

Semi-Supervised Graphical Deep Dictionary Learning for Hyperspectral Image Classification from Limited Samples

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Background

- In Hyperspectral Image Classification (HSI), the typical process involves manually labeling a subset of the acquired image samples to create the training set to predict the labels for the remaining unlabeled samples in the testing set.
- Since manual labeling is a labor-intensive and time-consuming task requiring expert knowledge, it is desirable to minimize the number of samples that need to be manually labeled. Therefore, practical hyperspectral imaging always strives for accurate classification from the fewest possible labeled samples.
- In this work, we aim to reach the high classification rates of current deep learning methods with the training data requirement of shallow learning techniques.
- Another unique aspect of hyperspectral imaging is that the samples are spatially correlated; i.e. contiguous locations are likely to belong to the same class unless they are along the edges.

Objectives

- In this work, we propose a unified semi-supervised feature learning framework that jointly learns the features for both the labeled and unlabeled samples in such a fashion that features are spatially correlated via a graph structure.
- The generated features are input to a third-party classifier for final classification.
- Our work is based on the framework of Deep Dictionary Learning (DDL) and Deep Transform Learning (DTL) [14].
- In DDL, X is the input data, D_1, D_2, D_3 are three layers of dictionaries and Z is the coefficient / representation.
- In DTL, T_1, T_2, T_3 are three layers of transforms and Z is the coefficient/representation.

Deep Dictionary Learning (DDL)

$$\min_{D_1, D_2, D_3, Z} \|X - D_1 D_2 D_3 Z\|_F^2$$

$$s.t. D_2 D_3 Z \geq 0, D_3 Z \geq 0 \text{ and } Z \geq 0$$

Deep Transform Learning (DTL)

$$\min_{T_1, T_2, T_3, Z} \|T_3 T_2 T_1 X - Z\|_F^2 + \lambda \sum_{i=1}^3 (\|T_i\|_F^2 - \log \det(T_i))$$

$$s.t. T_2 T_1 Z \geq 0, T_1 X \geq 0 \text{ and } Z \geq 0$$

Methodology

DDL based Formulation: Synthesis Version

$$\min_{D_1, D_2, D_3, Z} \left\| [X_S | X_U] - D_1 D_2 D_3 [Z_S | Z_U] \right\|_F^2$$

$$+ \mu \sum_{i=1}^2 \text{Trace}(Z L_i Z^T) + \gamma \sum_c \|\bar{Z}_c - Z_c\|_F^2$$

$$s.t. D_2 D_3 Z \geq 0, D_3 Z \geq 0 \text{ and } Z \geq 0$$

Solution via Alternating Minimization

$$\begin{aligned} \min_{D_1} \|X - D_1 D_2 D_3 Z\|_F^2 & \quad D_1 = X (D_2 D_3 Z)^\dagger \\ \min_{D_2} \|X - D_1 D_2 D_3 Z\|_F^2 & \quad D_2 = (D_1)^\dagger X (D_3 Z)^\dagger \\ \min_{D_3} \|X - D_1 D_2 D_3 Z\|_F^2 & \quad D_3 = (D_1 D_2)^\dagger X (Z)^\dagger \\ \min_{Z_U} \|X_U - D_1 D_2 D_3 Z_U\|_F^2 + m \text{Trace}(Z_S L_i Z_S^T) & \\ \downarrow & \\ (D_1 D_2 D_3)^T D_1 D_2 D_3 Z_U + \lambda Z_U (L_1 + L_2) & \\ = (D_1 D_2 D_3)^T X & \quad \min_{Z_c} \|X_c - D_1 D_2 D_3 Z_c\|_F^2 \\ & \quad + \mu \text{Trace}(Z_c L_i Z_c^T) + \gamma \|\bar{Z}_c - Z_c\|_F^2 \\ \downarrow & \\ [(D_1 D_2 D_3)^T D_1 D_2 D_3 + \gamma I] Z_c + \lambda Z_c (L_1 + L_2) & \\ = (D_1 D_2 D_3)^T X + \bar{Z}_c & \end{aligned}$$

