Diversity Analysis for Two-Way Multi-Relay Networks with Stochastic Energy Harvesting

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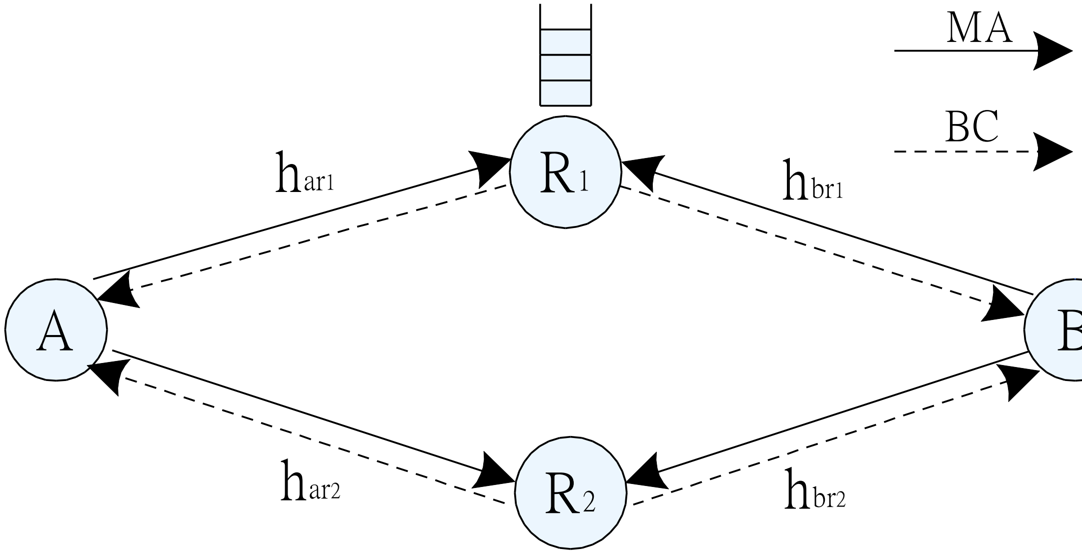
Outline

- EH two-way multi-relay networks with network coding
- Markov decision process with stochastic models
- Optimization of relay transmission policy
- Structure of optimal relay transmission policy
- Performance analysis of pairwise error probability

Two-Way Multi-Relay Networks with Network Coding



$$\gamma_{ar_1} = |h_{ar_1}|^2, \gamma_{br_1} = |h_{br_1}|^2 \quad \gamma_{ar_2} = |h_{ar_2}|^2, \gamma_{br_2} = |h_{br_2}|^2$$

- **Relay cooperation protocol:**

Amplify-and-Forward (AF), Space-Time Network Coding (STNC)

- **Channel assumptions:**
- Quasi-static and Rayleigh flat fading, $\mathcal{CN}(0,1)$
- Channels are reciprocal
- All nodes are half-duplex.

Transmission Protocol with STNC (MA Phase)

	MA Phase	
	Slot 1	Slot 2
A	s_{a1}	s_{a2}
B	s_{b1}	s_{b2}
R1	$\mathbf{y}_{sr_1}$	
R2	$\mathbf{y}_{sr_2}$	

$$\mathbf{y}_{sr_l} = h_{ar_l}\sqrt{P}\mathbf{s}_a + h_{br_l}\sqrt{P}\mathbf{s}_b + \mathbf{n}_{sr_l},$$
$$l \in \{1,2\}, \mathbf{n}_{sr_l} \sim \mathcal{CN}(0, N_0\mathbf{I})$$

$$x_{r_l} = \alpha_l \theta_l^{\mathrm{T}} \mathbf{y}_{sr_l}$$

$$\text{AF factor: } \alpha_l = \sqrt{\frac{P_{r_l}}{P\gamma_{ar_l} + P\gamma_{br_l} + N_0}}$$

Space Time Network Coding:

$$\Theta = \begin{pmatrix} \theta_1, & \theta_2, & \cdots, & \theta_L \end{pmatrix}$$
$$= \frac{1}{\sqrt{L}} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \theta_1 & \theta_2 & \cdots & \theta_L \\ \vdots & \vdots & \ddots & \vdots \\ \theta_1^{L-1} & \theta_2^{L-1} & \cdots & \theta_L^{L-1} \end{pmatrix}$$

$$\theta_l = \exp\left(j\frac{4l-1}{2L}\pi\right) \text{ for } l = 1, 2, \cdots, L$$

It meets full diversity criterion and minimum product criterion.

