Transmission Schemes for Enhanced DoF in Cache-Aided MIMO Communication Systems

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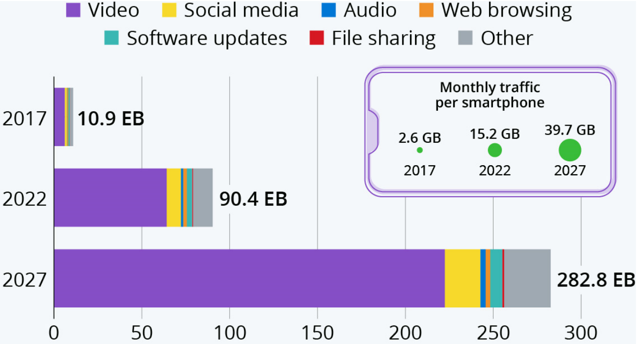
2 MIMO Coded Caching

3 General Transmission Design for MIMO-CC using Covariance Matrices

4 Refrences

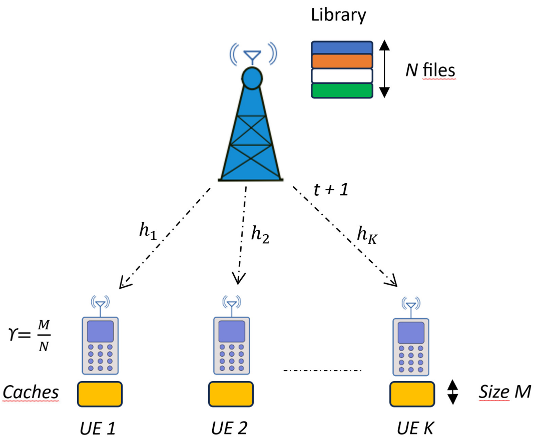
Coded Caching



- Mobile data traffic is growing continuously
  - multimedia content
  - mobile immersive viewing and extended reality.
- Wireless network infrastructure falls under considerable strain
  - demanding requirements: high throughput to ultra-low latency.

Coded Caching: SISO

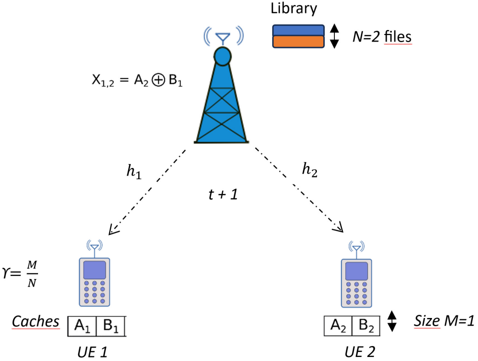
- Coded caching (CC) allows to use the UE memory as a new communication resource



- Offers a new performance (DoF) gain denoted by $t = K\gamma$.
- Enabled by:
  1) Cache-placement,
  2) Delivery phase
- The gain of CC is from multicasting codewords to groups of users of size $t + 1$[a],

[a] A. Maddah-Ali, "Fundamental limits of caching," IEEE Trans. Inf. Theory, vol. 60, no. 5, pp. 2856â2867, 2014

SISO Coded Caching: Example 1



- BS carefully construct codewords within each Tx vector $(t = 1)$

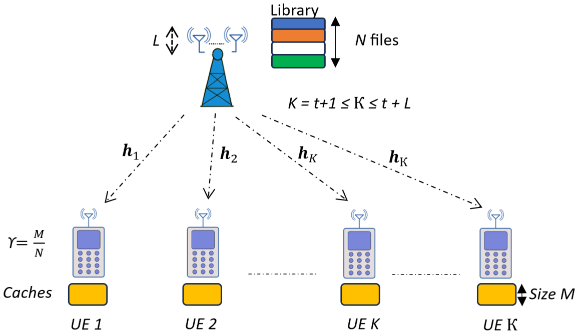
$$\mathrm{X}_{1,2} = A_2 \oplus B_1,$$

- Interference-free decoding of desired signal is possible per user

$$y_1 = h_1(A_2 \oplus B_1) + z_1, \text{ CC-aided Intf cancellation} : \Rightarrow \bar{y}_1 = h_1 A_2 + z_1,$$

$$y_2 = h_2(A_2 \oplus B_1) + z_2, \text{ CC-aided Intf cancellation} : \Rightarrow \bar{y}_2 = h_2 B_1 + z_2,$$