DTL based Formulation: Analysis Version

$$\min_{T_1, T_2, T_3, Z} \|T_3 T_2 T_1 [X_S | X_U] - [Z_S | Z_U]\|_F^2$$

$$+ \lambda \sum_{i=1}^3 (\|T_i\|_F^2 - \log \det(T_i))$$

$$+ \mu \sum_{i=1}^2 \text{Trace}(Z L_i Z^T) + \gamma \sum_c \|\bar{Z}_c - Z_c\|_F^2$$

$$s.t. T_2 T_1 Z \geq 0, T_1 X \geq 0 \text{ and } Z \geq 0$$

Proxy Variables
 $Z_3 = T_2 T_1 Z$
 $Z_2 = T_1 X$

$$\begin{aligned} \min_{T_1, T_2, T_3, Z_2, Z_3} & \|T_3 Z_3 - Z\|_F^2 + \alpha \|T_2 Z_2 - Z_3 - A\|_F^2 \\ & + \beta \|T_1 X - Z_2 - B\|_F^2 + \lambda \sum_{i=1}^3 (\|T_i\|_F^2 - \log \det(T_i)) \\ & + \mu \sum_{i=1}^2 \text{Trace}(Z L_i Z^T) + \gamma \sum_c \|\bar{Z}_c - Z_c\|_F^2 \\ s.t. & Z_3 \geq 0, Z_2 \geq 0 \text{ and } Z \geq 0 \end{aligned}$$

Solution via Alternating Minimization

$$\begin{aligned} \min_{T_3} & \|T_3 Z_3 - Z\|_F^2 + \lambda (\|T_3\|_F^2 - \log \det(T_3)) \\ \min_{T_2} & \alpha \|T_2 Z_2 - Z_3 - A\|_F^2 + \lambda (\|T_2\|_F^2 - \log \det(T_2)) \\ \min_{T_1} & \beta \|T_1 X - Z_2 - B\|_F^2 + \lambda (\|T_1\|_F^2 - \log \det(T_1)) \\ \min_{Z_3} & \|T_3 Z_3 - Z\|_F^2 + \alpha \|T_2 Z_2 - Z_3 - A\|_F^2 \quad s.t. Z_3 \geq 0 \\ \min_{Z_2} & \alpha \|T_2 Z_2 - Z_3 - A\|_F^2 + \beta \|T_1 X - Z_2 - B\|_F^2 \quad s.t. Z_2 \geq 0 \\ \min_{Z_U} & \|T_3 Z_3 - Z_U\|_F^2 + \mu \sum_{i=1}^2 \text{Trace}(Z_U L_i Z_U^T) \quad s.t. Z_U \geq 0 \\ \min_{Z_S} & \|T_3 Z_3 - Z_S\|_F^2 + \mu \sum_{i=1}^2 \text{Trace}(Z_S L_i Z_S^T) \\ & + \gamma \sum_c \|\bar{Z}_c - Z_c\|_F^2 \quad s.t. Z_S \geq 0 \end{aligned}$$

Update Step of Bregman Relaxation Variables

$$A \leftarrow T_2 Z_2 - Z_3 - A \quad B \leftarrow T_1 X - Z_2 - B$$

Results

Dataset	Metric	AttentionNet	GANCap	DCN-T	ECR	Proposed Synthesis	Proposed Analysis
Pavia University	OA	80.09	82.02	88.91	86.24	89.79	88.67
	AA	80.26	82.69	88.48	87.41	90.08	88.11
	Kappa	.78	.80	.85	.84	0.85	0.85
Indian Pines	OA	70.36	73.32	77.38	75.85	81.02	78.02
	AA	70.95	74.07	77.91	76.30	84.32	79.42
	Kappa	.69	.72	.77	.75	0.80	0.78