Transmission Protocol with STNC (BC Phase)

	BC Phase	
	Slot 1	Slot 2
R1	x_{r_1}	
R2		x_{r_2}
A	y_{r_1a}	y_{r_2a}
B	y_{r_1b}	y_{r_2b}

$$\begin{aligned} y_{r_la} &= h_{ar_l}x_{r_l} + n_{r_la} \\ &= h_{ar_l}\alpha_l\theta_l^{\mathrm{T}}(h_{ar_l}\sqrt{P}\mathbf{s}_a + h_{br_l}\sqrt{P}\mathbf{s}_b + \mathbf{n}_{sr_l}) + n_{r_la} \end{aligned}$$

$$\begin{aligned} \tilde{y}_{r_la} &= h_{ar_l}h_{br_l}\alpha_l\sqrt{P}\theta_l^{\mathrm{T}}\mathbf{s}_b + h_{ar_l}\alpha_l\theta_l^{\mathrm{T}}\mathbf{n}_{sr_l} + n_{r_la} \\ &= h_{ar_l}h_{br_l}\alpha_l\sqrt{P}\theta_l^{\mathrm{T}}\mathbf{s}_b + \tilde{n}_{r_la}, \\ &\quad \tilde{n}_{r_la} \sim \mathcal{CN}(0, (\gamma_{ar_l}\alpha_l^2 + 1)N_0) \end{aligned}$$

Instant Pairwise Error Propability (PEP)

Observing $\{\tilde{y}_{r_l a}\}_{l=1}^{2}$, Source A exploits MLD method to jointly decode $\mathbf{s}_b$

$$\hat{\mathbf{s}}_b = \arg \min_{\mathbf{s}_b \in \mathcal{A}_s^2} \sum_{l=1}^{2} \frac{\left\| \tilde{y}_{r_l a} - h_{ar_l} h_{br_l} \alpha_l \sqrt{P} \theta_l^{\mathrm{T}} \mathbf{s}_b \right\|^2}{(\gamma_{ar_l} \alpha_l^2 + 1) N_0}$$

Instant PEP (pairwise error probability) of Source A for one channel realization

$$\begin{aligned}
\Pr\left(\mathbf{s}_b \to \tilde{\mathbf{s}}_b \middle| \{\gamma_{ar_l}\}_{l=1}^{2}, \{\gamma_{br_l}\}_{l=1}^{2}\right) &= Q\left(\sqrt{2W_{R_1} + 2W_{R_2}}\right) \\
&= \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{W_{R_1} + W_{R_2}}{\sin^2 \theta}\right) d\theta \\
&< \exp\left(-W_{R_1}\right) \times \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{W_{R_2}}{\sin^2 \theta}\right) d\theta \\
&= P_{e,R_1} \times P_{e,R_2}
\end{aligned}$$

$$W_{R_l} = \frac{\gamma_{ar_l} \gamma_{br_l} \alpha_l^2 P \left|\theta_l^{\mathrm{T}} \Delta \mathbf{s}_b\right|^2}{4(\gamma_{ar_l} \alpha_l^2 + 1) N_0} = \frac{\gamma_{ar_l} \gamma_{br_l} P_{R_l} P \beta_l}{4\left[\left(P + P_{R_l}\right) \gamma_{ar_l} + P \gamma_{br_l} + N_0\right] N_0}$$

$$\Delta \mathbf{s}_b = \mathbf{s}_b - \tilde{\mathbf{s}}_b \neq 0, \quad \beta_l = \left|\theta_l^{\mathrm{T}} \Delta \mathbf{s}_b\right|^2$$

