Unsupervised Optimal Power Flow Using Graph Neural Networks

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Optimal Power Flow

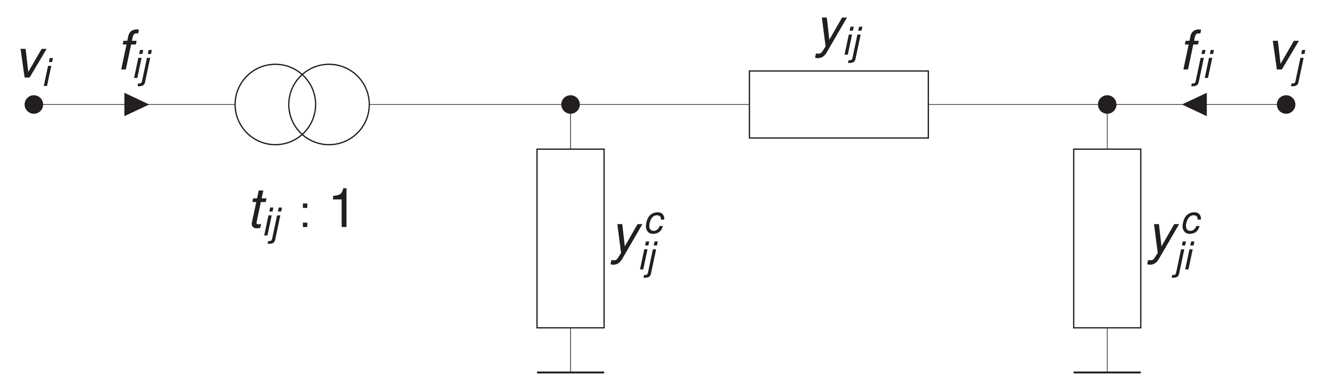


Figure: π-section branch model for edge (i,j) in transmission system.

$$
\begin{aligned}
&\underset{\mathbf{s}^g,\mathbf{v}}{\text{minimize}} && C(\mathbf{s}^g)\\
&\text{subject to} && \\
&\text{(generator limits)} && \mathbf{s}^g_{\min} \preceq \mathbf{s}^g \preceq \mathbf{s}^g_{\max}\\
&\text{(voltage limits)} && \mathbf{v}_{\min} \preceq |\mathbf{v}| \leq \mathbf{v}_{\max}\\
&\text{(line flow limits)} && |\mathbf{f}_f| \preceq \mathbf{f}_{\max}\\
&\text{(line flow limits)} && |\mathbf{f}_t| \preceq \mathbf{f}_{\max}\\
&\text{(angle difference limits)} && \boldsymbol{\theta}_{\min} \preceq \angle(\mathbf{v}_f \mathbf{v}_t^*) \preceq \boldsymbol{\theta}_{\max}\\
&\text{(Ohm's law)} && \mathbf{f}_f = (\mathbf{y} + \mathbf{y}^c_f)^* \frac{|\mathbf{v}_f|^2}{|\mathbf{t}|^2} - \mathbf{y}^* \frac{\mathbf{v}_f \mathbf{v}_t^*}{\mathbf{t}}\\
&\text{(Ohm's law)} && \mathbf{f}_t = (\mathbf{y} + \mathbf{y}^c_t)^* |\mathbf{v}_t|^2 - \mathbf{y}^* \frac{\mathbf{v}_f^* \mathbf{v}_t}{\mathbf{t}^*}\\
&\text{(energy conservation)} && \mathbf{s}^g - \mathbf{s}^d = (\mathbf{y}^s)^* |\mathbf{v}|^2 + C_f^T \mathbf{f}_f + C_t^T \mathbf{f}_t
\end{aligned}
$$