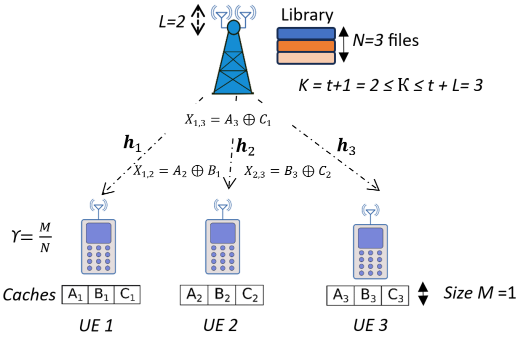
MISO Coded Caching



- The achievable DoF is increased to $t + L$ in a MISO setup [1],
  - The interference by user k can be removed by cache contents at up to t users.
  - The spatial multiplexing gain of L at Tx-side nulls intf at other UEs
  - From receiver perspective of each UE, y_k is an equivalent G-MAC.

[1]. S. P. Shariatpanahi, G. Caire, and B. Hossein Khalaj,"Physical-Layer Schemes for Wireless Coded Caching," IEEE Trans. Inf. Theory, vol. 65, no. 5, pp. 2792â2807, 2019

MISO Coded Caching: Example 1

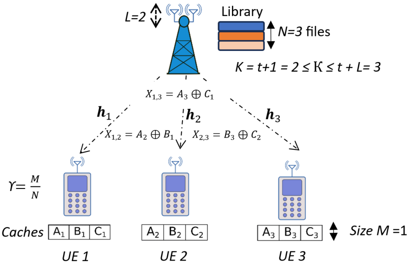


User	Available in Cache	Supp. by Beamformer	Useful data
1	A_1 , B_1, C_1	$C_2 \oplus B_3$	A_2 , A_3
2	A_2 , B_2, C_2	$A_3 \oplus C_1$	B_1, B_3
3	A_3 , B_3, C_3	$A_2 \oplus B_1$	C_1, C_2

Decoding Process for $\mathbf{X}$ at different network UEs

[2] A. T Ìolli, "Multi-antenna interference management for coded caching," IEEE Trans. Wireless Commun., vol. 19, no. 3, pp. 2091â2106, 2020

MISO Coded Caching: Example 1



$$\mathbf{x} = (A_2 \oplus B_1)\mathbf{w}_{1,2} + (A_3 \oplus C_1)\mathbf{w}_{1,3} + (C_2 \oplus B_3)\mathbf{w}_{2,3}$$

$$y_1 = \underline{(\mathbf{h}_1^H \mathbf{w}_{1,2})\tilde{X}_{1,2}} + \underline{(\mathbf{h}_1^H \mathbf{w}_{1,3})\tilde{X}_{1,3}} + (\mathbf{h}_1^H \mathbf{w}_{2,3})\tilde{X}_{2,3} + z_1$$

$$y_2 = \underline{(\mathbf{h}_2^H \mathbf{w}_{1,2})\tilde{X}_{1,2}} + (\mathbf{h}_2^H \mathbf{w}_{1,3})\tilde{X}_{1,3} + \underline{(\mathbf{h}_2^H \mathbf{w}_{2,3})\tilde{X}_{2,3}} + z_2$$

$$y_3 = (\mathbf{h}_3^H \mathbf{w}_{1,2})\tilde{X}_{1,2} + \underline{(\mathbf{h}_3^H \mathbf{w}_{1,3})\tilde{X}_{1,3}} + \underline{(\mathbf{h}_3^H \mathbf{w}_{2,3})\tilde{X}_{2,3}} + z_3$$