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Markov Decision Process with Stochastic Models

- State space $\mathcal{S} = \mathcal{S}_E \times \mathcal{S}_{AR} \times \mathcal{S}_{BR} \times \mathcal{S}_B$

  solar power state subspace: $\mathcal{S}_E = \{0, 1, \cdots, N_e - 1\}$

  channel state subspace: $\mathcal{S}_{AR} = \{0, 1, \cdots, N_c - 1\}$ $\mathcal{S}_{BR} = \{0, 1, \cdots, N_c - 1\}$

  battery state subspace: $\mathcal{S}_B = \{0, 1, \cdots, N_b - 1\}$

- Relay action space $\mathcal{W} = \{0, 1, \cdots, N_p - 1\} \left(N_p \le N_b\right)$

- Reward function

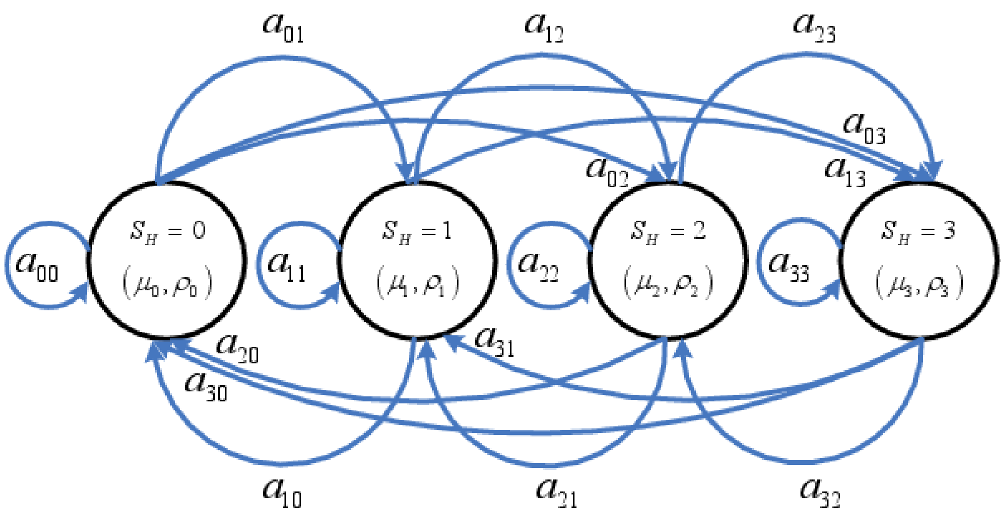
  Conditional PEP, i.e., the PEP conditioned on a fixed system state and relay action

Stochastic Solar Power Model

N_e-state Gaussian mixture hidden Markov model

solar power per unit area: $P_H \sim \mathcal{N}(\mu_e, \rho_e), \quad e \in \mathcal{S}_E = \{0, 1, \cdots, N_e - 1\}$

solar state transition probability: $P(S_E = j \mid S_E = i) = a_{ij}$



Ref: M.-L. Ku, Y. Chen, and K. J. R. Liu, "Data-Driven Stochastic Transmission Policies for Energy Harvesting Sensor Communications," *IEEE J. Select. Areas Commun.*, vol. 33, no. 8, pp. 1505-1520, Aug. 2015.

Harvested Energy Storage

- **Harvesting-store-and-use (HSU) protocol**
- **Quantization model**

basic transmission power: P_U

one basic energy quantum: $E_U = P_U \cdot \frac{T}{2}$

harvested energy during one policy period *T:* $E_H = P_H T s \eta$.