OPF as a Graph Signal Processing Problem

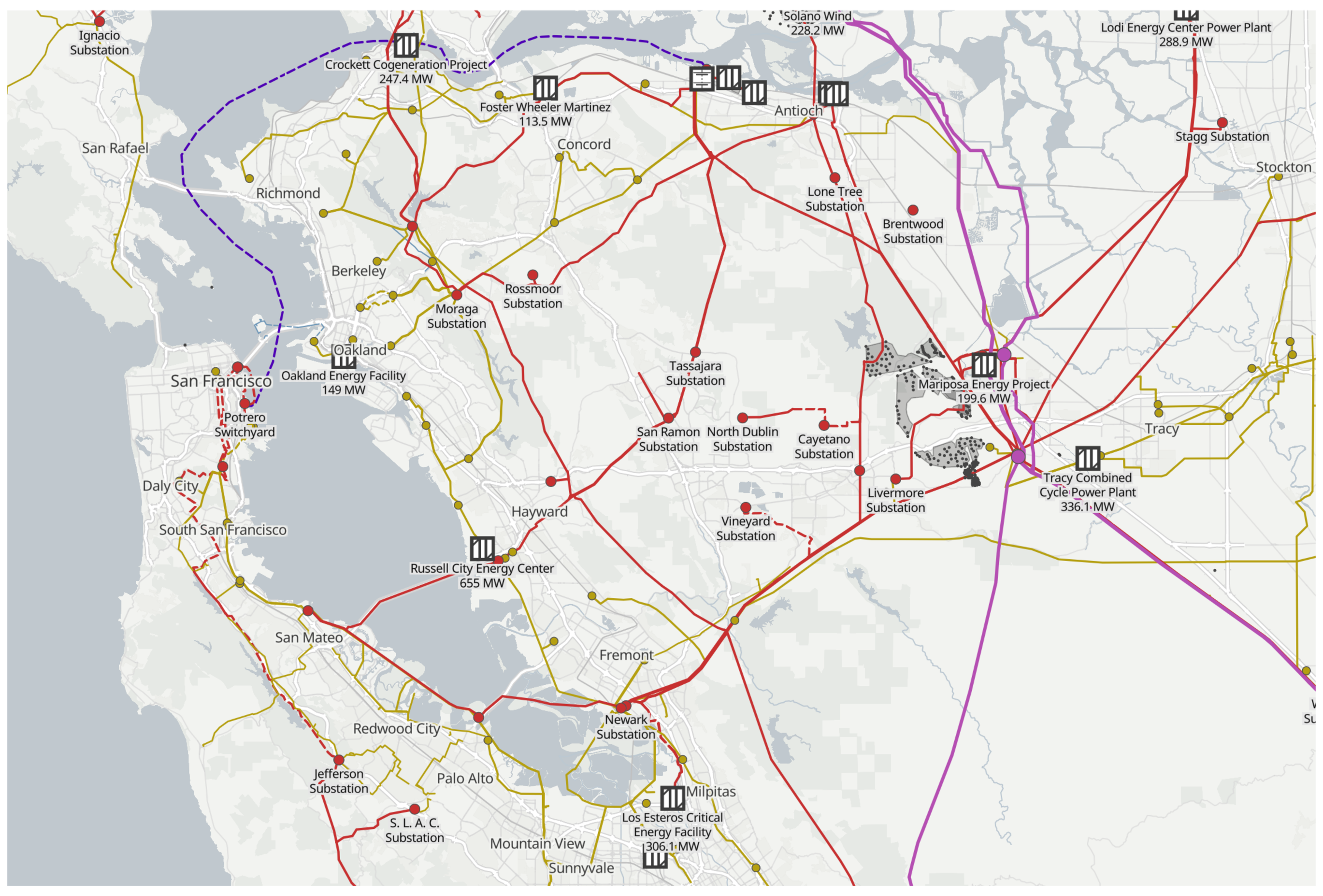


Figure: Map of San Francisco Bay power infrastructure from OpenStreetMap/OpenInfraMap.

- Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$:
  - $\Rightarrow$ nodes $i \in \mathcal{V}$ represent buses $\Rightarrow$ generators and loads.
  - $\Rightarrow$ edges $(i,j) \in \mathcal{E}$ represent branches $\Rightarrow$ lines and transformers.
- GNN $\mathbf{X} = \Phi(\mathbf{Z}, \mathcal{H}, \mathcal{G})$ with learnable parameters $\mathcal{H}$.
  - $\Rightarrow$ Input: $\mathbf{Z} = \left[\text{Re}(\mathbf{s}^d)\ \text{Im}(\mathbf{s}^d)\ \text{Re}(\mathbf{s}^g_{\min})\ \text{Im}(\mathbf{s}^g_{\max})\ \mathbf{v}_{\min}\ \mathbf{v}_{\max}\right] \in \mathbb{R}^{N\times 8}$
  - $\Rightarrow$ Output: $\mathbf{X} = \left[\text{Re}(\mathbf{s}^g)\ \text{Im}(\mathbf{s}^g)\ |\mathbf{v}|\ \angle\mathbf{v}\right] \in \mathbb{R}^{N\times 4}$
- Need to find parameters $\mathcal{H}$ that satisfy constraints.