User	Available in Cache	Supp. by Beamformer	Useful data
1	A_1, B_1, C_1	$C_2 \oplus B_3$	A_2, A_3
2	A_2, B_2, C_2	$A_3 \oplus C_1$	B_1, B_3
3	A_3, B_3, C_3	$A_2 \oplus B_1$	C_1, C_2

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MIMO CC: System Model

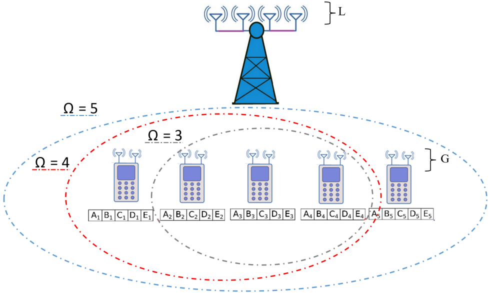


Figure: MIMO CC system model for arbitrary L and G and served users Ω

- Propose a flexible framework with enhanced DoF
  - The number of served UEs selected to optimize the achievable DoF for any given L, G, and t.
- Placement: split each file $W \in \mathcal{F}$ into $\binom{K}{t}$ subfiles $W_{\mathcal{P}}$, where $\mathcal{P} \subseteq [K]$, $|\mathcal{P}| = t$.
- Delivery: The server constructs and transmits the demand set via $\mathbf{x}(n) \in \mathbb{C}^L$,

$$\mathbf{y}_k(n) = \mathbf{H}_k \mathbf{x}(n) + \mathbf{z}_k(n)$$

  - $\mathcal{K}(n) \subseteq [K]$ with $\Omega \in \{t+1, \dots, t+L\}$ members ($|\mathcal{K}(n)| = \Omega$), and $n \in [\binom{K}{\Omega}]$,

Achievable DoF Analysis I

Theorem 1: Achievable Bound for the number of streams linealry decodable

Consider a MIMO-CC system with K users, CC gain t, L Tx-antennas, and G Rx-antennas. A subset $\mathcal{K}$ of users with size $\Omega \leq K$, where $t+1 \leq \Omega \leq t+L$ are served in each interval.

$$\beta \leq \min\left(G, \left(L-(\Omega-t-1)\beta\right)\binom{\Omega-1}{t}\right), \quad (1)$$

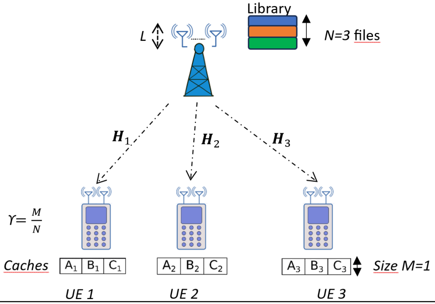
Every user in $\mathcal{K}$ is able to decode β parallel streams interference-free.

Interference caused by every stream sent to user k can be removed:

- By cache content at up to t UEs
- Suppressed by Tx-side precoding for the rest of $\Omega-t-1$ target users;

  $\Rightarrow$ The remaining spatial DoF : $L-(\Omega-t-1)\beta$.
- Maximum value of β directly depends on sufficient L;
  - To keep the direction of subpackets intended to the same null-space orthogonal
  - Otherwise; to keep the null-space of intended packets linearly

Achievable DoF Analysis I: Example



If $G = \beta = 2$, $\Omega = 3$, $t = 1$, then:

$$\mathbf{x} = (A_2 \oplus B_1)\mathbf{w}_{1,2} + (A_3 \oplus C_1)\mathbf{w}_{1,3} + (C_2 \oplus B_3)\mathbf{w}_{2,3}$$

$$y_{1,1} = \mathbf{u}_{1,1}^{\mathrm{H}}\mathbf{H}_1\mathbf{w}_{1,2}(\underline{A_2 \oplus B_1}) + \mathbf{u}_{1,1}^{\mathrm{H}}\mathbf{H}_1\mathbf{w}_{1,3}(\underline{A_3 \oplus C_1}) + \underline{\mathbf{u}_{1,1}^{\mathrm{H}}\mathbf{H}_1\mathbf{w}_{2,3}(C_2 \oplus B_3)} + z_{1,1},$$