EH probability in terms of the number of harvested energy quanta:

$$P\left(Q = q \mid S_E = e\right) \quad for \;\; q \in \{0, 1, \cdots, \infty\}$$

Battery State

- Available energy quanta in the relay battery:

$$b \cdot E_U, \; b \in \mathcal{S}_B = \{0, 1, \cdots, N_b - 1\}$$

- Battery transition model:

$$b' = b - w + q, \; w \in \{0, 1, \cdots, \min(b, N_p - 1)\}$$

- Battery state transition probability under the solar state and relay action

$$P_w(S_B = b' \mid S_B = b, S_E = e) = \begin{cases} P(Q = b' - b + w \mid S_E = e), \; b' = (b - w), \cdots, N_b - 2 \\ 1 - \sum_{q=0}^{N_b - 2 - b + w} P(Q = q \mid S_E = e), \; b' = N_b - 1 \end{cases}$$

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Channel State

- N_c-state Markov chain

$$\Gamma = \left\{0 = \Gamma_0, \Gamma_1, \cdots, \Gamma_{N_c} = \infty\right\} \qquad S_{AR} = i \Leftrightarrow \gamma_{AR} \in \left[\Gamma_i, \Gamma_{i+1}\right)$$

- Channel state stationary probability

$$P\left(H = i\right) = \int_{\Gamma_i}^{\Gamma_{i+1}} \frac{1}{\lambda} \exp\left(-\frac{\gamma}{\lambda}\right) d\gamma = \exp\left(-\frac{\Gamma_i}{\lambda}\right) - \exp\left(-\frac{\Gamma_{i+1}}{\lambda}\right).$$

- Channel state transition probability $h(\gamma) = f_D \sqrt{2\pi\gamma/\lambda} \exp(-\gamma/\lambda)$

$$P\left(H = j \mid H = i\right) = \begin{cases} \dfrac{h\left(\Gamma_{i+1}\right)}{P\left(H = i\right)}, \ j = i+1, i = 0, 1, \cdots, N_c - 2 \\ \dfrac{h\left(\Gamma_i\right)}{P\left(H = i\right)}, \ j = i-1, i = 1, 2, \cdots, N_c - 1 \\ 1 - \dfrac{h\left(\Gamma_i\right)}{P\left(H = i\right)} - \dfrac{h\left(\Gamma_{i+1}\right)}{P\left(H = i\right)}, \ j = i, i = 1, \cdots, N_c - 2 \end{cases}$$

Ref: H. S. Wang and N. Moayeri, "Finite-State Markov Channel-A Useful Model for Radio Communication Channels," *IEEE Trans. Wireless Commun.*, vol. 44, no. 1, pp. 163-171, Feb. 1995.

System States

- System state transition probability

$$S = \left(Q_e, H_{ar}, H_{br}, Q_b\right) \in \mathcal{S}$$

$$P_w\left\{s = \left(e', f', g', b'\right) \mid s = \left(e, f, g, b\right)\right\}$$
$$= P\left(S_E = e' \mid S_E = e\right) \cdot P\left(S_{AR} = f' \mid S_{AR} = f\right) \cdot P\left(S_{BR} = g' \mid S_{BR} = g\right)$$
$$\cdot P_w\left(S_B = b' \mid S_B = b, S_E = e\right)$$

Reward Function

Conditional PEP:

the PEP conditioned on a fixed system state and relay action

$$R_w\left(S=(e,b,f,g)\right) \triangleq P_{e,R_1}(w,f,g)$$
$$= \frac{\int_{\Gamma_g}^{\Gamma_{g+1}} \int_{\Gamma_f}^{\Gamma_{f+1}} \exp\left(-\gamma_1\right) \cdot \exp\left(-\gamma_2\right) \cdot \exp\left(-W_{R_1}\right) d\gamma_1 d\gamma_2}{P\left(S_{AR}=f\right) \cdot P\left(S_{BR}=g\right)}$$

$$\text{Let} P = P_{R_2} = P_U \, , P_{R_1} = wP_U \, , \eta = \frac{P_U}{N_0},$$

$$W_{R_1} = \frac{\gamma_1 \gamma_2 w \eta^2 \beta_1}{4\left[(w+1)\,\eta\gamma_1 + \eta\gamma_2 + 1\right]}$$

