# CONTRASTIVE DEEP NONNEGATIVE MATRIX FACTORIZATION FOR COMMUNITY DETECTION

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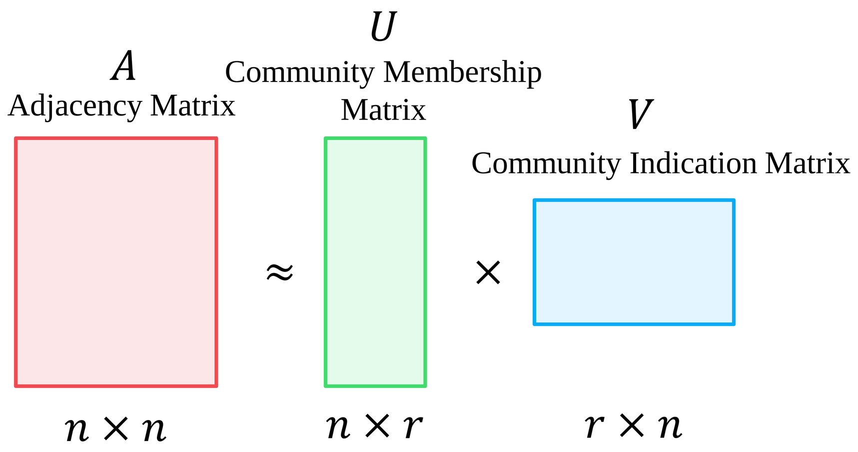
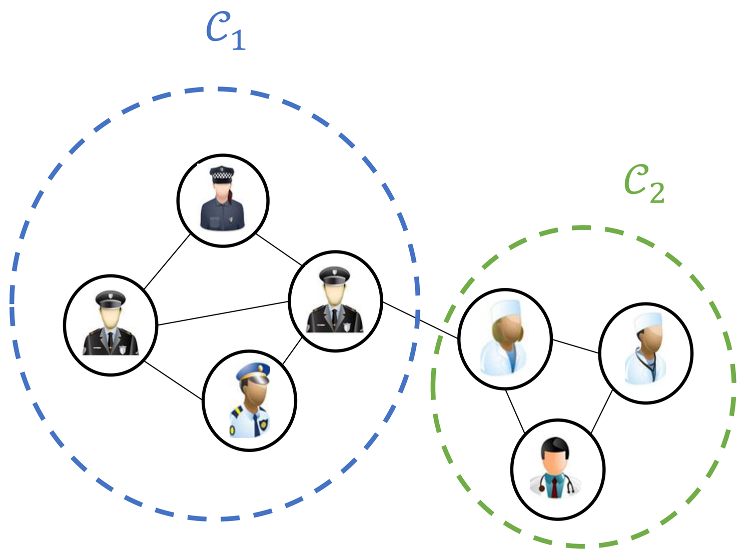
## BACKGROUND