$$y_{1,2} = \mathbf{u}_{1,2}^{\mathrm{H}}\mathbf{H}_1\mathbf{w}_{1,2}(\underline{A_2 \oplus B_1}) + \mathbf{u}_{1,2}^{\mathrm{H}}\mathbf{H}_1\mathbf{w}_{1,3}(\underline{A_3 \oplus C_1}) + \underline{\mathbf{u}_{1,2}^{\mathrm{H}}\mathbf{H}_1\mathbf{w}_{2,3}(C_2 \oplus B_3)} + z_{1,2},$$

- Subpackets delivered using linearly independent multicast beamformers,
  $\Rightarrow \mathbf{w}_{1,2} \in \mathrm{Null}(\mathbf{U}_3^{\mathrm{H}}\mathbf{H}_3)$, and $\mathbf{w}_{1,3} \in \mathrm{Null}(\mathbf{U}_2^{\mathrm{H}}\mathbf{H}_2)$;
  $\min_L(2 \times (L-2)) \rightarrow L = 3$ is sufficient, and Thus, $\beta = 2 \times (3-2) = 2$.

Achievable DoF Analysis II

Corollary 1: Achievable DoF

The DoF of $\beta\Omega$ is necessarily achievable in every given MIMO setup, as long as β and Ω satisfy the given condition in Theorem 1. Using Theorem 1, the maximum achievable DoF for the proposed MIMO-CC transmission design is given by solving

$$\begin{aligned} &\mathrm{DoF}_{\max}(\beta^*, \Omega^*) = \max_{\beta,\Omega}\ \Omega\beta, \\ &s.t.\ \ \beta \leq \min\left(G, \frac{L\binom{\Omega-1}{t}}{1+(\Omega-t-1)\binom{\Omega-1}{t}}\right), \end{aligned} \tag{2}$$

where β^* and Ω^* represent the optimal parameters chosen to achieve $\mathrm{DoF}_{\max}$.

Corollary 2: Simplified Achievable DoF

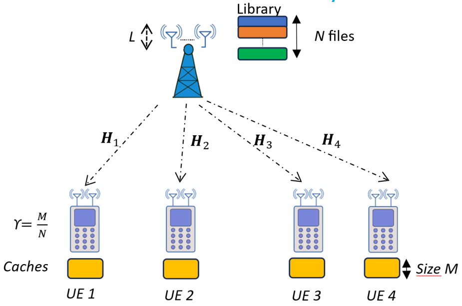
One can obtain the upperbound for Ω w.r.t β in MIMO setup from Theorem I, and search across one dimension.

$$\mathrm{DoF}_{\max}(\beta^*) = \max_{\beta \leq G} \left(\beta\left\lceil \frac{L}{\beta}\right\rceil + \beta t\right) \tag{3}$$

where the bound for Ω is:

$$\Omega \leq \left\lceil \frac{L}{\beta}\right\rceil + t \tag{4}$$

Achievable DoF Analysis II



$$\mathrm{DoF}_{\max}(\beta^*) = \max_{\beta \le G} \left(\beta \left\lceil \frac{L}{\beta} \right\rceil + \beta t\right)$$

Given Theorem 1 and $G = 2$, $t = 1$, $L = 3$, Then;

- $\Omega = 2,\ \beta \le \min\left(2, \frac{3\binom{2-1}{1}}{1+(2-1-1)\binom{2-1}{1}}\right) = 1 \Rightarrow \mathrm{DoF} = \left(1\left\lceil \frac{3}{1} \right\rceil + 1\right) = 4$
- $\Omega = 3,\ \beta \le \min\left(2, \frac{3\binom{3-1}{1}}{1+(3-1-1)\binom{3-1}{1}}\right) = 2 \Rightarrow \mathrm{DoF} = \left(2\left\lceil \frac{3}{2} \right\rceil + 2\right) = 6 = \mathrm{DoF}_{\max}$
- $\Omega = 4,\ \beta \le \min\left(2, \frac{3\binom{4-1}{1}}{1+(4-1-1)\binom{4-1}{1}}\right) = 1 \Rightarrow \mathrm{DoF} = \left(1\left\lceil \frac{3}{1} \right\rceil + 1\right) = 4$