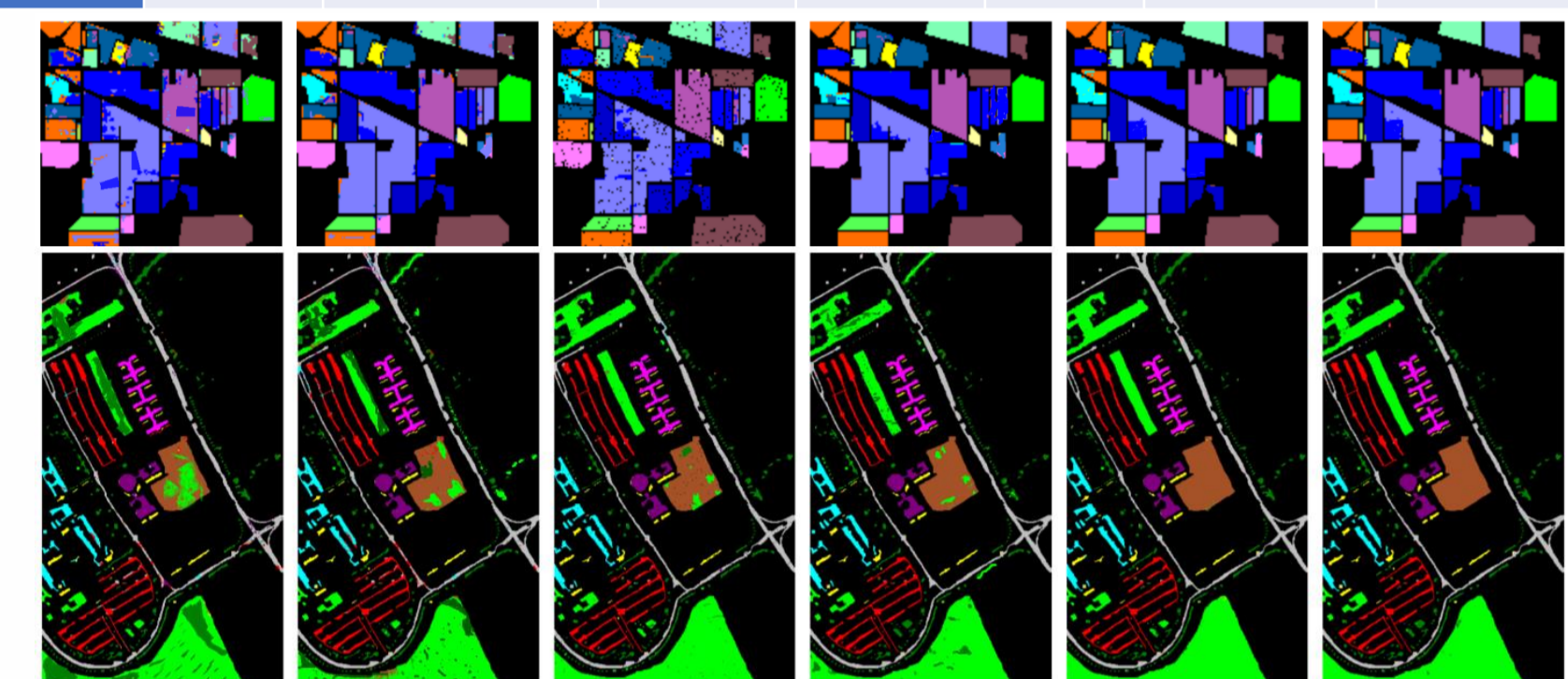


Fig. 1. Pictorial view of classification. Top - Indian Pines. Bottom - Pavia. Left to Right - AttentionNet, GANCap, DCN-T, ECR, Proposed Synthesis, and Proposed Analysis