Shallow NMF based Network Topology → Community Detection

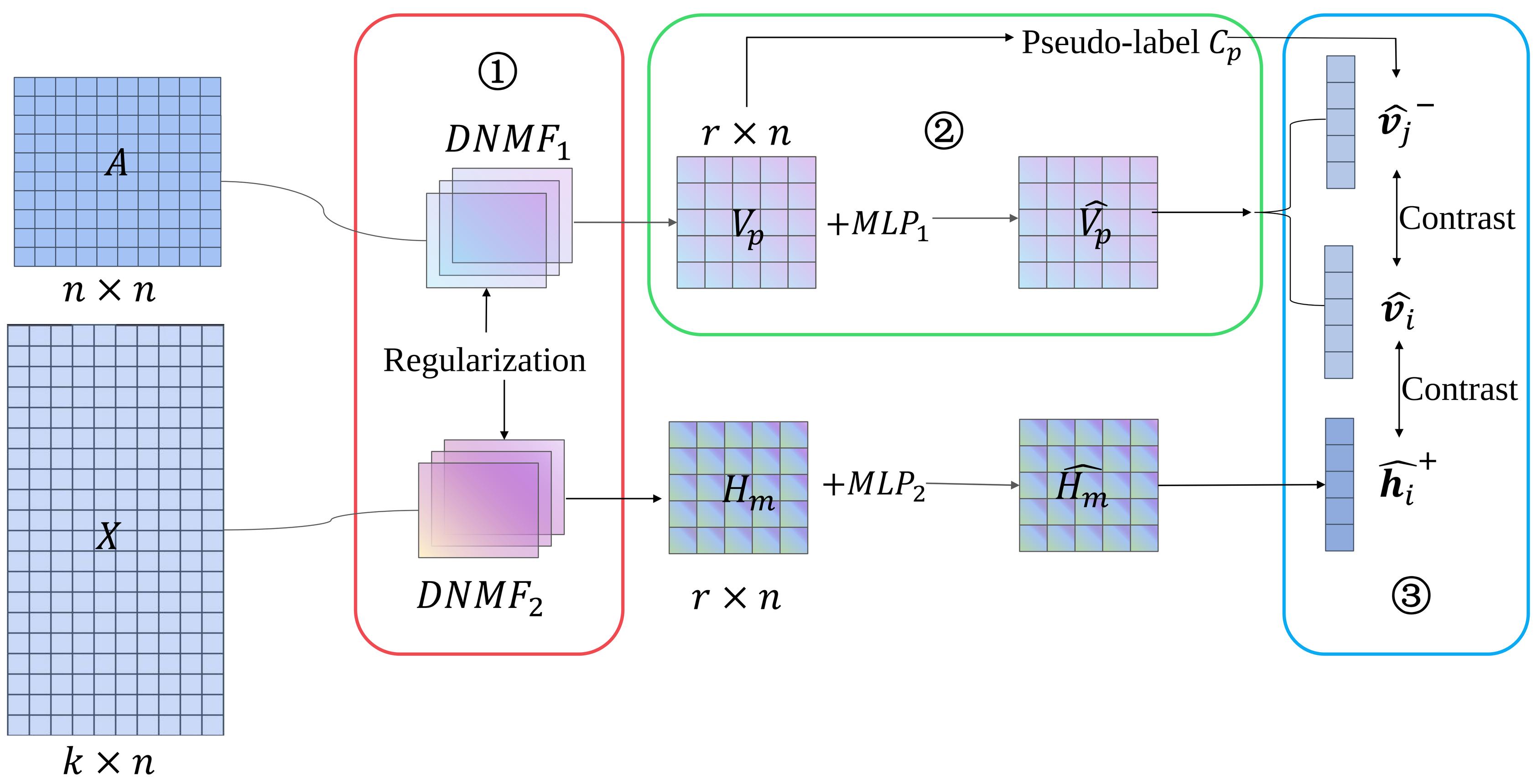



Three shortcomings of existing methods:

1) Directly transform the original network into community membership;
2) **Only pay attention to the topology of the network and ignore its node attributes;**
3) **Hard to learn the global structure information necessary for community detection.**

## PROPOSED METHOD: CDNMF




Total objective function of Our **CDNMF**:

$$\min_{U_i, V_p, W_j, H_m} L = L_{DNMF} + \beta L_{reg} + \gamma L_{cl}$$

Key: fusing information about **network topology (A)** and **node attributes (X)** in the framework of NMF using the idea of contrastive learning

① **DNMF Layer**

$$\min_{U_i, V_p, W_j, H_m} L_{DNMF} = L_A + L_X$$

1.1) Decompose the adjacency matrix A of the graph

$$\min_{U_i, V_p} L_A = \|A - U_1 U_2 \ldots U_p V_p\|_F^2$$

$$+ \alpha \left( \sum_{i=1}^{p} \|f(U_i)\|_F^2 + \|f(V_p)\|_F^2 \right)$$

1.1.1) Reconstruction loss

$$\min_{U_i, V_p} L_D = \|A - U_1 U_2 \ldots U_p V_p\|_F^2$$

$$\text{s.t. } V_p \geq \mathbf{0}, U_i \geq \mathbf{0}, \forall i = 1, 2, \ldots, p$$