$$
\begin{aligned}
&\underset{\mathcal{H}}{\text{minimize}} && C(\Phi(\mathbf{Z}, \mathcal{H}, \mathcal{G})) &\\
&\text{subject to} && &\\
&\text{(inequality constraints)} && g_i(\mathbf{X}) \preceq 0 & i = 1, \dots, n\\
&\text{(equality constaints)} && h_i(\mathbf{X}) = 0 & i = 1, \dots, m
\end{aligned}
$$

Penalty Function Relaxation

- Relax inequality $g_i(\mathbf{X})$ by adding penalty ϕ_i.
- Relax equality $h_i(\mathbf{X})$ by adding penalty ψ_i.

$$\mathcal{H}^* = \underset{\mathcal{H}}{\text{argmin}}\ \mathbb{E}_Z \left[\mathcal{L}(\Phi(\mathbf{Z}, \mathcal{H}, \mathcal{G}))\right]$$

$$\mathcal{L}(\mathbf{X}) = C(\mathbf{X}) + \sum_{i=1}^{n} \lambda_i \phi_i(\mathbf{X}) + \sum_{i=1}^{m} \mu_i \psi_i(\mathbf{X})$$

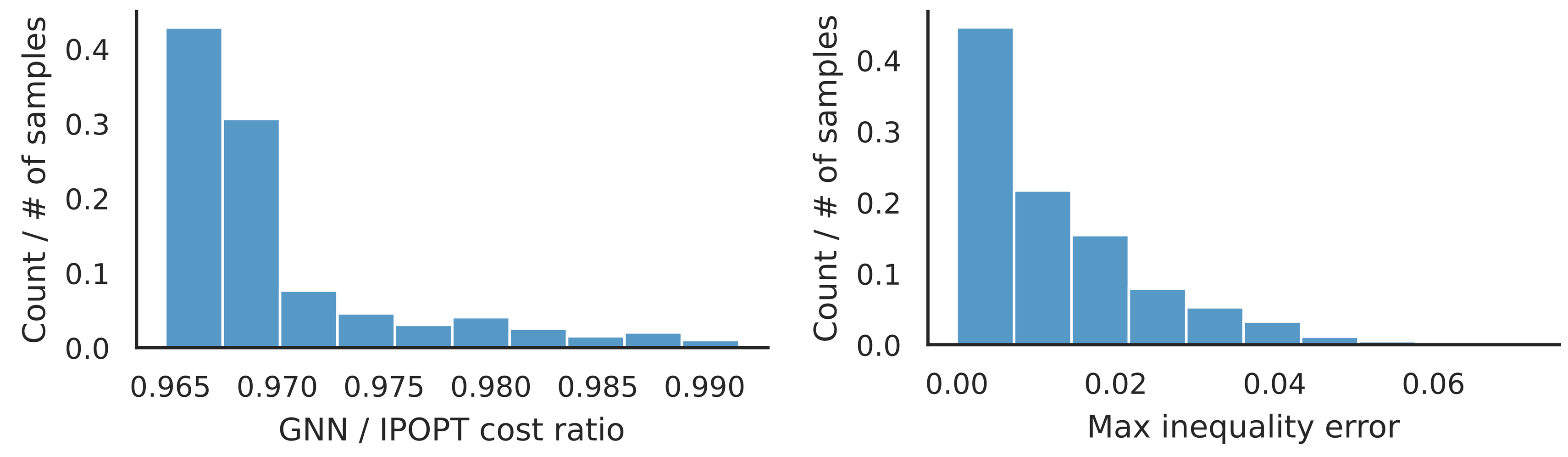
- The problem is unconstrained with hyperparameters λ_i, μ_i.

Numerical Experiments

- Evaluated performance on the IEEE-30 and IEEE-118 systems.
- Generated data by perturbing reference load $\mathbf{s}^d_{\text{ref}}$:

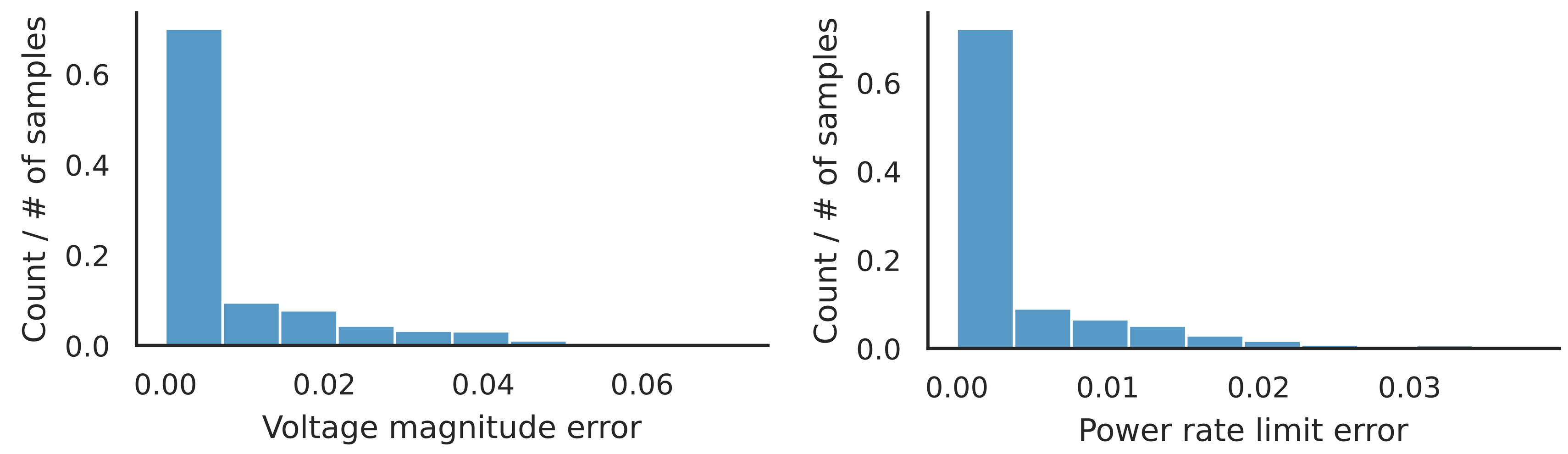
$$\mathbf{s}^d \sim \text{Uniform}(0.9\mathbf{s}^d_{\text{ref}},\ 1.1\mathbf{s}^d_{\text{ref}})$$

- Training on 10,000 unlabeled demand samples.
- Compare GNN against interior point methods (IPOPT).



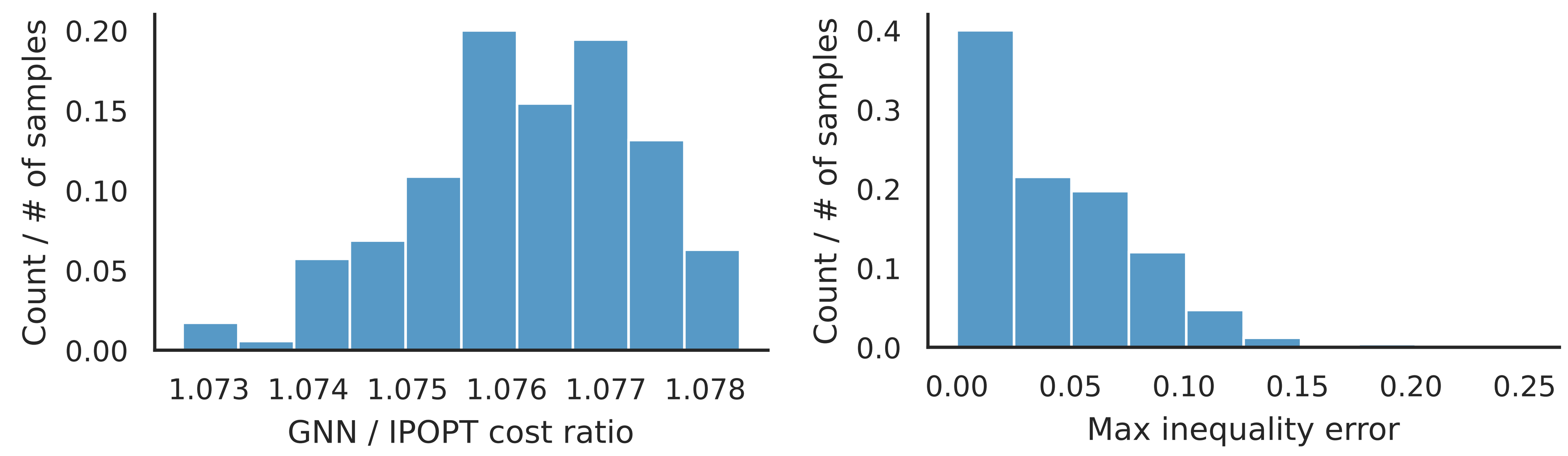
(a) (b)

Figure: GNN performance on the IEEE-30 test dataset.