Asymptotic Approximations of Reward Function

When $w = 0,\ P_{e,R_1}(w=0,f,g) = 1.$

When $w \geq 1$ and $\eta = \dfrac{P_U}{N_0} \gg 1, W_{R_1} \approx \dfrac{w\eta\beta_1\gamma_1\gamma_2}{4\left(\gamma_1+\gamma_2\right)}.$

Considering $\dfrac{1}{2}\min(x,y) \leq \dfrac{xy}{x+y} \leq \min(x,y)$

$$P_{e,R_1}^{(up)}(w \geq 1,f,g) \approx \begin{cases} \frac{8\eta^{-1}}{w\beta_1\left(1-e^{-\Gamma_1}\right)}, & \min(f,g)=0; \\ 0, & \min(f,g) \geq 1. \end{cases}$$

$$P_{e,R_1}^{(lo)}(w \geq 1,f,g) \approx \begin{cases} \frac{4\eta^{-1}}{w\beta_1\left(1-e^{-\Gamma_1}\right)}, & \min(f,g)=0; \\ 0, & \min(f,g) \geq 1. \end{cases}$$

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Optimization of Relay Transmission Policy

Define the policy $\pi(s): \mathcal{S} \to \mathcal{A}$ as the relay action in the state s

the expected discount long-term reward

$$V_{\pi}(s_0) = E_{\pi}\left[\sum_{k=0}^{\infty} \lambda^k R_{\pi(s_k)}(s_k)\right], \quad s_k \in \mathcal{S}, \quad \pi(s_k) \in \mathcal{A}.$$

the optimal policy can be found through the Bellman equation

$$V_{\pi^*}(s) = \min_{w \in W}\left(R_w(s) + \lambda \sum_{s' \in S} P_w(s' \mid s) V_{\pi^*}(s')\right), \quad s \in \mathcal{S}.$$

the well-known value iteration approach can be applied to find the optimal policy

$$V_w^{i+1}(s) = R_w(s) + \lambda \sum_{s' \in S} P_w(s' \mid s) V^{(i)}(s'), \quad s \in \mathcal{S}, \quad w \in \mathcal{W};$$

$$V^{i+1}(s) = \min_{w \in W}\left(V_w^{i+1}(s)\right), \quad s \in \mathcal{S}.$$

$$\left|V^{i+1}(s) - V^{i}(s)\right| \le \varepsilon$$

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Non-Conservative Property of Optimal Relay Transmission Policy

Proposition 1: For any fixed system state $s = (e, f, g, b \geq 1) \in \mathcal{S}$ with the non-empty battery, in high SNR regimes, i.e., $\frac{P_U}{N_0} \gg 1$, the optimal relay power action w^* must be larger than or equal to one.

Long term value of State s in the *i-th* iteration:

$$V_w^{(i+1)}(s) = R_w(f,g) + \lambda \cdot \mathbb{E}_{e,f,g,b}\left[V^{(i)}\left(e', f', g', \min\left(b - w + q, N_b - 1\right)\right)\right]$$

The difference between the long-tem values of the two relay action:

$$\begin{aligned} & V_{w\geq 1}^{(i+1)}(e,f,g,b) - V_{w=0}^{(i+1)}(e,f,g,b) \\ & = R_{w\geq 1}(f,g) - R_{w=0}(f,g) \\ & \quad + \lambda \cdot \mathbb{E}_{e,f,g,b}\left[V^{(i)}\left(e', f', g', \min\left(b-w+q, N_b-1\right)\right) - V^{(i)}\left(e', f', g', \min\left(b+q, N_b-1\right)\right)\right]. \end{aligned}$$