1.1.2) Nonnegative penalty loss

$$f(B) = \begin{cases} B_{ij}, B_{ij} < 0 \\ 0, B_{ij} \geq 0 \end{cases}$$

1.2) Similarly decompose the attribute matrix X of the graph

$$\min_{W_j, H_m} L_X = \|X - W_1 W_2 \ldots W_m H_m\|_F^2$$

$$+ \alpha \left( \sum_{j=1}^{m} \|f(W_j)\|_F^2 + \|f(H_m)\|_F^2 \right).$$

② **Debiased Negative Sampling Layer**

$$c_i^* = \operatorname{argmax}(V_p(:, i)) \qquad \widetilde{\mathcal{N}}_i = \{v_m\} \, (c_m^* \neq c_i^*)$$

Constructing **community-level discrimination** based on the interpretable results $V_p$ from DNMF.

③ **Graph Contrastive Learning Layer**

$$\min_{V_p, H_m} L_{cl} = -\frac{1}{n} \sum_{i=1}^{n} l(V_p(:, i), H_m(:, i))$$

For each node, we have

$$l(V_p(:, i), H_m(:, i))$$

$$= \log \frac{e^{\theta(V_p(:,i), H_m(:,i))/\tau}}{e^{\theta(V_p(:,i), H_m(:,i))/\tau} + \sum_{k=1}^{n} \mathbb{1}_{[k \in \widetilde{\mathcal{N}}_i]} e^{\theta(V_p(:,i), V_p(:,k))/\tau}}$$

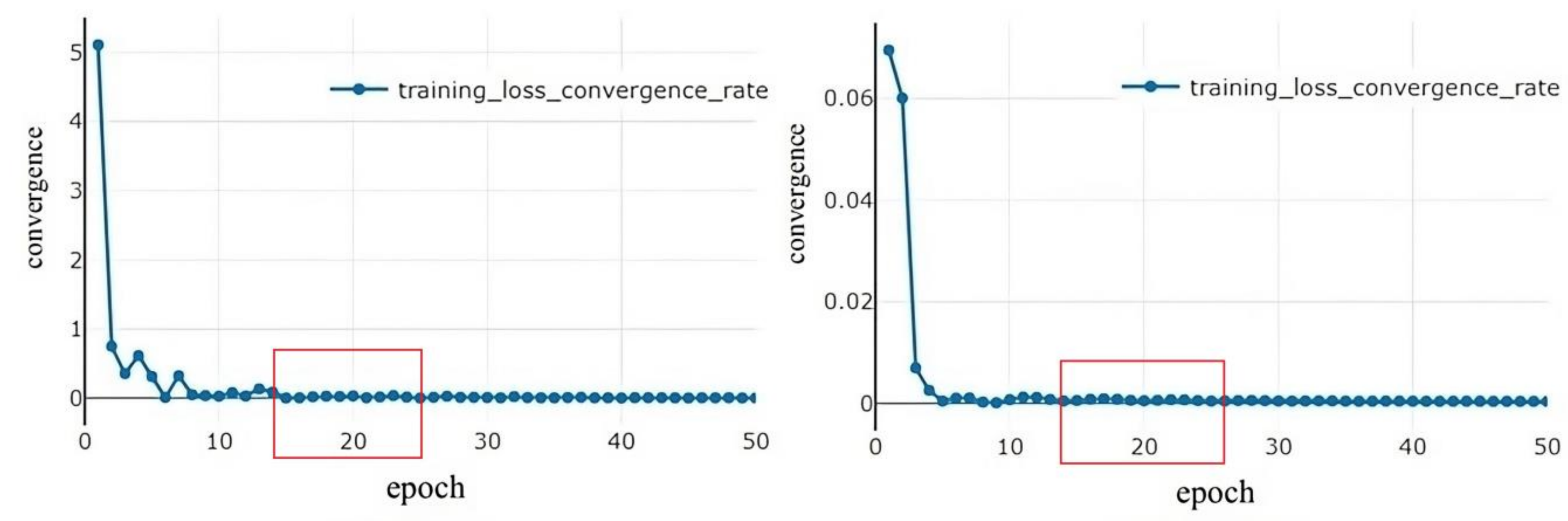
**Consistent with the community semantics implied by graph topology and node attributes, and constructs embedding spaces with significant community structure.**