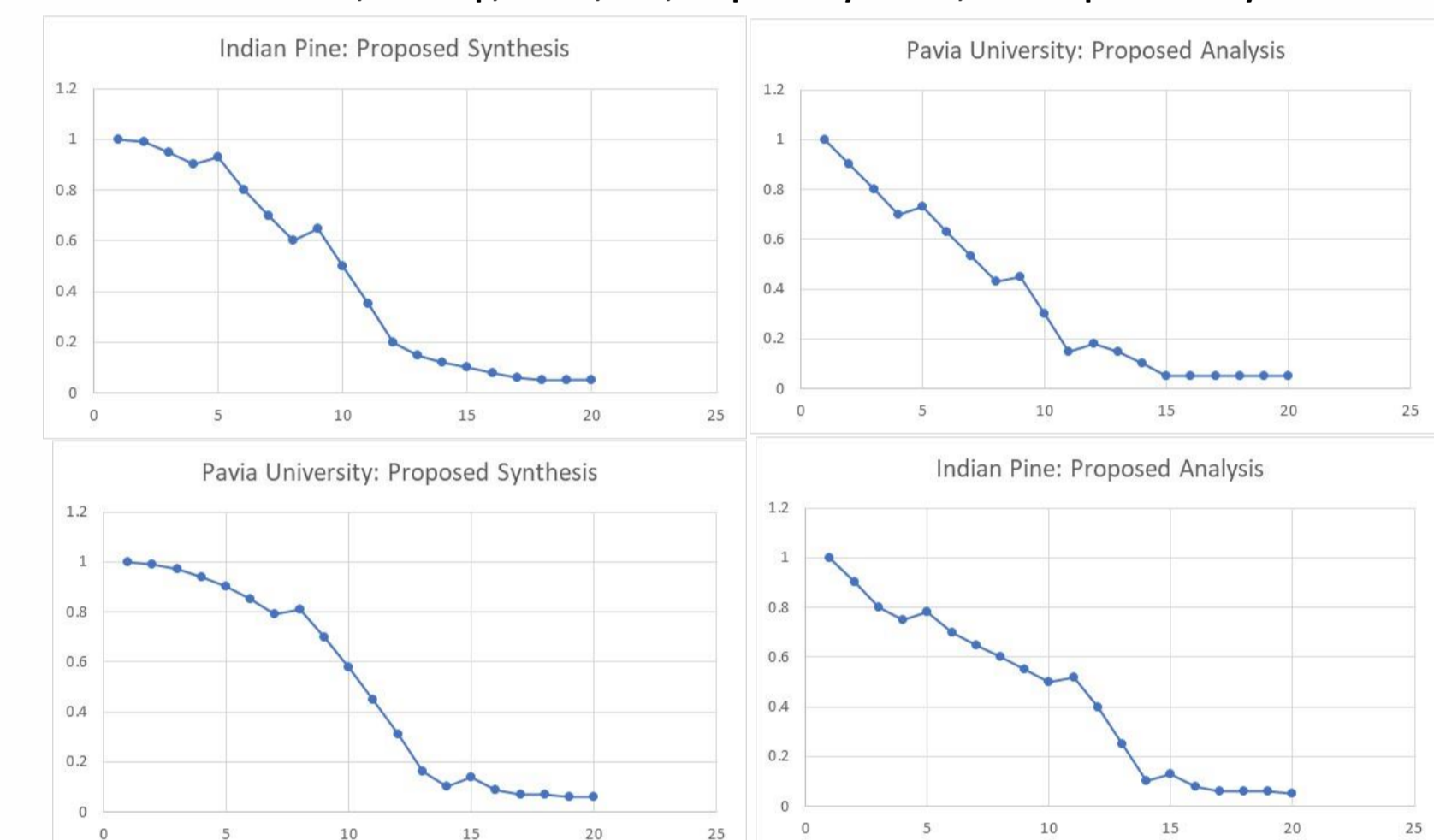


Fig. 2. Empirical Convergence. Y axis: Normalized Objective. X axis: Iteration Number

