Low-Complexity MIMO Detector with 1024-QAM

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Outline:

Introduction
- Motivation
- System Model
- Popular Detectors

Proposed Work
- Low-Complexity LORD
- Optimizing Search Region
- Optimizing LLR Saturation

Results
- Complexity Study
- Simulation Scenario
- Simulation Results

Summary & future work
Motivation

• Quadrature Amplitude Modulation (QAM) is rising
  – 1024QAM and beyond
  – Mainly in microwave backhaul but also in WiFi

• Broadcom announced new 5G WiFi chips
  – NitroQAM™ (1024-QAM) technology
  – 8x8 MU-MIMO

• Detection with 1024QAM
  – Near-optimal detectors are complex
  – Their low complexity versions degrade performance

• Low complexity LORD detector has limitations
  – Optimize search region
  – Optimize LLR saturation
MIMO system combined with OFDM

Received signal at resource element is given by:  \[ \mathbf{y} = \mathbf{Hx} + \mathbf{n} \]

\[ \mathbf{H} = N_r \times N_t \quad \text{channel matrix} \]

\[ \mathbf{x} \quad \text{transmitted QAM symbols} \]

\[ \mathbf{n} \quad \text{complex additive white Gaussian noise with zero mean and variance} \quad \sigma^2 = \frac{N_t}{\text{SNR}} \]

We consider the case \( N_r = N_t = 2 \)

\[ \mathbf{y} = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{n} \]

\( \mathbf{h}_1 \): channel coefficients of user of interest
\( \mathbf{h}_2 \): channel coefficients of interferer

\[ E[x_1 \cdot x_1^*] = E[x_1 \cdot x_1^*] = 1 \]

\( x_1 \) and \( x_2 \) are drawn from a 1024QAM constellation \( \mathcal{M} \)
Maximum Likelihood (ML) Detection

A hard-output ML detector solves:

$$\min_{x \in S} ||y - Hx||^2$$

where $S$ is the lattice of symbol vectors ($|S| = |\mathcal{M}|^2$)

Let $b_x = [x_b]_{b=1}^K$ be the bit vector of $x$, $x_b \in \{0,1\}$ and $K = \log_2(|S|)$

A soft-output ML detector calculates the log-likelihood ratio (LLR) of bit $b$ as:

$$\lambda_b = \min_{x \in S_{b,1}} \frac{||y - Hx||^2}{\sigma^2} - \min_{x \in S_{b,0}} \frac{||y - Hx||^2}{\sigma^2}$$

$S_{b,1}$ corresponds to points in $S$ having in the bit position $b$ a value of 1

$S_{b,0}$ corresponds to points in $S$ having in the bit position $b$ a value of 0
The MMSE detector solves for an equalized output $\hat{y}$:

$$\hat{y} = (H^*H + (1/\text{SNR})I_2)^{-1}H^*y$$

And the LLRs can be computed as:

$$\lambda^t_b = \min_{x(t) \in \mathcal{M}_{b,t,1}} \frac{|\hat{y}(t) - x(t)|^2}{\sigma^2_{\text{MMSE}}} - \min_{x(t) \in \mathcal{M}_{b,t,0}} \frac{|\hat{y}(t) - x(t)|^2}{\sigma^2_{\text{MMSE}}}$$

where $t \in \{1,2\}$ is the symbol index

$\sigma^2_{\text{MMSE}} = \sigma^2W(t,t)$ is a scaled variance, where $W = (H^*H + (1/\text{SNR})I_2)^{-1}$

$\mathcal{M}_{b,t,1}$ corresponds to points in $\mathcal{M}$ having in the bit position $\hat{b}$ of symbol $t$ a 1

$\mathcal{M}_{b,t,0}$ corresponds to points in $\mathcal{M}$ having in the bit position $\hat{b}$ of symbol $t$ a 0
Layered Orthogonal Lattice Detector (LORD)

QR decomposition in the preprocessing step:

\[
\tilde{y} = Q^* y = Rx + Q^* n = Rx + \tilde{n}
\]

\[
\begin{bmatrix}
\tilde{y}_1 \\
\tilde{y}_2
\end{bmatrix}
= \begin{bmatrix}
r_{1,1} & r_{1,2} \\
0 & r_{2,2}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ \begin{bmatrix}
\tilde{n}_1 \\
\tilde{n}_2
\end{bmatrix}
\]