Achievable DoF Analysis: Illustration

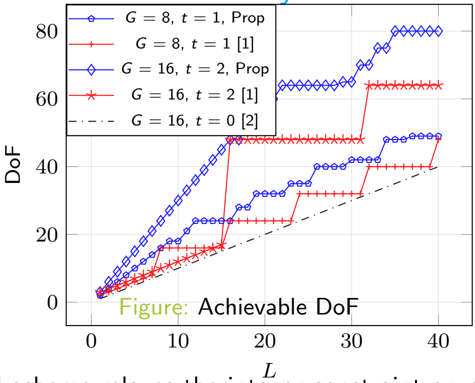


Figure: Achievable DoF

- The proposed scheme relaxes the integer constraint on $\frac{L}{G}$
- Gain boost of MIMO-CC setups: with smaller TX-side SM gains than [3][3].
- This flexibility becomes evident when enhancing the RX SM per UE
- Proposed DoF consistently achieves higher levels v.s MIMO scheme $(t = 0)$

[3] E. Parrinello, "Fundamental Limits of Coded Caching with Multiple Antennas, Shared Caches and Uncoded Prefetching," IEEE Trans. Inf. Theory, vol. 66, no. 4, pp. 2252â2268, 2020.

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General Transmission Design for MIMO-CC using Covariance Matrices

- Consider a generic multicast transmission signal vector $\mathbf{x}(i)$, built as

$$\mathbf{x}(i) = \sum_{\mathcal{T} \subseteq \mathcal{K}(i), |\mathcal{T}|=t+1} \tilde{\mathbf{x}}_{\mathcal{T}}(i) \,, \forall i \,, \tag{5}$$

  - $\tilde{\mathbf{x}}_{\mathcal{T}}(i)$ is the corresponding multicast signal at time instant i, chosen from a complex Gaussian codebook $\tilde{\mathbf{x}}_{\mathcal{T}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{K}_{\tilde{\mathbf{x}}_{\mathcal{T}}})$.
  - $\mathbf{x} = \sum_{\mathcal{T} \subseteq \mathcal{K}, |\mathcal{T}|=t+1} \tilde{\mathbf{x}}_{\mathcal{T}}$, and $\mathbf{K}_{\mathbf{x}} = \sum_{\mathcal{T} \subseteq \mathcal{K}, |\mathcal{T}|=t+1} \mathbf{K}_{\tilde{\mathbf{x}}_{\mathcal{T}}}$.
  - This allows us to rewrite (10) as follows:

$$\mathbf{y}_k = \mathbf{H}_k \sum_{\mathcal{T} \in \mathcal{S}_k^{\mathcal{K}}} \tilde{\mathbf{x}}_{\mathcal{T}} + \mathbf{H}_k \sum_{\mathcal{T} \in \bar{\mathcal{S}}_k^{\mathcal{K}}} \tilde{\mathbf{x}}_{\mathcal{T}} + \mathbf{z}_k \,, \tag{6}$$

General Transmission Design for MIMO-CC using Covariance Matrices: Example

- Given $L = 4$, $\Omega = 3$ users, $t = 1$ (i.e., $\mathrm{DoF} = 6$), and files A–C requested by users 1-3, respectively. The MC transmission vector is

$$\mathbf{x} = \mathbf{x}_{1,2} + \mathbf{x}_{1,3} + \mathbf{x}_{2,3}, \tag{7}$$

- Each signal is proportional $\mathbf{x}_{1,2} \propto (A_2 \oplus B_1)$, $\mathbf{x}_{1,3} \propto (A_3 \oplus C_1)$, $\mathbf{x}_{2,3} \propto (B_3 \oplus C_2)$.[4]
- Then, the received signals at users 1-3:

$$\begin{aligned}
\mathbf{y}_1 &= \underline{\mathbf{H}_1\mathbf{x}_{1,2}} + \underline{\mathbf{H}_1\mathbf{x}_{1,3}} + \mathbf{H}_1\mathbf{x}_{2,3} + \mathbf{z}_1 \ , \\
\mathbf{y}_2 &= \underline{\mathbf{H}_2\mathbf{x}_{1,2}} + \underline{\mathbf{H}_2\mathbf{x}_{2,3}} + \mathbf{H}_2\mathbf{x}_{1,3} + \mathbf{z}_2 \ , \\
\mathbf{y}_3 &= \underline{\mathbf{H}_3\mathbf{x}_{1,3}} + \underline{\mathbf{H}_3\mathbf{x}_{2,3}} + \mathbf{H}_3\mathbf{x}_{1,2} + \mathbf{z}_3 \ ,
\end{aligned}$$

- UE 1, both $\mathbf{x}_{1,2}$ and $\mathbf{x}_{1,3}$: desired signals, and $\mathbf{x}_{2,3}$: Gaussian interference

  $\Rightarrow$y$_1$: an equivalent G-MAC channel, equal rate decoding of its msgs

$$R^1_{MAC} = \min\left(R_{\{1,2\}}, R_{\{1,3\}}, \frac{1}{2}R_{\{\{1,2\},\{1,3\}\}}\right), \tag{8}$$

[4] As a special case, in (7), the transmission vector $\mathbf{x} = (A_2 \oplus B_1)\mathbf{w}_{1,2} + (A_3 \oplus C_1)\mathbf{w}_{1,3} + (B_3 \oplus C_2)\mathbf{w}_{2,3}$ [4].

MULTIGROUP MULTICAST TRANSMISSION

Symmetric rate

Assuming that all requests for files are served⇒ The worst-case delivery rate.

$$R_{sym} = \frac{K}{\sum_n T_n} = \frac{K\binom{K}{t}\binom{K-t-1}{\Omega-t-1}}{\sum_n \frac{1}{R_{\text{max-min}}(n)}}, \quad (9)$$

The goal: Design transmission parameters, R_{sym} is maximized in the delivery scheme.

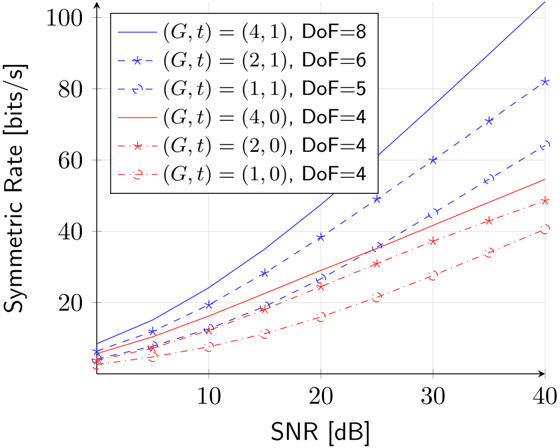
Max-Min Rate Optimization Problem

$$R_{\max-\min}(n) = \max_{\mathbf{K}_{\mathbf{x}_\mathcal{T}}, R_\mathcal{T}, \mathcal{T}\in \mathsf{B}} \min_{k\in\mathcal{K}} \min_{\mathsf{B}\subseteq \mathsf{S}_k^\mathcal{K}} \left[\frac{1}{|\mathsf{B}|}\sum_{\mathcal{T}\in\mathsf{B}} R_\mathcal{T}\right]$$

$$s.t. \quad \sum_{\mathcal{T}\in\mathsf{B}} R_\mathcal{T} \le \log\left|\mathbf{I} + \mathbf{H}_k \sum_{\mathcal{T}\in\mathsf{B}} \mathbf{K}_{\mathbf{x}_\mathcal{T}} \mathbf{H}_k^H \mathbf{Q}_k^{-1}\right|, \ \forall k\in\mathcal{K}, \mathsf{B}\subseteq \mathsf{S}_k^\mathcal{K}$$

$$\sum_{\mathcal{T}\in\mathsf{S}^\mathcal{K}} \mathrm{Tr}(\mathbf{K}_{\mathbf{x}_\mathcal{T}}) \le P_T, \quad \text{where} \quad \mathbf{Q}_k = (N_0\mathbf{I} + \mathbf{H}_k \sum_{\mathcal{T}\in\bar{\mathsf{S}}_k^\mathcal{K}} \mathbf{K}_{\mathbf{x}_\mathcal{T}} \mathbf{H}_k^H)$$