We have： $V_{w\geq 1}^{(i+1)}(e,f,g,b) - V_{w=0}^{(i+1)}(e,f,g,b) < 0$

Thus, the optimal action： $w^* \geq 1, \text{ if } b > 0$

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Expected Reward Analysis

Expected reward w.r.t. the optimal policy: π^*

$$\bar{R} = \sum\nolimits_{s\in\mathcal{S}} p_{\pi^*}(s=(e,b,f,g)) \times R_{w^*=\pi^*(s)}(s=(e,b,f,g))$$
$$= \sum_{s\in\mathcal{S},b=0} p_{\pi^*}(s) \times R_{w^*=0}(s) + \sum_{s\in\mathcal{S},b\geq 1} p_{\pi^*}(s) \times R_{w^*\geq 1}(s)$$
$$= P_{\pi^*}(b=0)\cdot P_{e,R_2} + \sum_{s\in\mathcal{S},b\geq 1} p_{\pi^*}(s)\cdot P_{e,R_2}\cdot P_{e,R_1}(w^*\geq 1, f, g)$$

The asymptotic approximations of PEP w.r.t. the optimal policy:

$$\bar{R}^{(\mathrm{up})} \approx P_{\pi^*}(b=0)\cdot P_{e,R_2} + \sum_{s\in\mathcal{S}_0} \frac{8\cdot p_{\pi^*}(s)\cdot P_{e,R_2}}{w^*\beta_1\left(1-e^{-\Gamma_1}\right)\eta}$$

$$\bar{R}^{(\mathrm{lo})} \approx P_{\pi^*}(b=0)\cdot P_{e,R_2} + \sum_{s\in\mathcal{S}_0} \frac{4\cdot p_{\pi^*}(s)\cdot P_{e,R_2}}{w^*\beta_1\left(1-e^{-\Gamma_1}\right)\eta}$$

where $\mathcal{S}_0 = \{s=(e,f,g,b), \min(f,g)=0, b\geq 1, s\in\mathcal{S}\}$

Diversity Order

$$P_{PEP,R_2} = \frac{1}{\pi}\int_0^{+\infty}\int_0^{+\infty}\int_0^{\pi/2} \exp(-\gamma_1)\cdot\exp(-\gamma_2)\cdot\exp\left(-\frac{W_{R_2}}{\sin^2\theta}\right) d\theta d\gamma_1 d\gamma_2$$

$$\text{where } \; W_{R_2} = \frac{\gamma_1\gamma_2\eta^2\beta_2}{4(2\eta\gamma_1+\eta\gamma_2+1)}$$

$$\text{Since } \; P_{PEP,R_2} \propto \frac{\eta^{-1}}{\alpha_2}, \; (\eta = P_U/N_0 \geq 1)$$

$$\bar{R} \propto \frac{P_{\pi^*}(b=0)}{\beta_2}\eta^{-1} + \frac{\sum_{s\in\mathcal{S}_0} p_{\pi^*}(s)}{w^*\left(1-e^{-\Gamma_1}\right)\beta_1\beta_2}\eta^{-2}$$