Exhaustively search layer 2. For each possibility \( \bar{x}_2, \bar{x}_1 = \text{slice} \left( \left( \tilde{y}_1 - r_{1,2} \bar{x}_2 \right) / r_{1,1} \right) \)

The searched lattice of vectors \( \bar{x} = [\bar{x}_1, \bar{x}_2] \) is \( \hat{S} \) (\( |\hat{S}| = |\mathcal{M}| \))

Searching layer 2, LLRs of \( x_2 \) can be computed

\[
\lambda_{b}^2 = \min_{x \in \hat{S}_{b,2,1}} \frac{||y - Hx||^2}{\sigma^2} - \min_{x \in \hat{S}_{b,2,0}} \frac{||y - Hx||^2}{\sigma^2}
\]

\( \hat{S}_{b,2,1} \) corresponds to points in \( \hat{S} \) having in the bit position \( \hat{b} \) of symbol 2 a value 1

\( \hat{S}_{b,2,0} \) corresponds to points in \( \hat{S} \) having in the bit position \( \hat{b} \) of symbol 2 a value 0

To compute LLRs of \( x_1 \) the layers are swapped and the same operation is repeated

Output identical to ML detector with 2x2 MIMO
Turbo LORD (T-LORD)

T-LORD is a generalization of LORD

It builds on the maximum-a-posteriori (MAP) detector instead of the ML detector

Used with iterative detection and decoding ($T$ iterations)

MAP detector accepts a-priori LLRs $\xi$ from the decoder

The modified distance metric is:

$$
\varphi(x) = -\frac{||y - Hx||^2}{\sigma^2} + \sum_{k=1}^K b_x(k)\xi(k)
$$

The a-posteriori LLRs can then be calculated as:

$$
\lambda^t_b = \max_{x \in S_{b,t,1}} \varphi(x) - \max_{x \in S_{b,t,0}} \varphi(x)
$$
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Summary & future work
Searching $|\mathcal{M}| = 1024$ lattice points is still computationally demanding

LC-LORD only explores a subset of the constellation at the root layer
- A reduced QAM $\theta$
- A square subset centered on equalized output $\tilde{y}_2/r_{2,2}$

LLRs can not be computed when all points in $\theta$ have the same bit value at a specific bit
- An LLR saturation mechanism is required
- Especially for high order bits when Gray mapping is employed

LC-LORD need not be applied on all carriers
- Worst carriers can be isolated an treated with regular LORD
- This depends on the implementation constraints
- Criteria for sorting worst carriers is:

$$\min_{l=1,2} r^l(2,2)$$

where $l$ denotes the antenna index at the root layer
Optimizing Search Region Location

LC-LORD fails when the actual transmitted symbol lies outside $\theta$

This is worse with correlated channels
- $\mathbf{H}$ ill-conditioned
- $r_{2,2}$ tends to zero

One solution uses the hard-output of MMSE detection as a center of search on both layers

This is called MMSE-LC-LORD

Note that operations on both layers are now dependent
- Can not be fully parallelized

Constellation Schematic - Black Circles Indicate that Third MSB is 1
This proposed solution is based on:
- Layer ordering
- Zero-forcing with decision feedback (ZFDF)

Find equalized output on layer 2

\[ \bar{x}^1_2 = \text{slice} \left( \frac{\tilde{y}_2}{r_{2,2}} \right) \]

Get its corresponding projection on layer 1

\[ \bar{x}^1_1 = \text{slice} \left( \frac{(\tilde{y}_1 - r_{1,2}\bar{x}^1_2)}{r_{1,1}} \right) \]

We obtain \( \bar{\mathbf{x}}^1 = [\bar{x}^1_1, \bar{x}^1_2] \)

Permute layers and apply same procedure to obtain \( \bar{\mathbf{x}}^2 = [\bar{x}^2_1, \bar{x}^2_2] \)