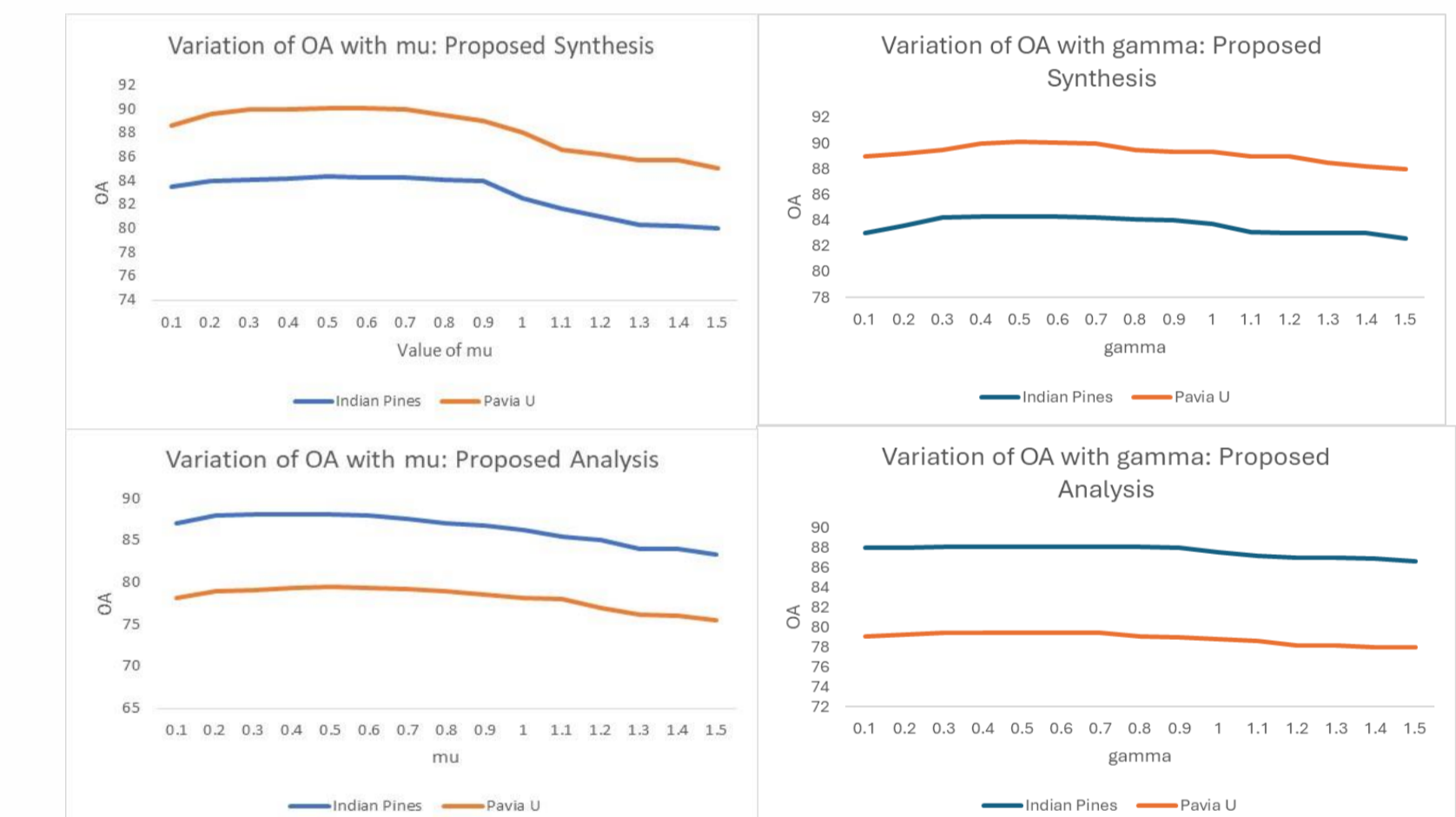


Fig. 3. Variation of Overall Accuracy (OA) with μ (μ) and γ (γ)

Method	Time (in seconds rounded)		Pavia University					Indian Pines				
	Proposed Synthesis	Proposed Analysis	Proposed Synthesis	Proposed Analysis	Proposed Synthesis	Proposed Analysis	Proposed Synthesis	Proposed Analysis	Proposed Synthesis	Proposed Analysis		
AttentionNet	250	103										
GANCap	304	129										
DCN-T	327	112										
ECR	571	206										
Proposed Synthesis	418	177	Layer: 1	87.87	86.29	77.18	76.13	Layer: 2	88.92	88.00	80.51	78.55
			Layer: 3	89.79	88.67	81.02	79.42	Layer: 4	89.06	88.14	80.60	78.95
Proposed Analysis	933	365	Layer: 5	86.15	86.45	78.83	77.68					

Conclusion and References

The goal of this work was to propose a technique that can pragmatically solve hyperspectral image classification problems. It takes into account two unique aspects of hyperspectral image classification - 1. The total number of samples to be labeled is fixed; and 2. The samples are spatially correlated. The first aspect results in a semi-supervised formulation. The second aspect is modeled by graph regularization.

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