Theorem: If $P_{\pi^*}(b=0) = 0, \; d = 2.$

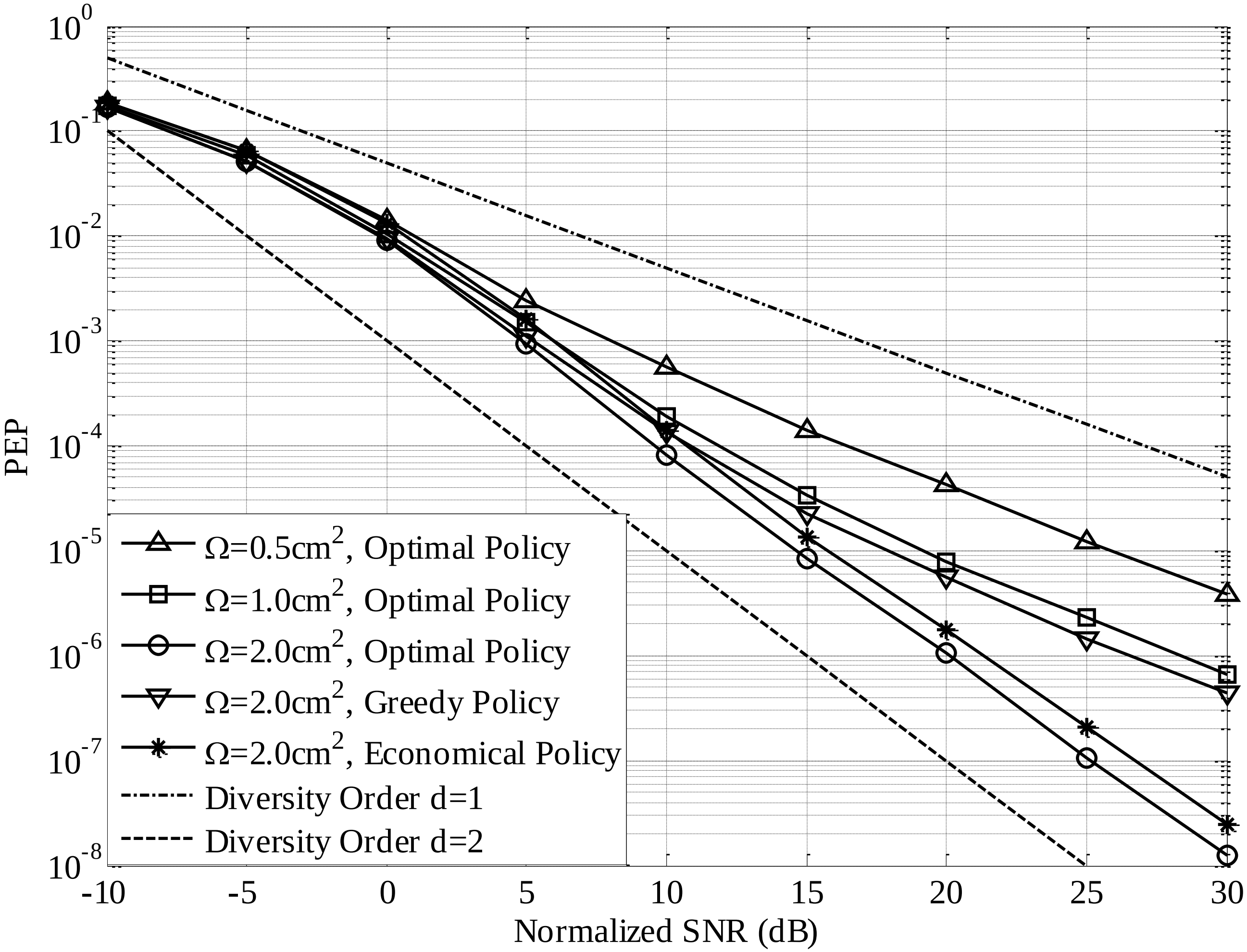
If $P_{\pi^*}(b=0) > 0, \; d = 1.$

Simulation Results

SIMULATION PARAMETERS

Modulation type	QPSK
Basic transmission power (P_U)	10mW
Policy management period (T)	300s
Energy conversion efficiency (η)	20%
Channel simulation model	Jakes' model
Normalized Doppler frequency (f_D)	0.05
Channel quantization thresholds (Γ)	$\{0, 0.3, 0.6, 1.0, 2.0, 3.0, \infty\}$
Discount factor (λ)	0.99

Simulation Results of Optimal PEP



$$P_A = P_B = P_{R_1} = P_u$$

$$\Omega = 0.5cm^2:$$

$$P_{\pi^*}(b=0) \approx 10^{-1}$$

$$\Omega = 1cm^2:$$

$$P_{\pi^*}(b=0) \approx 10^{-2}$$

$$\Omega = 2cm^2:$$

$$P_{\pi^*}(b=0) \approx 10^{-10}$$

Thank you!