The centers of reduced search on both layers are the components of \( \mathbf{x}_{\text{center}} \)

\[ \mathbf{x}_{\text{center}} = \min_{\mathbf{x} \in \{\bar{\mathbf{x}}^1, \bar{\mathbf{x}}^2\}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \]
Proposed Iterative LC-LORD (Iter-LC-LORD)

This solution adds an iterative behavior

The Center Generator is a hard-output LC-LORD - single layer operation - hard output constitutes updated centers of search

Updated search centers are closer to ML hard output

Might get stuck in local minima

Algorithm halts after a maximum of $J$ iterations

With T-LORD center updates can take place on every detection/decoding iteration
Proposed Region-Thresholding LC-LORD (RegTh-LC-LORD)

With LC-LORD one of the two terms below can go missing

\[ \lambda^t_b = \max_{x \in S_{b,t,1}} \varphi(x) - \max_{x \in S_{b,t,0}} \varphi(x) \]

LLR saturation in Literature:
- Saturate LLR to a threshold value
- Substitute missing term by maximum Euclidean norm within \( \theta \)

Proposed approach (RegTh-LC-LORD):
- Locate the closest point to the center of \( \theta \) having opposite bit value (in green)
- Project on other layer + slice
- Substitute missing term by the distance from resultant vector to received vector

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Complexity Study

Preprocessing complexity
- QR decomposition (can be avoided in 2x2 MIMO)
- Handling search region boundaries

Search routine complexity
- Summarized in table in terms of Euclidean distance computations (visited nodes)
- Table shows the worst case
- When search center is close to boundaries of $\mathcal{M}$, $\theta$ gets clipped
- In Iter-LC-LORD $\theta$s of subsequent iterations partially overlap and computations can be saved

<table>
<thead>
<tr>
<th>Approach</th>
<th>Description</th>
<th>Nodes Visited</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML</td>
<td>Full Complexity LORD</td>
<td>$2 \times</td>
</tr>
<tr>
<td>LC-LORD</td>
<td>Low Complexity LORD</td>
<td>$2 \times</td>
</tr>
<tr>
<td>LO-LC-LORD</td>
<td>Layer Ordered LC-LORD + Region Thresholding</td>
<td>$2 \times</td>
</tr>
<tr>
<td>Iter-LC-LORD</td>
<td>Iterative LC-LORD + Region Thresholding</td>
<td>$(j + 2) \times</td>
</tr>
<tr>
<td>MMSE-LC-LORD</td>
<td>MMSE-based LC-LORD + Region Thresholding</td>
<td>$2 \times</td>
</tr>
<tr>
<td>RegTh-LC-LORD</td>
<td>LC-LORD + Region Thresholding</td>
<td>$2 \times</td>
</tr>
<tr>
<td>MMSE</td>
<td>Soft-output MMSE</td>
<td>$</td>
</tr>
</tbody>
</table>
Simulation Scenario

- A 2x2 MIMO simulation chain was implemented
  - System model in introduction
  - All studied detectors were implemented
  - Iterative detection/decoding

- Turbo coding/decoding
  - Code rate 1/2
  - 8 iterations

- Two channel types
  - Uncorrelated (rich scattering)
  - Highly correlated ($\alpha = 0.9$)

- Performance measure
  - Frame Error Rate (FER)

- Parameters
  - $|\theta| = 225$
  - $J = 8$ (1.8 on average)
  - $T = 4$
FER – Uncorrelated

Detectors Performance with Uncorrelated Channels and 15% Full Complexity Carriers, for $T = 1$ (solid) and $T = 4$ (dotted)
FER – Correlated

Detectors Performance with Correlated Channels and 15% Full Complexity Carriers, for $T = 1$ (solid) and $T = 4$ (dotted)
FER – Correlated

Detectors Performance with Correlated Channels and 30% Full Complexity Carriers, for $T = 1$ (solid) and $T = 4$ (dotted)
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Summary and Future Work

- 2×2 MIMO systems that use 1024-QAM were studied.
- Building on the LORD detector, several algorithms were proposed.
- Optimizing the location of a reduced region of search.
- Optimizing LLR saturation.
- The optimizations resulted in an enhanced performance, at a reduced complexity.

- The proposed approaches are to be studied with higher order MIMO, where LORD loses optimality.
Thanks for listening

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