General Transmission Design for MIMO-CC Cov Matrices: Simulation



- Covariance of the transmission signal, aiming for improved performance while achieving the enhanced DoF in Theorem 1.
- DoF scales with t, and G, when $L=4$ for all setups.

General Transmission Design for MIMO-CC Cov Matrices: Simulation

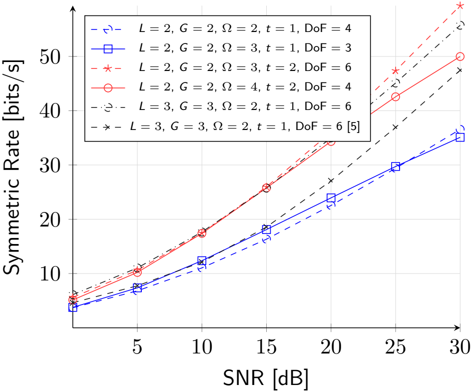


Figure: MIMO multicast design for $L = G$

- Cov-based design achieves the enhanced DoF, if params selected based on Theorem 1.
- Proposed approach achieves a DoF boost even with smaller Tx-spatial SM.

General Transmission Design for MIMO-CC Cov Matrices: Simulation

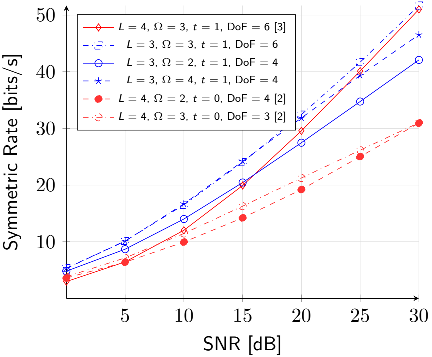


Figure: MIMO multicast design for $L > G$ - $G = 2, t = \{1, 0\}$

- Compare Extended-Shared cache model, achieving DoF $Gt + L$ for divisible ratio of $\frac{L}{G}$
  $\Rightarrow L = 3$, $t = 1$, $G = \beta = 2$, DoF $= 6$, while [3] requires higher Tx SM $L = 4$[5],

[5] E. Parrinello, "Fundamental Limits of Coded Caching with Multiple Antennas, Shared Caches and Uncoded Prefetching," IEEE Trans. Inf. Theory, vol. 66, no. 4, pp. 2252â2268, 2020.

Conclusion and Future Work

- We proposed a flexible CC scheme for MIMO setups
  - Optimizing the number of users served in each transmission to maximize the achievable DoF.
- Proposed a high-performance beamformer design for MIMO-CC setups
  - Formulating the problem of maximizing the symmetric rate w.r.t transmit covariance matrices for the multicast signals.
  - Solved the non-convex problem using SCA and verified the performance enhancement through simulations.
- Optimality Proof of the proposed bound; (Converse Theorem)
- Generalized form of the proposed DoF with different SM per UE G_k;

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[1] M. J. Salehi, E. Parrinello, S. P. Shariatpanahi, P. Elia, and A. Tolli, "Low-Complexity High-Performance Cyclic Caching for Large MISO Systems," vol. 21, no. 5, pp. 3263–3278, 2022.

[2] E. Biglieri, R. Calderbank, A. Constantinides, A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO wireless communications*. Cambridge university press, 2007.

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[5] M. Salehi, M. NaseriTehrani, and A. Tölli, "Multicast beamformer design for mimo coded caching systems," in *2023-2023 IEEE ICASSP*. IEEE, 2023, pp. 1–5.