELED MULTI-BERNOULLI FILTERING

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1. Introduction

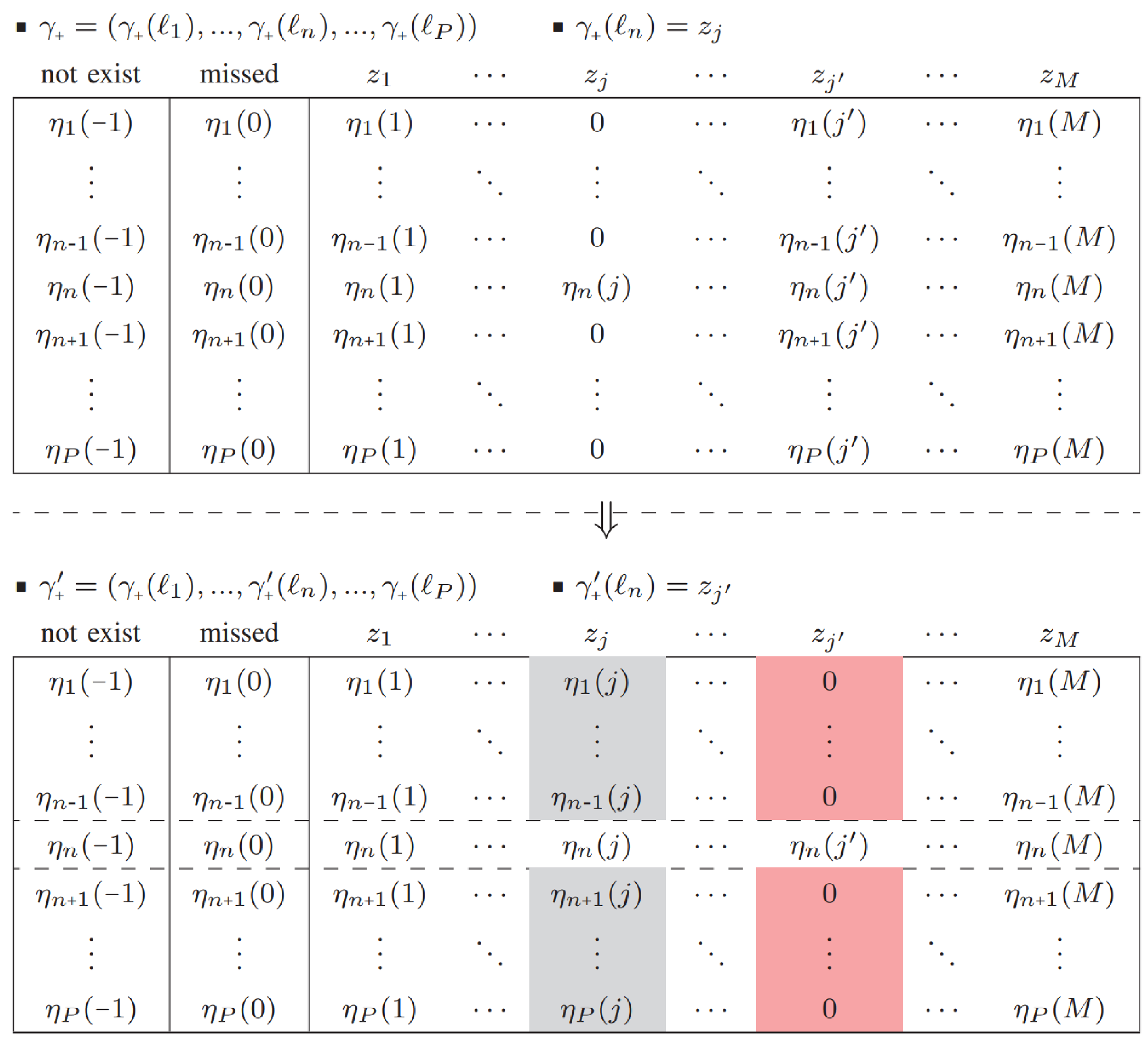
- **Generalized Labeled Multi-Bernoulli (GLMB) densities** arise in a host of multi-object system applications, but computing the GLMB filtering density requires solving NP-hard problems [1].
- This work [2] develops a **linear complexity Gibbs sampling (GS)** framework for **GLMB density computation**.
- Specifically, we propose a **tempered Gibbs sampler** that exploits the structure of the GLMB filtering density to achieve the **first $O(T(P+M))$ complexity**.
  - T: the number of *sampling iterations*
  - P: the number of *hypothesized objects*
  - M: the number of *measurements*