## EXPERIMENTS

① Excellent **community detection results**

| Method | Cora | | Citeseer | | PubMed | |
|---|---|---|---|---|---|---|
| | ACC | NMI | ACC | NMI | ACC | NMI |
| NMF | 0.4103 | 0.2851 | 0.3074 | 0.1319 | 0.5133 | 0.1606 |
| ONMF | 0.3811 | 0.2416 | 0.3330 | 0.1423 | 0.5575 | 0.1582 |
| BNMF | 0.4191 | 0.2521 | 0.3324 | 0.0825 | 0.5110 | 0.0714 |
| NSED | 0.4234 | 0.2928 | 0.3448 | 0.1492 | 0.5201 | 0.1729 |
| LINE | 0.4044 | 0.2376 | 0.3019 | 0.0573 | 0.4990 | 0.1357 |
| Node2Vec | 0.3674 | 0.1978 | 0.2521 | 0.0486 | 0.4067 | 0.0635 |
| MNMF | 0.1647 | 0.0035 | 0.1890 | 0.0031 | 0.3397 | 0.0002 |
| LP-FNMTF | 0.2861 | 0.0261 | 0.2327 | 0.0143 | 0.5437 | 0.1532 |
| K-means++ | 0.3230 | 0.2210 | 0.4160 | 0.1910 | 0.4150 | 0.2300 |
| VGAER | 0.4530 | 0.2970 | 0.3020 | 0.2170 | 0.3010 | 0.2230 |
| DNMF | 0.4849 | 0.3572 | 0.3635 | 0.1582 | 0.5389 | 0.1709 |
| DANMF | 0.5499 | 0.3764 | 0.4242 | 0.1831 | 0.6393 | 0.2221 |
| Ours | **0.6081** | **0.4006** | **0.4756** | **0.2559** | **0.6653** | **0.2330** |

② High **running efficiency** and **fast convergence**




(a) Cora (b) Citeseer

③ Necessity of **contrastive schema**

| Methods | Cora | | | | Citeseer | | | |
|---|---|---|---|---|---|---|---|---|
| | ACC | Δ | NMI | Δ | ACC | Δ | NMI | Δ |
| Ours $[L(A)]$ | 0.5835 | 2.46% | 0.3781 | 2.25% | 0.4598 | 1.58% | 0.1672 | 8.87% |
| Ours $[L(X)]$ | 0.5162 | 9.19% | 0.3501 | 5.05% | 0.3499 | 12.6% | 0.1749 | 8.10% |
| Ours | **0.6081** | | **0.4006** | | **0.4756** | | **0.2559** | |

## CONCLUSIONS AND FUTURE DIRECTIONS

- We introduce the idea of **contrastive learning** (CL) into the **nonnegative matrix factorization** (NMF) for community detection (CD) **for the first time**, solving the problems of the existing work. The two modules are **mutually reinforcing** and **naturally coupled**.
- Different matrix factorization methods (such as **ONMF**, **BNMF**, **DANMF**, etc.) and other contrastive learning algorithms (such as **MVGRL**, **GRACE**, etc.) could be explored to further derive more community detection algorithms **under the CDNMF framework**.

## Paper, Code, Blog and Contact

Github (Paper & Code) | My Blog | Twitter