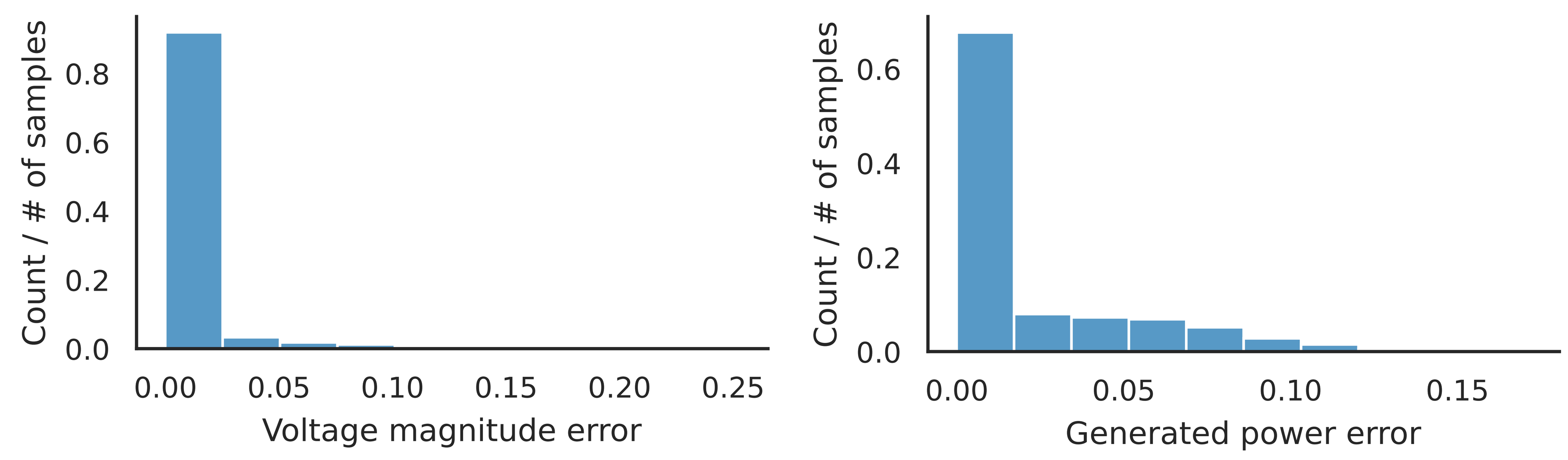
(a) (b)

Figure: IEEE-30 constraint violations of (a) voltage magnitude and (b) flow limits.



(a) (b)

Figure: GNN performance on the IEEE-118 test dataset.



(a) (b)

Figure: IEEE-118 constraint violations of (a) voltage magnitude and (b) generation.

Results and Discussion

- Compared GNN against an interior point method (IPOPT).
  - $\Rightarrow$ On IEEE-30: GNN outperformed IPOPT.
  - $\Rightarrow$ On IEEE-118: higher cost and frequently infeasible.
- Difference between system topologies explains this:
  - $\Rightarrow$ IEEE-30 has binding branch flow limits.
  - $\Rightarrow$ IEEE-118 does not have limits on power flow.

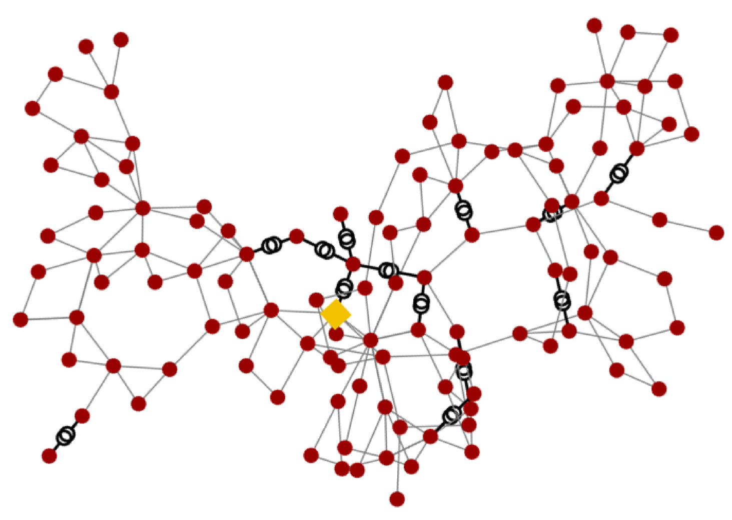


Figure: IEEE-118 Test System Graph