2. Problem Statement

- GLMB truncation amounts to **selecting significant γ_+** [3].
  - γ_+: the positive 1-1 mapping for the association between objects $\ell \in \mathbb{L}$ and measurements $z \in Z$ with $\gamma_+(\ell) = -1$ (not exist) or 0 (undetected), i.e., $\gamma_+ : \mathbb{L} \to \{-1{:}Z\}$
- A (discrete) probability distribution π on $\{-1{:}M\}^P$ is defined by
$$\pi(\gamma_+) \propto 1_{\Gamma_+}(\gamma_+)\prod_{i=1}^{P}\eta_i(\gamma_+(\ell_i)).$$
  - $1_Y(X)$: the set inclusion function
- Systematic-scan GS (**SGS**) samples from the stationary distribution π by constructing a Markov chain with transition kernel
$$\pi(\gamma'_+|\gamma_+) = \prod_{i=1}^{P}\pi_i(\gamma'_+(\ell_i)|\gamma'_+(\ell_{1:i-1}),\gamma_+(\ell_{i+1:P})),$$
where the i-th conditional, defined on $\{-1{:}M\}^P$, is given by
$$\pi_i(\cdot\,|\gamma_+(\ell_{\bar{i}})) = \frac{\tilde{\pi}_i(\cdot|\gamma_+(\ell_{\bar{i}}))}{\langle\tilde{\pi}_i(\cdot|\gamma_+(\ell_{\bar{i}})),1\rangle}.$$
  - $\bar{i}$: $1,2,\dots,i-2,i-1,i+1,i+2,\dots,P-1,P$
  - $\tilde{\pi}_i(j|\gamma_+(\ell_{\bar{i}})) = \begin{cases}\eta_i(j), & j<1\\ \eta_i(j)\left(1-1_{\{\gamma_+(\ell_{\bar{i}})\}}(j)\right), & j\in\{1{:}M\}\end{cases}$
- The conditionals are characterized by the $P \times (M+2)$ matrix, so **the time complexity of its computation is $O(TP^2M)$**.

3. Methodology

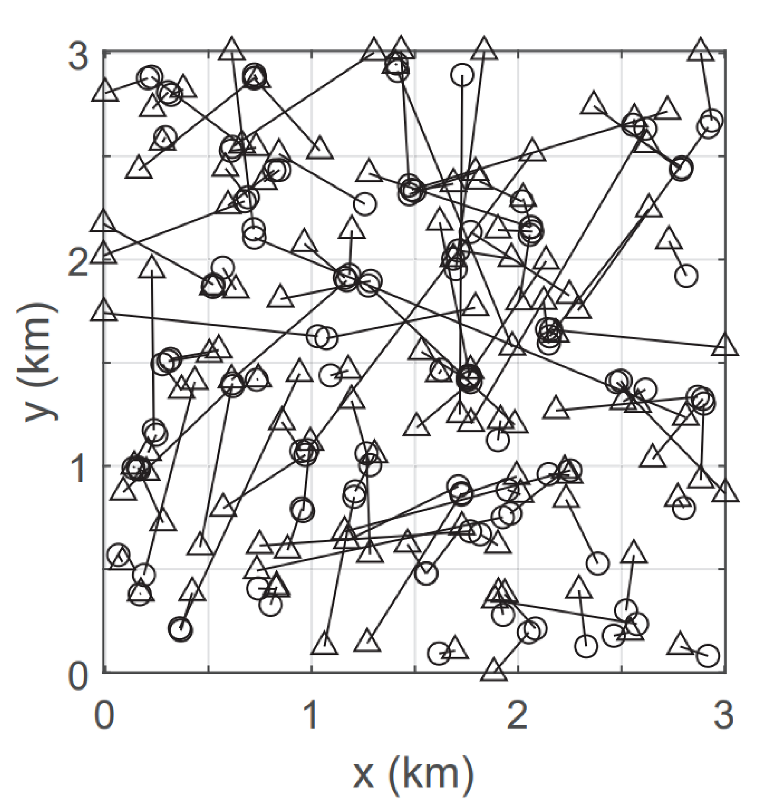
- Tempered GS (**TGS**) generates the next iterate γ'_+ by randomly selecting a coordinate to update with an additional **mechanism to improve mixing and sample diversity** [4].
- A coordinate $i \in \{1{:}P\}$ is chosen according to the distribution
$$\rho(i|\gamma_+) = \frac{\phi_i(\gamma_+(\ell_i)|\gamma_+(\ell_{\bar{i}}))}{\pi_i(\gamma_+(\ell_i)|\gamma_+(\ell_{\bar{i}}))}.$$
  - $\phi_i(\cdot\,|\gamma_+(\ell_{\bar{i}}))$: the bounded proposal distribution on $i \in \{-1{:}M\}$, e.g., for $\alpha,\beta \in (0,1]$ and for any function $f, f^\beta(\cdot) = [f(\cdot)]^\beta$
$$\phi_i(j|\gamma_+(\ell_{\bar{i}})) = \alpha\pi_i(j|\gamma_+(\ell_{\bar{i}})) + \frac{(1-\alpha)\pi_i^\beta(j|\gamma_+(\ell_{\bar{i}}))}{\langle\pi_i^\beta(\cdot|\gamma_+(\ell_{\bar{i}})),1\rangle}$$
- Given the selection of the i-th coordinate, its state is updated by sampling from the proposal, i.e.,
$$\gamma'_+(\ell_i) \sim \phi_i(\cdot\,|\gamma_+(\ell_{\bar{i}})).$$
- The structure of the problem allows TGS to be implemented with an **$O(T(P+M))$ complexity** via the **positive 1-1 constraint**.
  - e.g.,



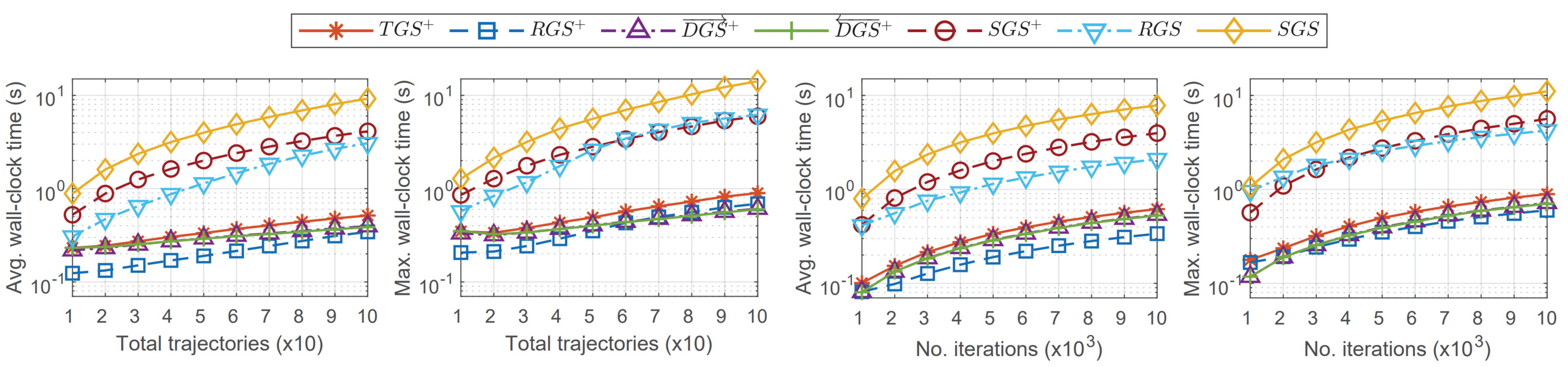
4. Experiments

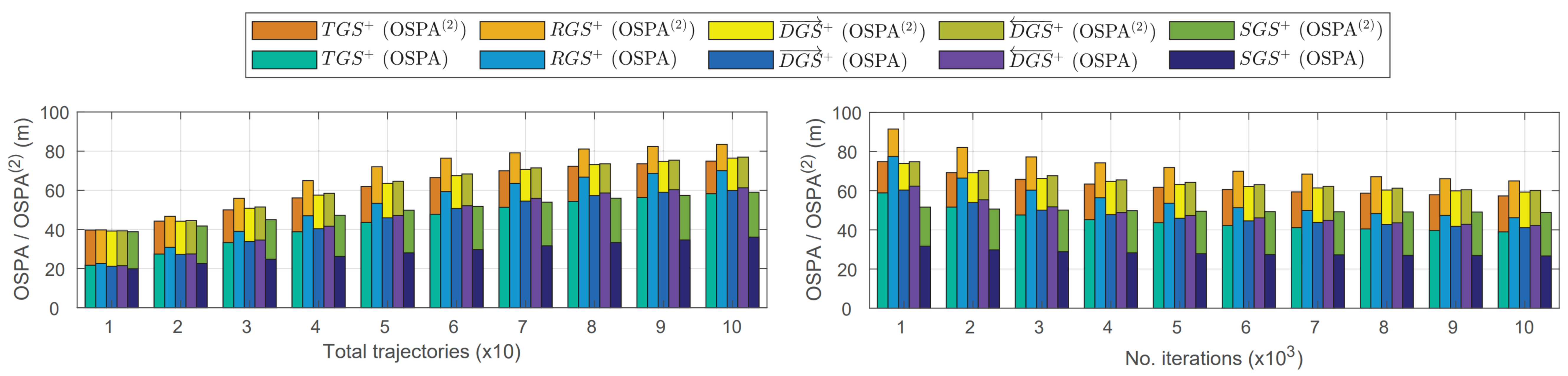
Parameter settings.

Parameter	Setting (default)
Number of Iterations T	1000 - 10000 (5000)
Total Trajectories N_X	10 - 100 (50)
Detection Rate P_D	0.78 - 0.96 (0.86)
Clutter Rate λ_c	50 - 140 (90)
Mixture Rate α	0.1 - 0.9 (0.5)
Tempering Level β	0.1 - 0.9 (0.5)



Random scenario





(a) Varying N_X (b) Varying T

5. Conclusion

- This innovation enables the GLMB filter implementation to be reduced to an **$O(T(P+M+\log T)+PM)$ complexity** from $O(TP^2M)$.
- The proposed framework provides the **flexibility for trade-offs** between tracking performance and computational load.

References

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4. G. Zanella & G. Roberts, "Scalable importance tempering and Bayesian variable selection," J. Roy. Stat. Soc.: Ser. B (Stat. Methodol.), 2019

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